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High-energy gravitational scattering and black-hole quantum hair (revised title)

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Revised outline

- The string-black hole correspondence & stringholes
- Stringhole production in high-energy gravitational scattering
- Scattering off a stringhole and quantum hair
- ~~Stringholes and a model for the big bounce~~

The String-BH correspondence

Entropy of free string states
(FV, BM, 1969)

# of physical string states @ vanishing string coupling ($\alpha' M^2 = N$, C = central charge, c=1).

$$d(N) = N^{-p} e^{2\pi\sqrt{\frac{CN}{6}}} = M^{-2p} e^{2\pi\sqrt{\frac{\alpha' C}{6\hbar}} M}$$

Neglecting numerical factors this gives, at large M,

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \; ; \; l_s = \sqrt{2\alpha'\hbar} \; ; \; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

Physical interpretation of S_{st}: the number of "string bits" contained in the total length of the string, $L = \alpha' M$.

Semiclassical BH entropy

Bekenstein-Hawking formula for arbitrary D

$$S_{BH} = \frac{\mathcal{A}}{4 l_D^{D-2}} \;\; ; \;\; \mathcal{A} \sim R_S^{D-2} \sim (G_D M)^{\frac{D-2}{D-3}} \Rightarrow S_{BH} = \frac{M R_S}{\hbar}$$

$$l_D^{D-2} = G_D \hbar$$

can be compared with previous

$$S_{st} \sim \frac{\alpha' M}{l_s} = \frac{M}{M_s} \;\; ; \;\; l_s = \sqrt{2\alpha' \hbar} \;\; ; \;\; M_s = \sqrt{\frac{\hbar}{2\alpha'}}$$

The two entropies look very different but can we trust both results everywhere in parameter space?

Let's assume for the moment that we can.

The correspondence curve

S_{BH} grows faster than S_{st} but latter starts higher at small M. Hence, the two entropies must meet at some finite value of M:

$$\frac{S_{BH}}{S_{st}} = \frac{M R_S/\hbar}{M/M_s} = \frac{M_s R_S}{\hbar} = \frac{R_S}{l_s}$$

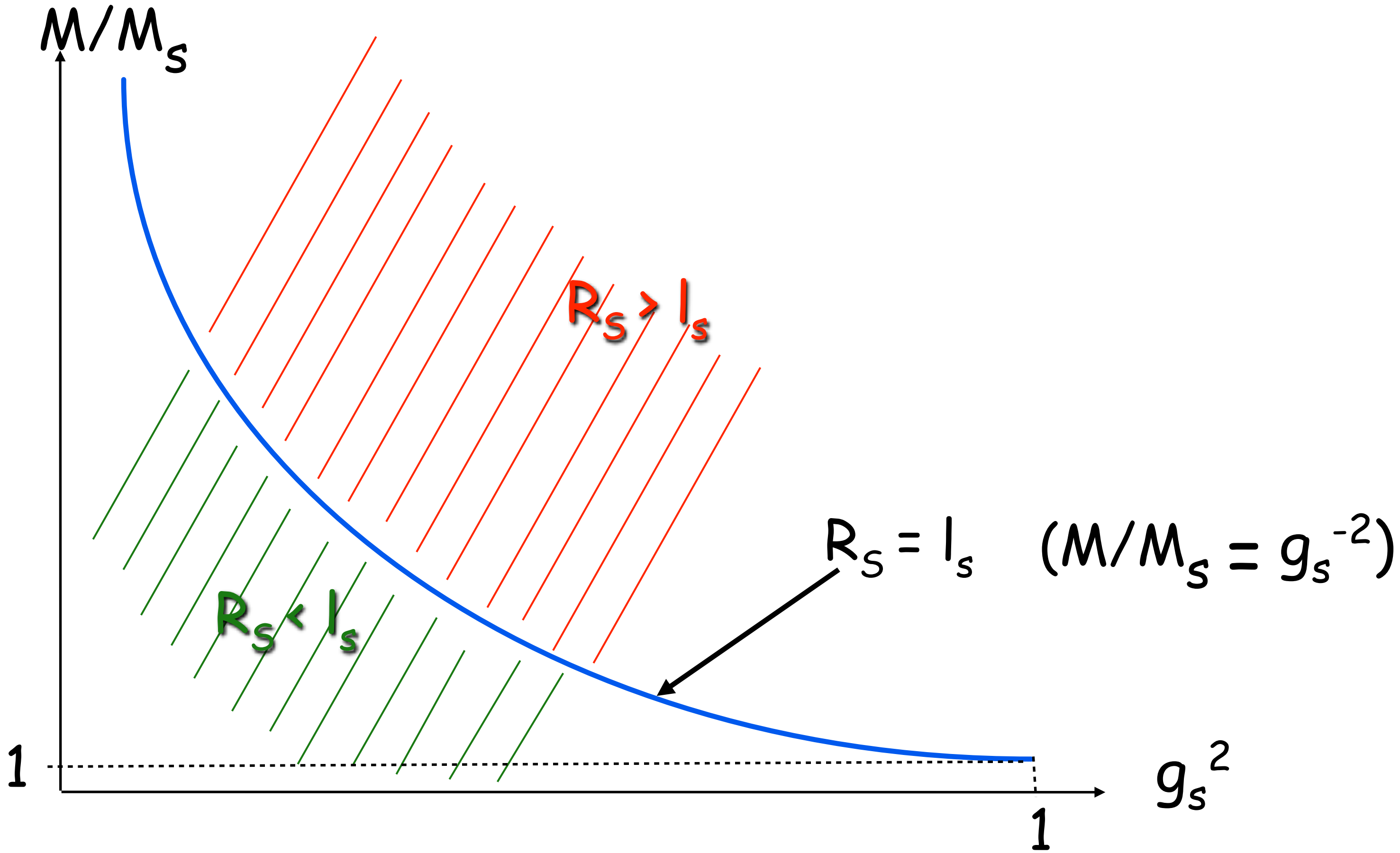
S_{BH} wins over S_{st} for $R > l_s$, the opposite is true for $R < l_s$. They coincide at $R = l_s$ (where $T_{BH} \sim T_{Hag}$) and take the value:

$$S_{BH} = S_{st} = \frac{l_s^{D-2}}{l_D^{D-2}} = g_s^{-2} \gg 1 \Rightarrow M = M_* \equiv g_s^{-2} M_s$$

NB: at very small string coupling $M_* \gg M_P \gg M_s$

$S_{BH} = S_{st}$ defines a hyperbola in the (g_s, M) plane called the correspondence curve.

The correspondence curve



Below the correspondence curve

Below the correspondence curve (CC) the Schwarzschild radius of the string is smaller than the string length scale. The latter is believed to be the minimal size of any string.

Hence such strings are simply NOT BHs.

