(Non)-geometry and non-geometric fluxes

David ANDRIOT

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(Partial) reviews: arXiv:0708.3984, 1106.4015, 1303.0251

arXiv:1306.4381, 1402.5972 by D. A. and André Betz

Quantum Gravity in Paris,
LPT Orsay and IHES, 18/03/2014, France

Introduction

A standard scenario to relate to string theory to 4D physics

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	theory
world-sheet (2D)	Superstring (CFT)

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- 10D β-supergravity: restores standard geometric description
  $\hookrightarrow$ allows compactification $\Rightarrow$ (new?) phenomenology

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Four-dimensional non-geometric fluxes

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$\Leftrightarrow$ some "structure constants" in gauging algebra

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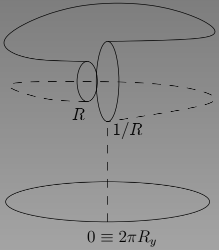

G. Moutsopoulos PhD 2008

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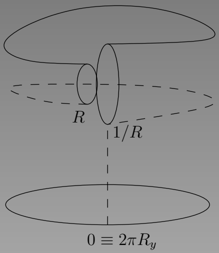

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No compactification on such spaces...

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$$T^3 + H_{123} \overset{T_1}{\longleftrightarrow} \text{twisted torus } ({f^1}_{23}) \overset{T_2}{\longleftrightarrow} \text{non – geometric config.}$$

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If T-duality // non-isometry: base direction 3
non-geometric, not even locally; characteristic of R-flux

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Local picture:
$$\begin{array}{c} T^2 \\ | \\ S^1 \end{array} \longrightarrow \begin{array}{c} T^2 \times T^{2\prime} \\ | \\ S^1 \end{array} \qquad T^2, T^{2\prime}: \text{ T-duals}$$

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$$\text{Local picture:} \quad \begin{array}{c} T^2 \\ | \\ S^1 \end{array} \quad \longrightarrow \quad \begin{array}{c} T^2 \times T^{2\prime} \\ | \\ S^1 \end{array} \qquad T^2, T^{2\prime} : \text{ T-duals}$$

non-geometry from projection

Doubled geometry and Double Field Theory (DFT)

Idea: fields glue with a stringy symmetry
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Double also the base: doubled geometry

hep-th/0512005 by A. Dabholkar and C. Hull, ... by C. Hull and R. A. Reid-Edwards

$2d$ space with $X^M = (\tilde{x}_m, x^m)$, ∂_M, $\tilde{x}_m$: dual or winding coord.

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Difficult to construct on concrete examples beyond torus

arXiv:1106.6291 by M. B. Schulz

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Recent work proposes gen. diffeos ${\mathcal{F}_M}^N$, can $\rightarrow$ O(d,d)

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$$A'_M(X') = {\mathcal{F}_M}^N A_N(X)$$

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Generalized Complex Geometry (GCG)

World-sheet perspective

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- Asymmetric orbifolds: asymmetric (L/R) orbifold action on $\mathcal{X}^m$; no target space description; "R-flux situations"?

hep-th/0210209, hep-th/0512005 by A. Dabholkar and C. Hull

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arXiv:1010.1263, 1106.0316, 1112.4611 by R. Blumenhagen, et al.
arXiv:1207.0926, 1312.1621, 1402.7306 by D. Mylonas, P. Schupp, R. Szabo
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  Star-products, 3-products (check commutativity, associativity); doubled formalisms

More on closed string non-commutativity

arXiv:1211.6437 by D. A., M. Larfors, D. Lüst, P. Patalong

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Use toroidal example:

$$T^3 + H_{123} \overset{T_1}{\longleftrightarrow} \text{twisted torus } ({f^1}_{23}) \overset{T_2}{\longleftrightarrow} \text{non} - \text{geometric config.}$$

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$[Y, Y] = 0$ $\qquad$ $[Z, Z] \neq 0$

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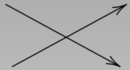

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non-geometry/non-commutativity only occurs upon projection

β-supergravity: back to 10D geometry, Q, R

Lagrangian $\tilde{\mathcal{L}}_\beta(f, Q, R)$

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β-supergravity: back to 10D geometry, Q, R

Lagrangian $\tilde{\mathcal{L}}_\beta(f, Q, R)$

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10D theory with non-geometric fluxes, uplift 4D ✓

More structure

More structure

- For standard ∇_m and Levi-Civita connection Γ^m_{np}

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Structures appear clearly using Generalized (Complex) Geometry formalism; also Lie algebroid structure.

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Field redefinition always possible off-shell/locally

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$$g = f_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f_0} \end{pmatrix} , \ b = f_0 \begin{pmatrix} 0 & -Kz^3 & 0 \\ Kz^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} , \ f_0(z^3) = \frac{1}{1+(Kz^3)^2}$$

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Geometry restored : T^3, Q-flux ✓.

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True for a class of non-geometric/geometric backgrounds: glue with "β-transforms" $\in O(n,n)$ and diffeos.

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Global aspects: non-geom.: g geom.: $\tilde{g}$

Toroidal example :

$$g = f_0 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{f_0} \end{pmatrix} ,\ b = f_0 \begin{pmatrix} 0 & -Kz^3 & 0 \\ Kz^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,\ f_0(z^3) = \frac{1}{1 + (Kz^3)^2}$$

$$\hookrightarrow \tilde{g} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} ,\ \beta = \begin{pmatrix} 0 & Kz^3 & 0 \\ -Kz^3 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} ,\ {Q_3}^{12} = K$$

Geometry restored : T^3, Q-flux ✓.

Similar for Q-brane ⇒ Bianchi identity and Poisson equation.
True for a class of non-geometric/geometric backgrounds:
glue with "β-transforms" $\in O(n,n)$ and diffeos.

Alternative description of 10D non-geometric backgrounds;
compactification possible.

Conclusion

Different perspectives on non-geometry:
- 4D non-geometric fluxes in gauged supergravity
- 10D non-geometric backgrounds (stringy symmetry); toroidal example, Q-brane
- T-fold, doubled geometry, and DFT
- Generalized Complex Geometry and Gen. tangent bundle
- World-sheet: asymmetric orbifolds, non-commutativity and non-associativity

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β-supergravity: 10D theory with non-geometric fluxes Q, R
non-geometric bckgd of standard supergravity $\rightarrow$ geometric
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More about it, Generalized Geometry, NS-branes: talk 27/03

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Study various geometric structures in different perspectives.

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Study various geometric structures in different perspectives.

Extension beyond the NSNS sector: RR non-geometric fluxes? exotic D-branes? Exceptional geometry/field theory could help
SUSY with non-geometric fluxes, SU(3)$\times$ SU(3) structures
$\hookrightarrow$ Get new pheno. interesting backgrounds, de Sitter vacua.