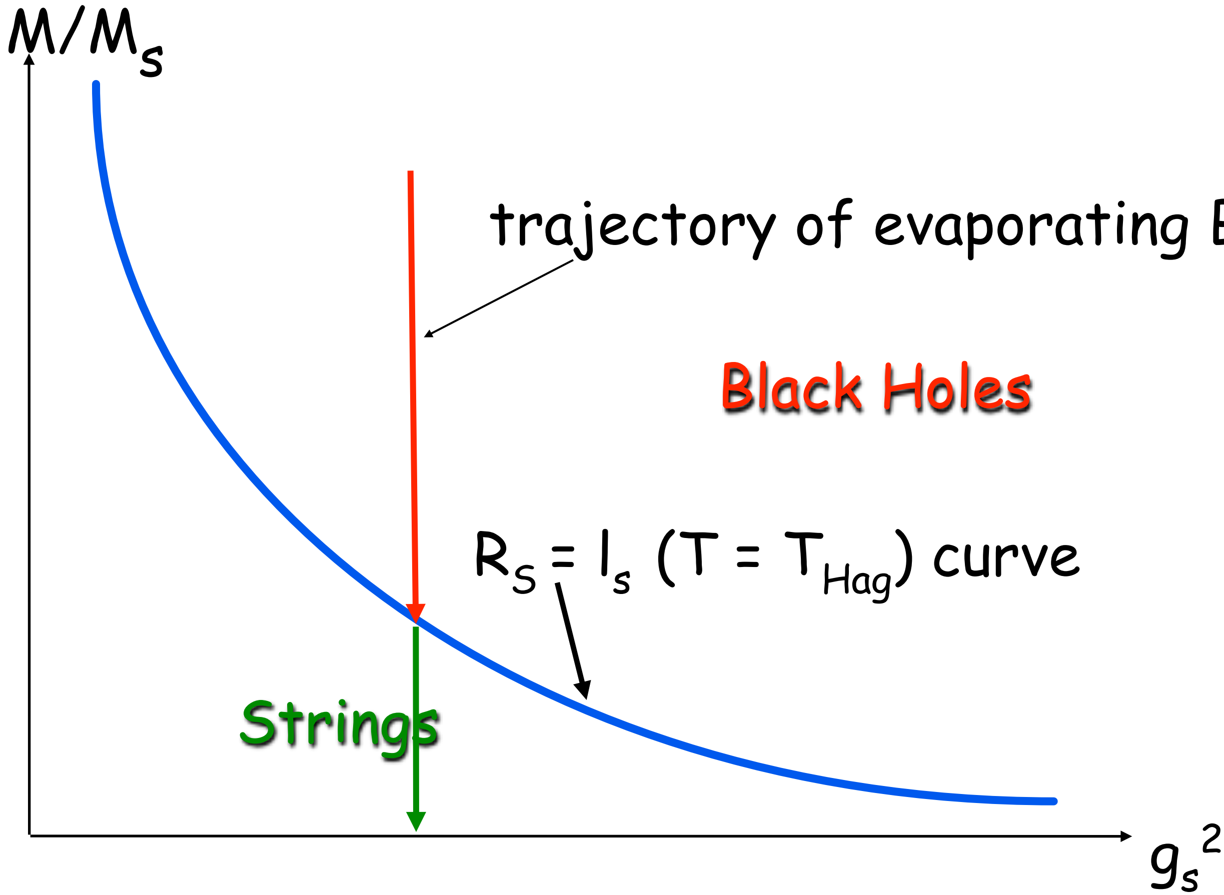
Interpretation: in QST there are no BHs whose R_S is smaller than l_s, i.e. whose Hawking temperature is higher than M_s ($T = M_s$ is believed ST's maximal temperature)

So far, everything looks consistent!

It can even solve the problem of end-point of evaporation!

Evaporation of a BH at fixed g_s (Bowick et al. 1987)

Singularity at the end of evaporation avoided?



Approaching the correspondence curve: the random-walk puzzle

If we want to identify BH with FS above the CC, their properties should match as we approach the curve.

By definition the two entropies match (up to O(1) factors) but there is still a "random-walk puzzle".

S_{st} can be understood in terms of a "random walk" but then a string on the CC being much longer (heavier) than l_s (M_s), will have a typical size much bigger than its Schwarzschild radius l_s.

But then it has nothing to do with a BH!

Size distribution of free strings

The resolution of the RW puzzle is quite simple. One has to compute the distribution of the string sizes for a given M

(NB: M fixes length not size!).

This was done by T. Damour & GV (2000). The entropy of strings of given M and size R is given by

(c_1, c_2 are positive # O(1), calculation reliable for $R > R_S$):

$$S(M,R) \equiv \log d(M,R) = a_0 \frac{M}{M_s} f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right);$$

$$a_0 = 2\pi\sqrt{\frac{D-2}{6}}\ ;\ f\left(\frac{R}{l_s}, \frac{\alpha' M}{l_s}\right) = \left(1 - \frac{c_1 l_s^2}{R^2}\right)\left(1 - \frac{c_2 R^2}{(\alpha' M)^2}\right)$$

Entropy is maximized for: $\frac{R}{l_s} \sim \sqrt{\frac{M}{M_s}} = \text{random walk value}$

But there is still an S of order M/M_s in strings of size $O(l_s)$!

We shall call such strings lying on the CC "stringholes"

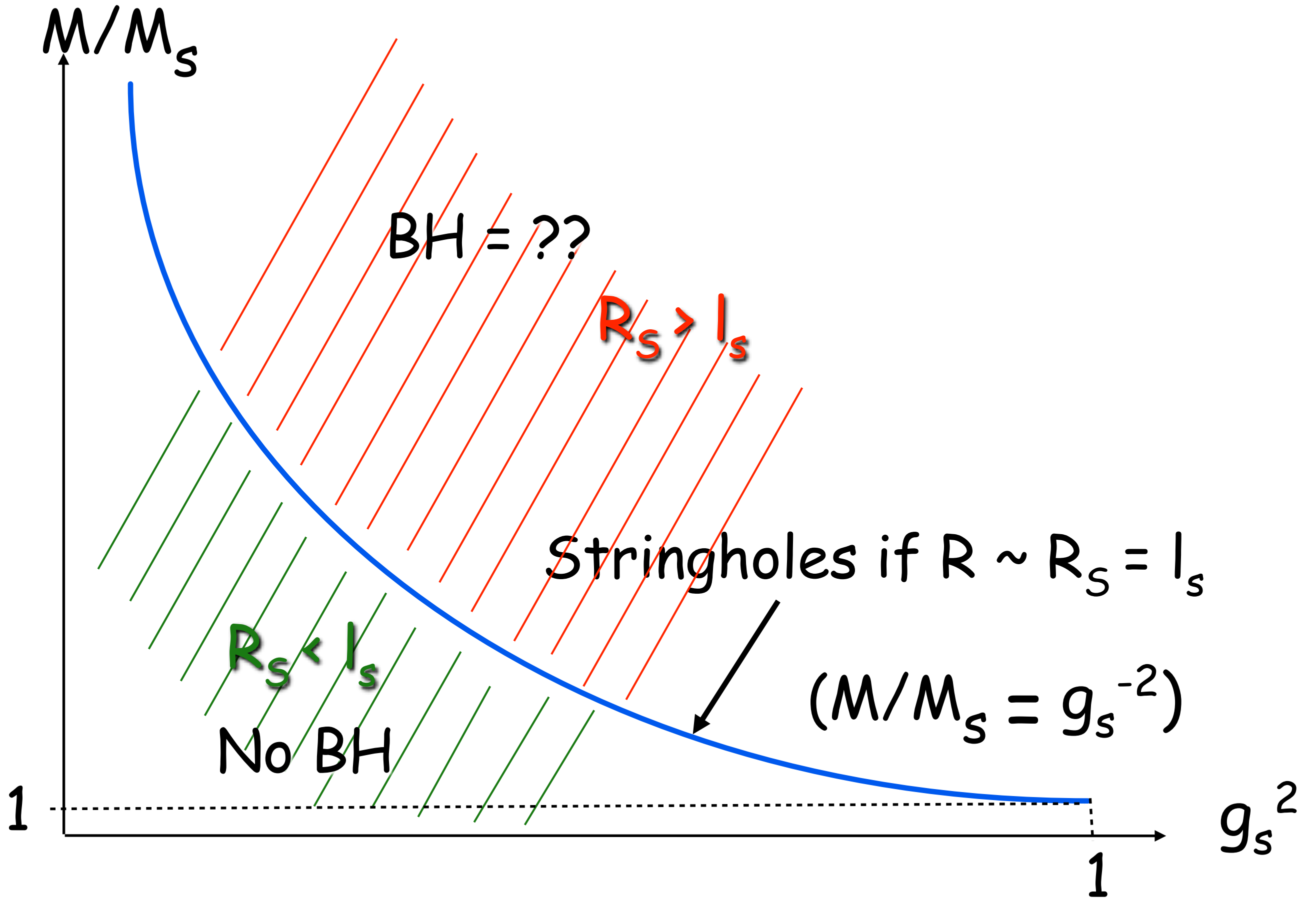
Stringholes can also be understood as string states in which only oscillators with $n > N^{1/2}$ are excited. It is easy to compute the asymptotic behavior of such a restricted partition function and to find that it also gives an exponential degeneracy though with a smaller coefficient in the exponent.

$$P(z,K) = \prod_{k=K}^{\infty} \left(\frac{1}{1-z^k} \right) = \sum_N p(N,K) z^N$$

$$p(N, K \sim \sqrt{N}) \sim e^{c_K \sqrt{N}}$$

$$r^2 \sim l_s^2 \sum_{n > \sqrt{N}} \frac{1}{n} \langle a_n^\dagger a_n \rangle \sim l_s^2$$

Stringholes



Above the correspondence curve

It is reassuring that the string-coupling corrections become of O(1) just when we can reproduce BH properties up to factors O(1).

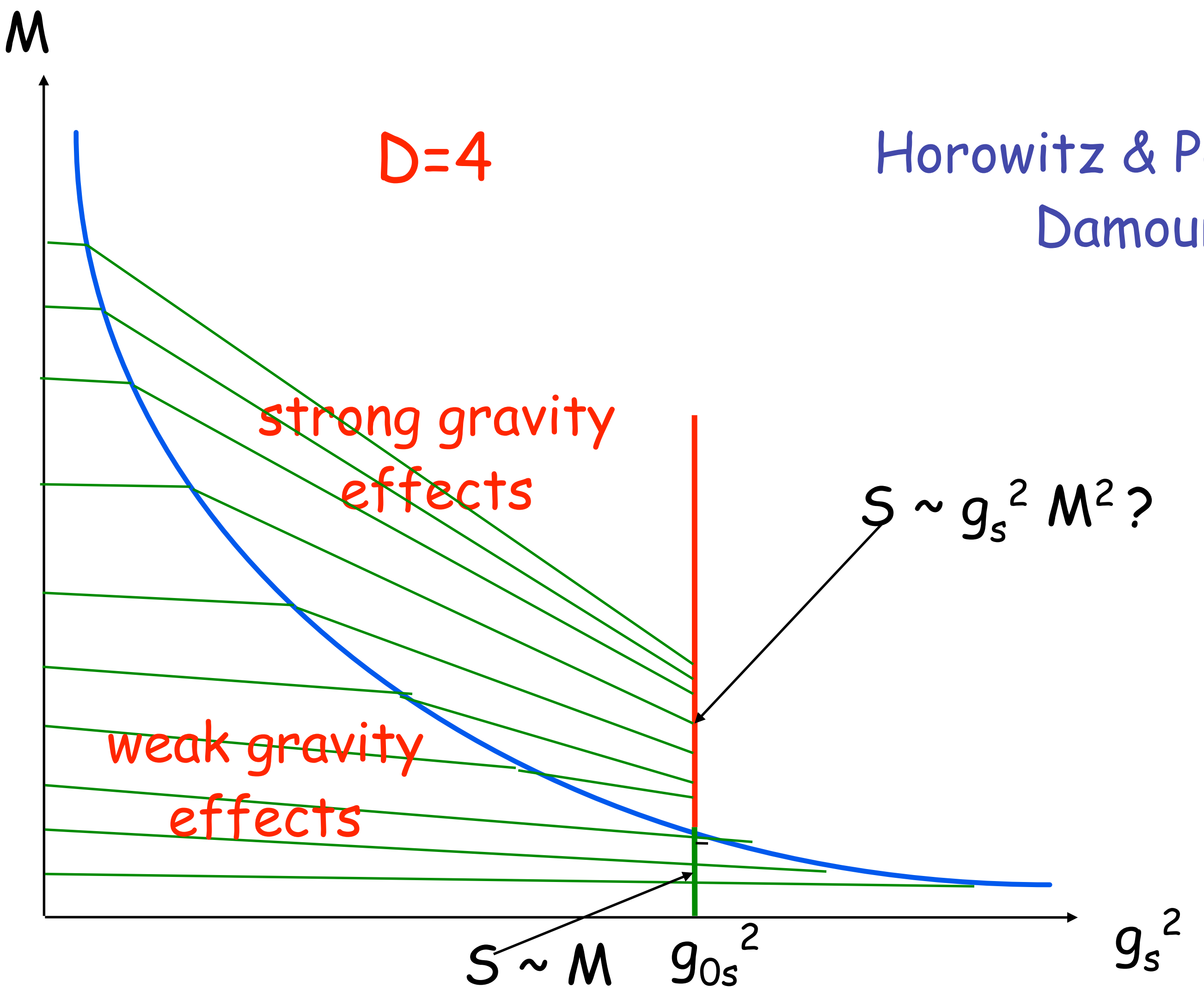
As we go farther and farther above the CC the discrepancy between free-string and BH entropy becomes larger and larger but also the corrections get out of hand.

In order to see whether we can have agreement there we would have to compute the effect of interactions when they become non-perturbative.

This is a hard & unsolved problem.

Here is a (contrived?) example of what could possibly do the job (see below for a different hint).

Gravity-induced increase in density of states



Horowitz & Polchinski, '97, '98
Damour & GV, '00

Transplanckian-energy strings collisions: stringhole production

(GV: 0410.166 and references therein)

A nice theoretical laboratory for studying deep questions about quantum string gravity.

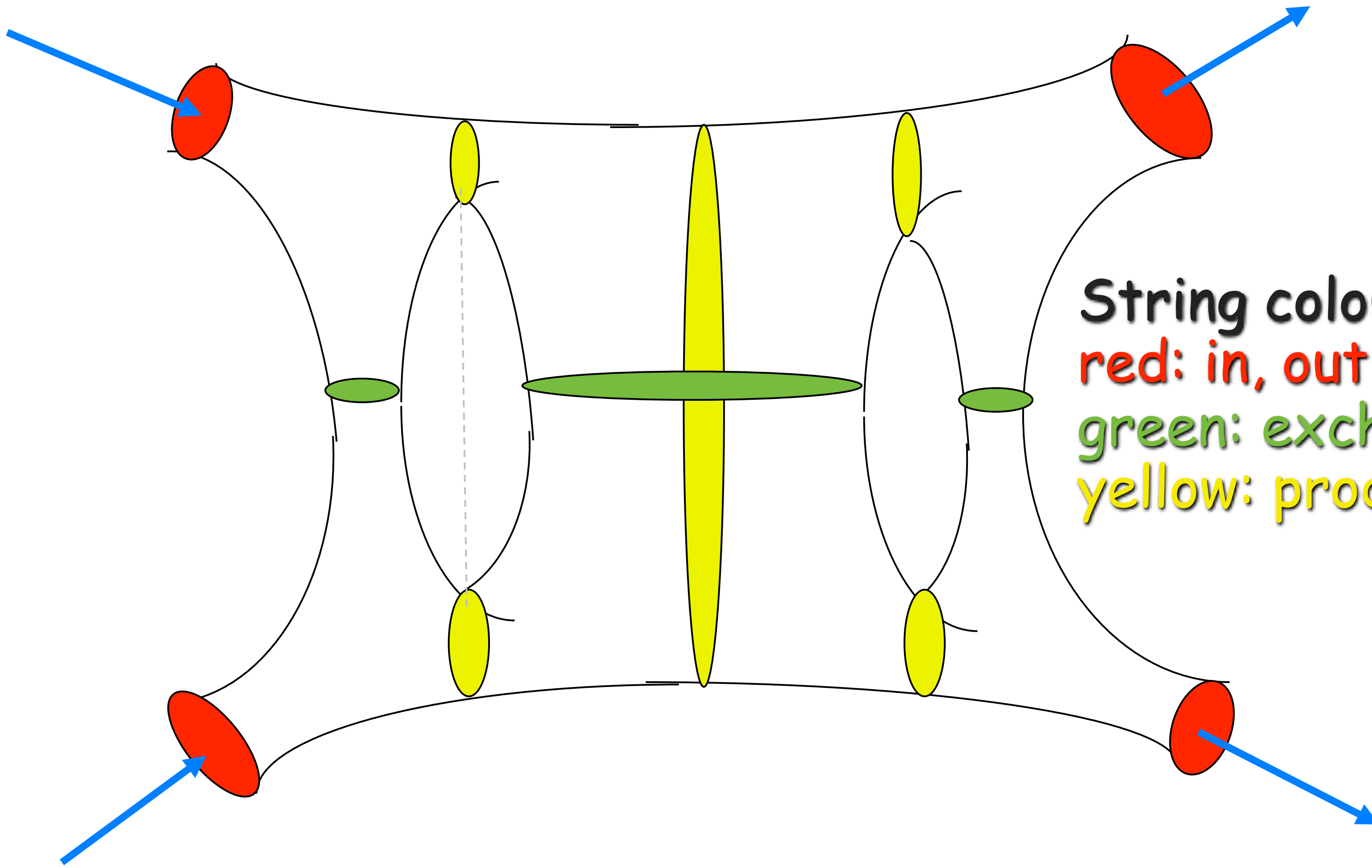
We can hardly imagine a simpler pure initial state that could lead to BH formation and whose unitary evolution we would like to understand/follow.

Calculations performed in flat spacetime & D =10.

An effective metric emerges at the end.

Recently extended to HE string-brane collisions (DDRV 2010 + ..). No time to review the subject.

TPE (closed)string-string collisions
(a two-loop contribution)



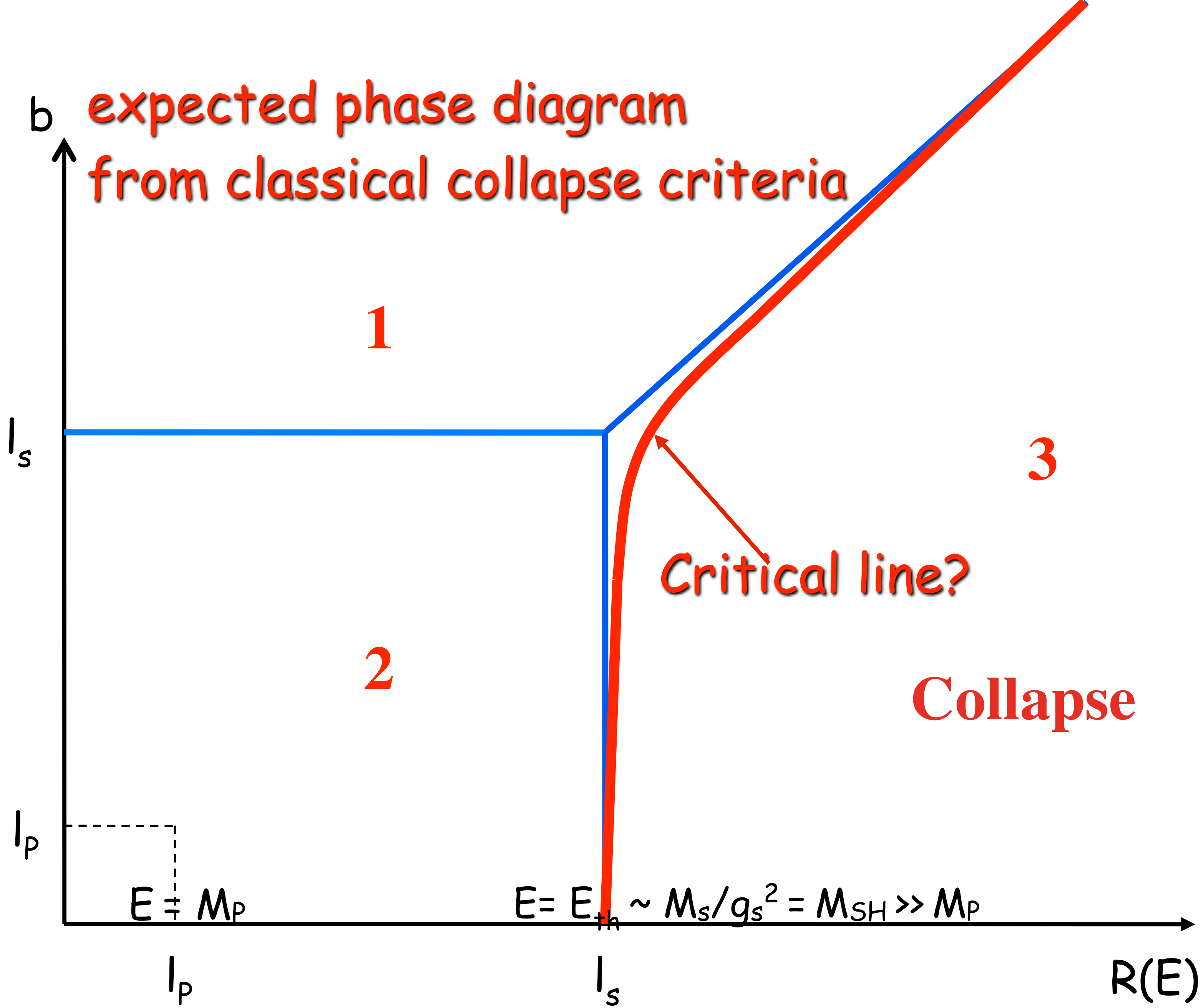
String colour code:
red: in, out
green: exchanged
yellow: produced

Parameter-space for high-energy string-string collisions

3 relevant length scales (neglecting l_P @ g_s << 1)

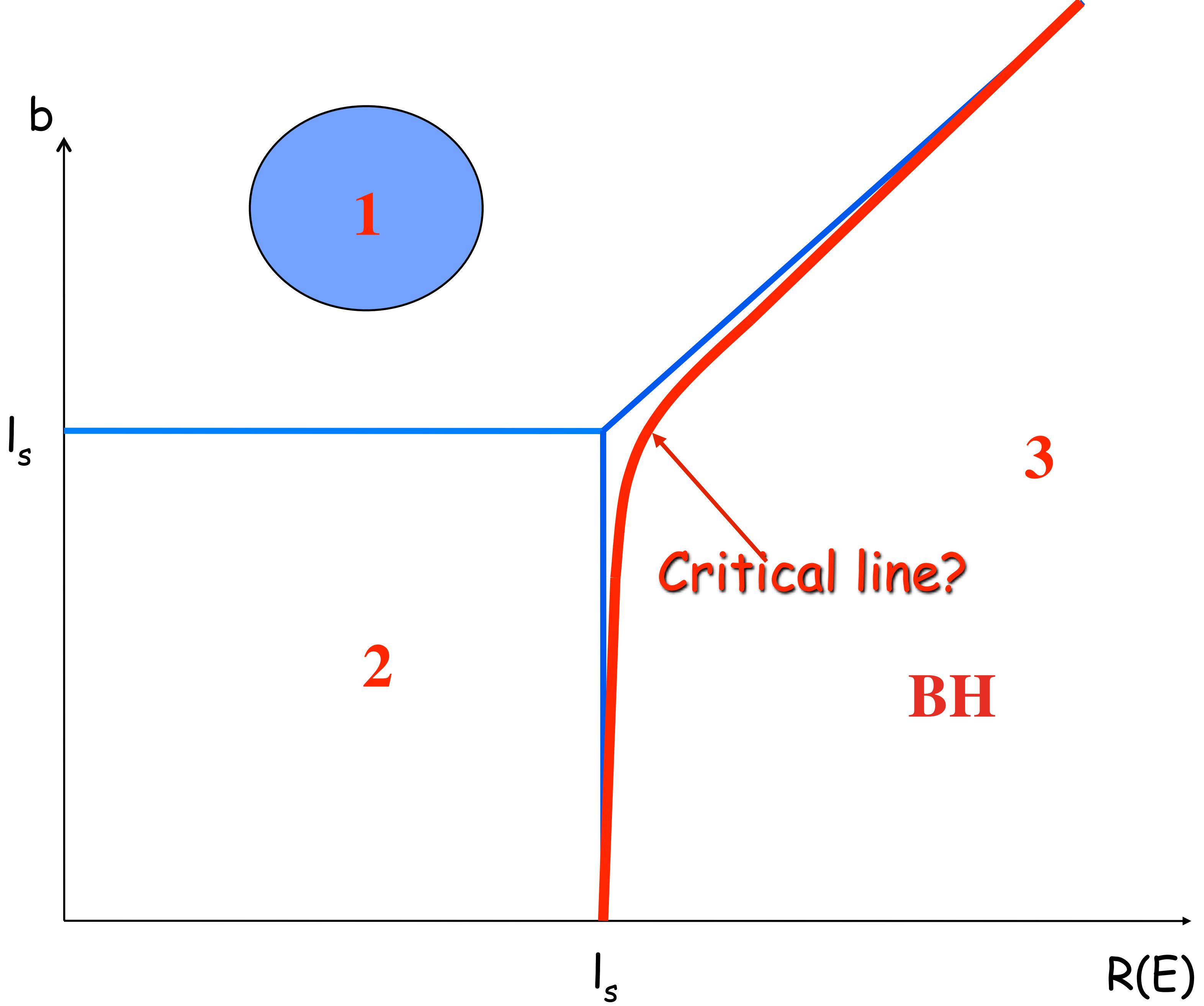
$$b \sim \frac{2J}{\sqrt{s}} \quad ; \quad R_D \sim \left(G\sqrt{s}\right)^{\frac{1}{D-3}} \quad ; \quad l_s \sim \sqrt{\alpha' \hbar} \quad ; \quad G\hbar = l_P^{D-2} \sim g_s^2 l_s^{D-2}$$

NB: Playing with s and g_s we can make R_D/l_s arbitrary



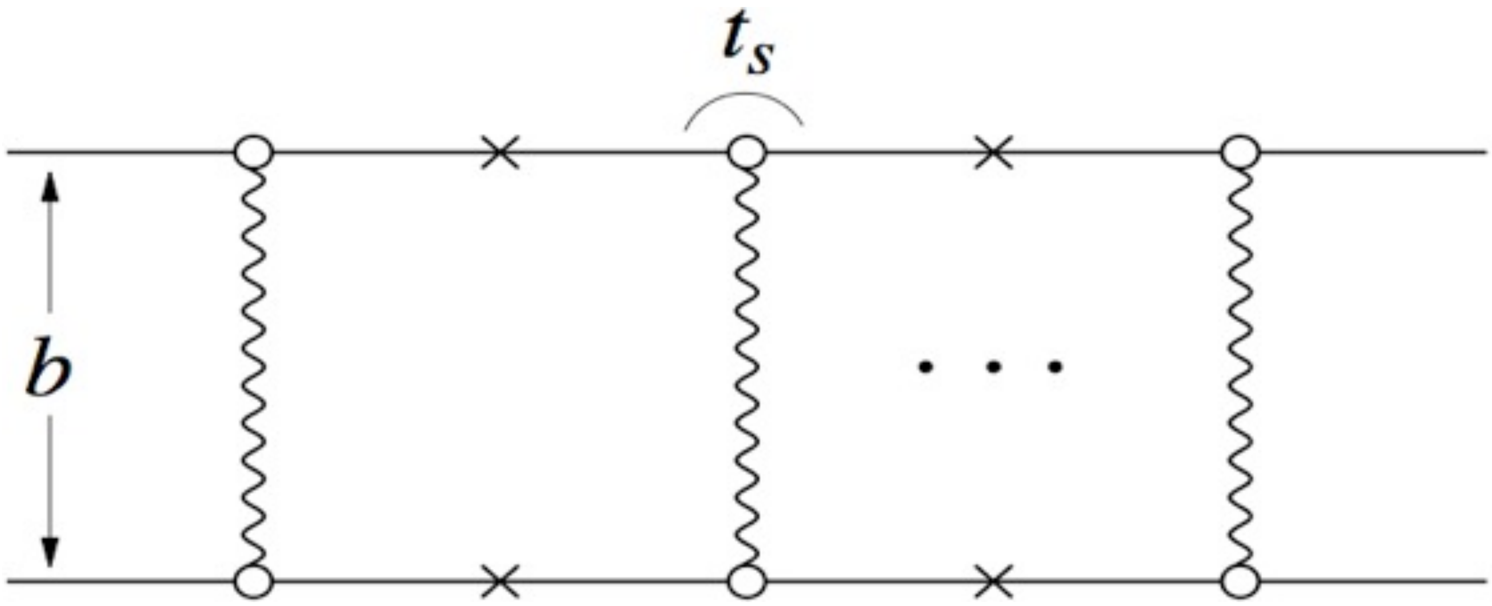
expected phase diagram
from classical collapse criteria
b
1
2
3
l_s
l_P
Critical line?
Collapse
$E = M_P$
$E = E_{th} \sim M_s/g_s^2 = M_{SH} >> M_P$
l_P
l_s
R(E)

The weak-gravity regime


b
1
l
s
Critical line?
3
2
BH
l
s
R(E)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Leading eikonal diagrams (crossed ladders included)



Point-particle limit @ large b

$$S(E,b) \sim exp\left(i\frac{Gs}{\hbar}c_D b^{4-D}\right) \;;\; S(E,q) = \int d^{D-2}b\, e^{-iqb} S(E,b) \;\;;\;\; s = 4E^2 \;,\; q \sim \theta E$$

The integral is dominated by a saddle point at:

$$b_s^{D-3} \sim \frac{G\sqrt{s}}{\theta} \;;\; \theta \sim \left(\frac{R_S}{b}\right)^{D-3} \;;\; R_S^{D-3} \sim G\sqrt{s}$$

Generalization of Einstein's deflection formula to ultra-relativistic collisions and arbitrary D. It corresponds precisely to the relation between b and θ in the metric generated by a relativistic point-particle of energy E. This is an effective metric , NOT a class. one!

- At fixed θ, larger E probe larger b (i.e. the IR). How come?
- (Gs/h) b^{4-D} gives the average loop-number. The total q = θ E is shared among as many exchanged gravitons so that:

$$q_{ind} \sim \frac{\hbar q}{G\ s\ b^{4-D}} \sim \frac{\hbar\ \theta\ b^{D-4}}{R^{D-3}} \sim \frac{\hbar}{b_s}$$

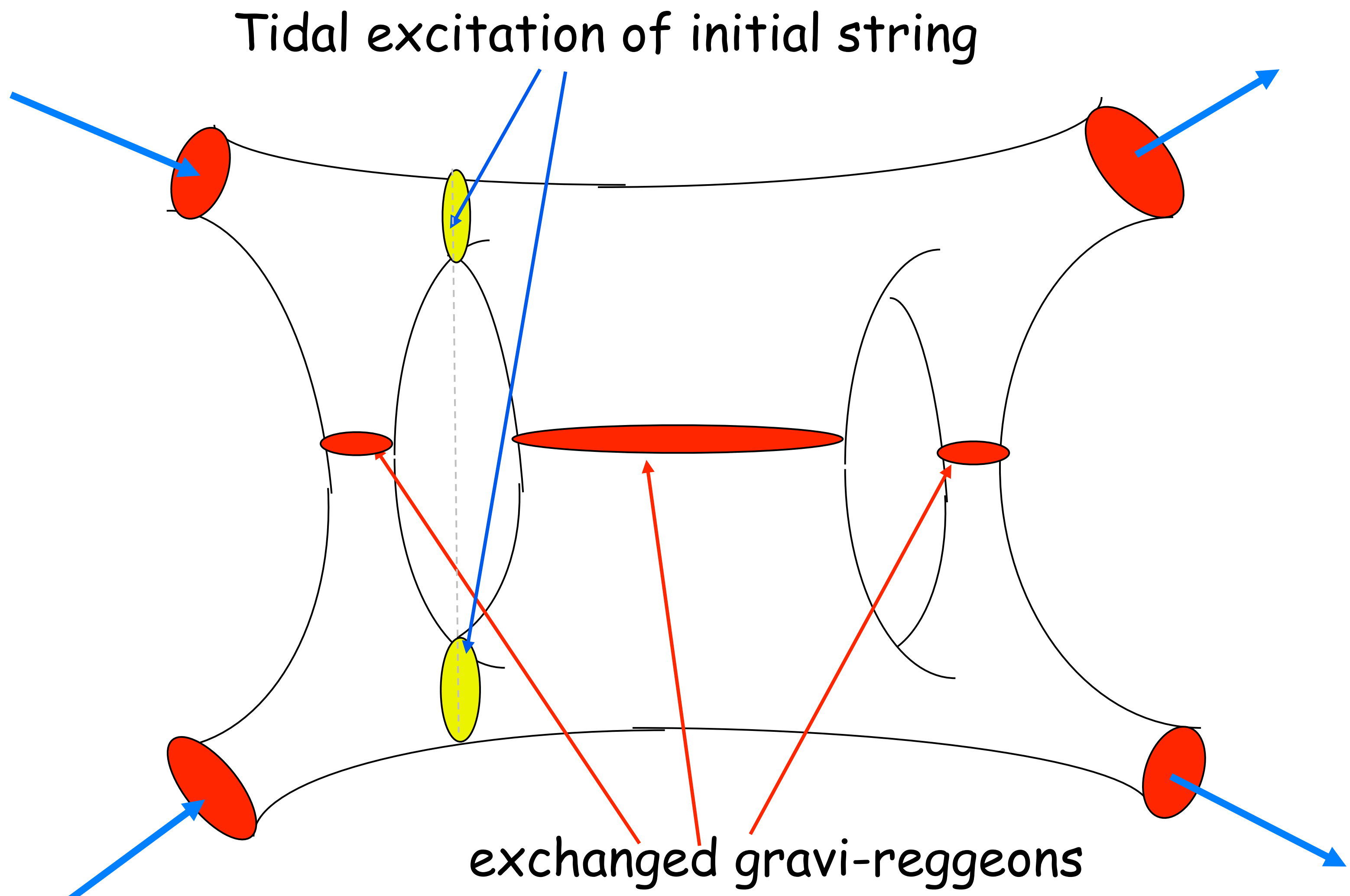
String-string scattering @ large b

(new effects because of imaginary part)

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D}\left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

Graviton exchanges can excite one or both strings. Reason (Giddings '06): a string moving in a non-trivial metric feels tidal forces as a result of its finite size. A simple argument gives the critical impact parameter b_t below which the phenomenon kicks-in (as found by direct calculation by ACV). It is parametrically larger than l_s.

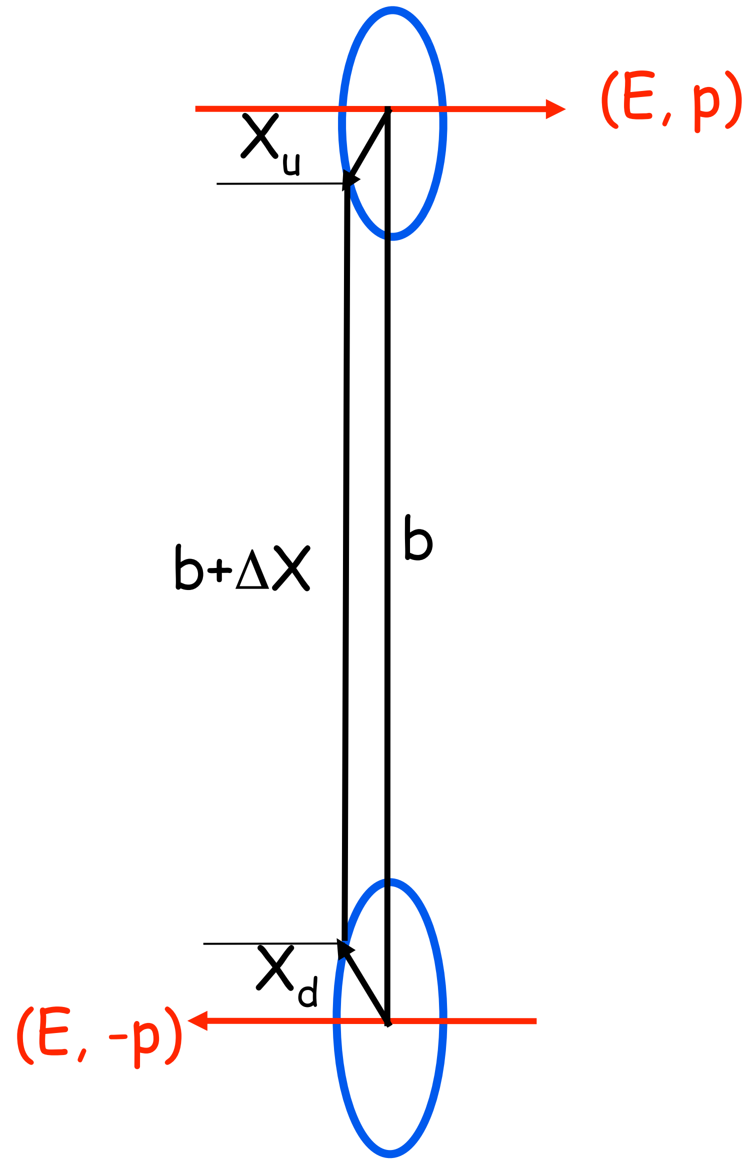
$$b_t \sim \left(\frac{Gsl_s^2}{\hbar}\right)^{\frac{1}{D-2}}$$


Tidal excitation of initial string
exchanged gravi-reggeons

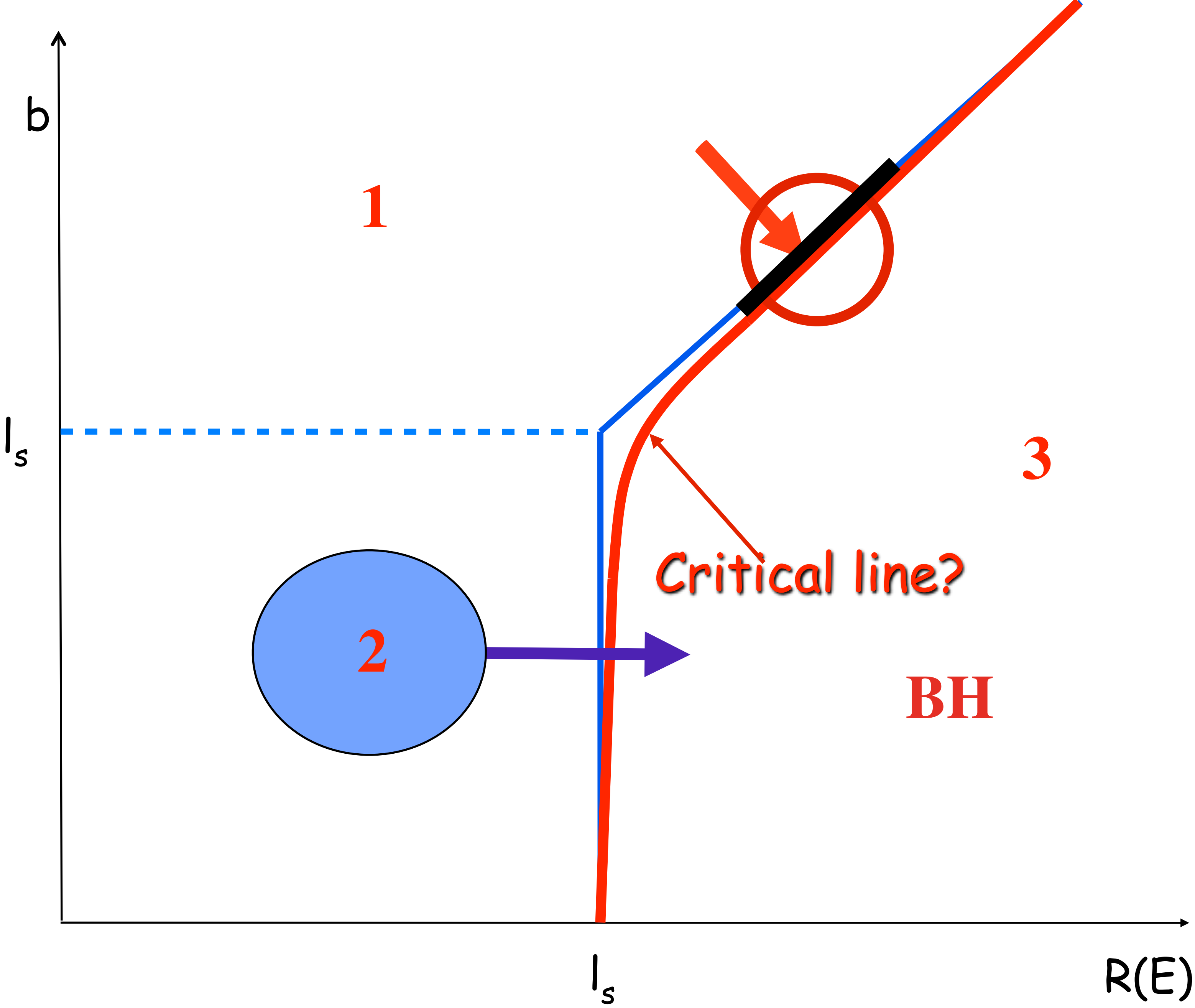
These effects are neatly captured, at the leading eikonal level, by replacing the impact parameter **b** by a shifted impact parameter, displayed by each string's position operator (stripped of its zero modes) evaluated at $\tau = 0$ (= collision time) and averaged over σ.

This leads to a unitary operator eikonal formula for the S-matrix

More details later...



The string-gravity regime: approaching stringhole production



b
1
l_s
3
Critical line?
2
BH
l_s
R(E)

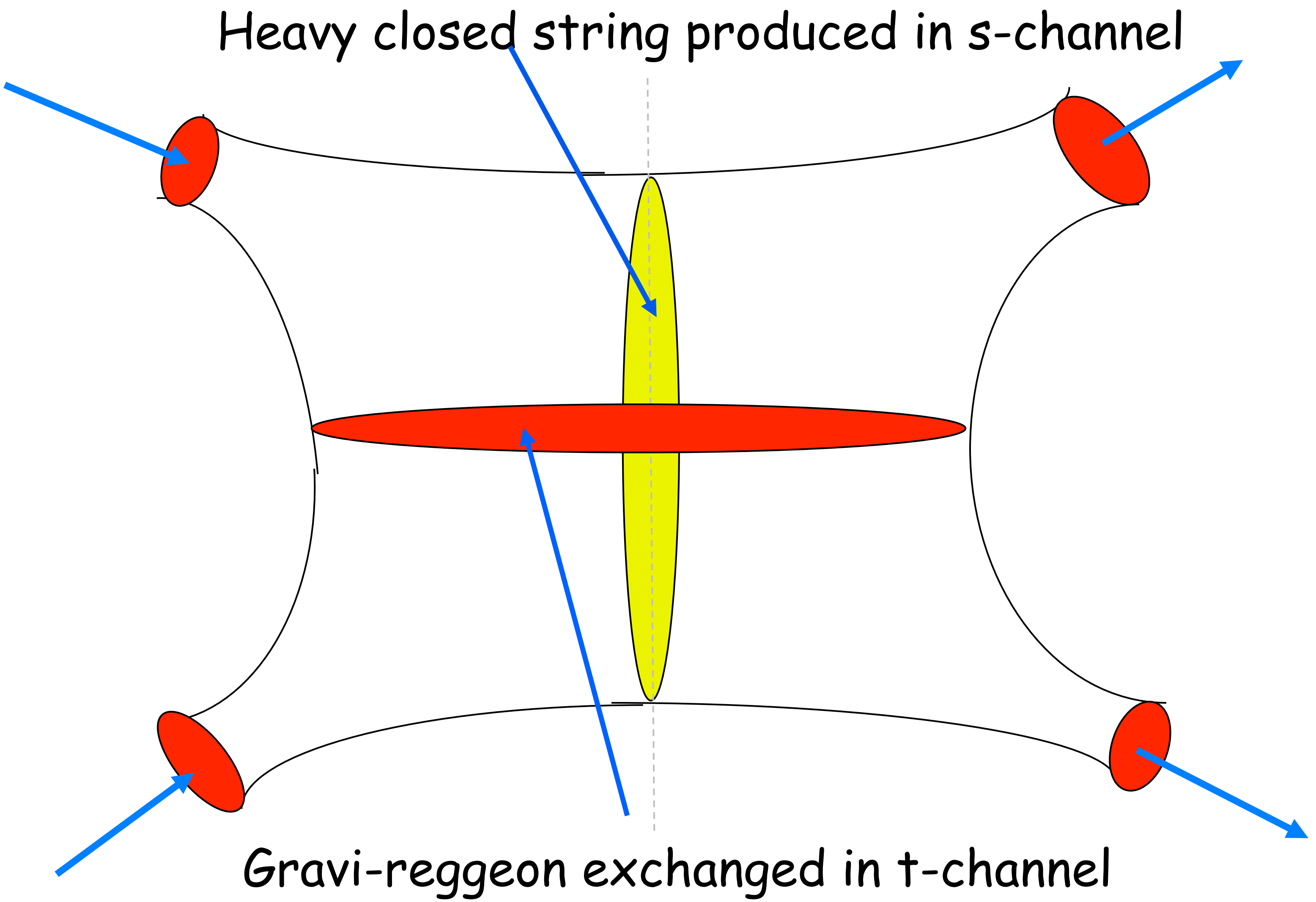
String-string scattering @ b,R < l_s

$$S(E,b) \sim exp\left(i\frac{A_{cl}}{\hbar}\right) \quad ; \quad \frac{A_{cl}}{\hbar} \sim \frac{Gs}{\hbar} c_D b^{4-D} \left(1 + O((R/b)^{2(D-3)}) + O(l_s^2/b^2) + O((l_P/b)^{D-2}) + \dots\right)$$

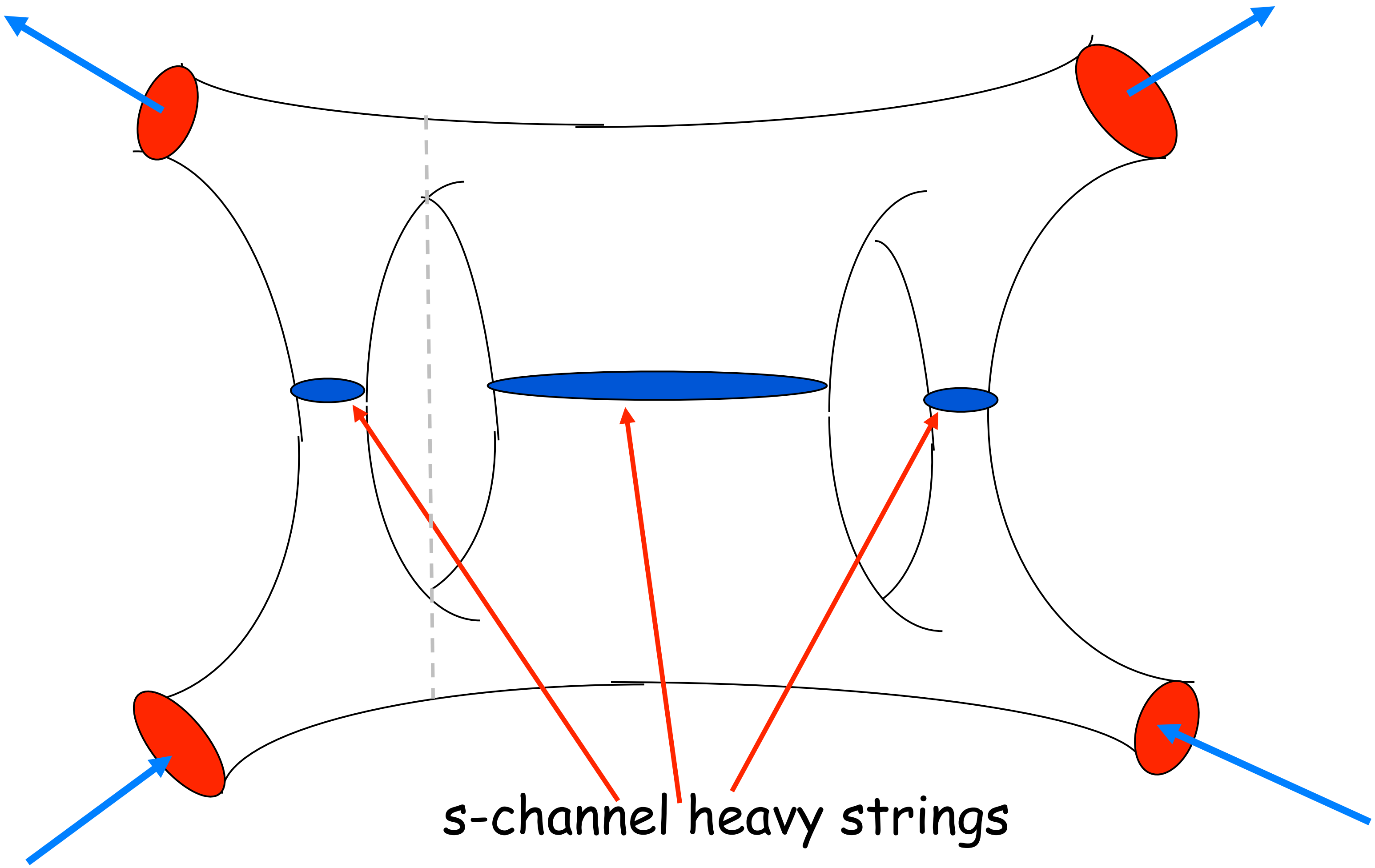
Because of (good old DHS) duality even single graviton exchange does not give a real scattering amplitude. The imaginary part is due to formation of closed-strings in the s-channel.

It is exponentially small at large impact parameter (hence irrelevant in region 1, important in region 2)

Im A is due to closed strings in s-channel (DHS duality)



At higher loop order many strings produced in s-channel



Turning the previous diagram by 90°

$$\mathrm{Im} A_{cl}(E,b) \sim \frac{G\ s\ l_s^{4-D}}{\hbar} \exp\left(-\frac{b^2}{l_s^2\ \log s}\right)$$

At impact parameters below the string scale one starts producing more and more strings. Their average number grows like $Gs \sim E^2$ (Cf. # of exchanged strings) so that, above $E = M_s/g$, the average energy of each final string starts decreasing as the incoming energy grows

$$\langle E_{final} \rangle \sim \frac{M_s^2}{g^2 \sqrt{s}} \to M_s \text{ at } \sqrt{s} = E_{th} \text{ with } \langle n \rangle \to g_s^{-2} \sim S_{SH}$$

Similar to what we expect in BH physics!

Fast growth of <n> & consequent softening: an interesting signature even below the actual threshold of BH production?

Str.-str. vs. str.-brane scattering @ b, $R_s < l_s$

In string-string scattering:

$$\langle n_{closed} \rangle \sim \frac{E R_S}{\hbar} \left(\frac{R_S}{l_s} \right)^{D-4} \quad \Rightarrow \quad \langle E_{closed} \rangle \sim M_s \left(\frac{l_s}{R_S} \right)^{D-3} \sim \frac{M_s^2}{g_s^2 E}$$

If extrapolated to $R_S > l_s$ this gives only massless string modes (Hawking radiation?). Can it be trusted?

In string-brane scattering (DDRV, in progress):

$$\langle n_{open} \rangle \sim \frac{E l_s}{\hbar} \left(\frac{R_p}{l_s} \right)^{7-p} \quad \Rightarrow \quad \langle E_{open} \rangle \sim M_s \left(\frac{l_s}{R_p} \right)^{7-p} \sim M_s (g_s N)^{-1}$$

Now the calculation should be reliable even for $R_p > l_s$. This is where we should be able to make contact with a CFT living on the brane system.

A hint on the nature of BHs in String Theory?

If extrapolation to $R_S > l_s$ can be qualitatively trusted it would indicate that above the correspondence line it becomes entropically preferable to break up the heavy string/black hole into its massless decay products.

Can these form a gravitationally bound system (~geon?)

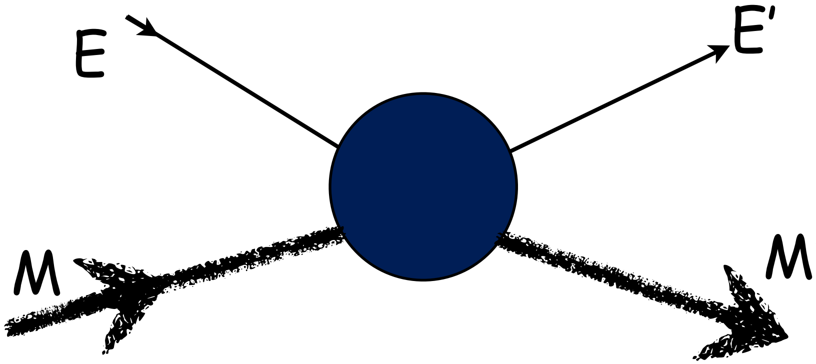
As argued by Dvali and Gomez the number of massless quanta ("gravitons") whose energies add up to the total mass M, and which can bind gravitationally in a region of size R_S, is of order $M R_S/h$, i.e. of order S_{BH}.

Our results appear to lend some credibility to their picture (not necessarily in its details).

Stringholes are hippies!

(GV: 1212.2606)

Scattering of a massless string on a heavy one



kinematical region:

$$M_s M \ll s - M^2 = -2p \cdot P = 2EM \ll M^2$$

Light string acting as a hair-detecting probe

Leading eikonal generalizing ACV and DDRV (R. Russo private comm.)

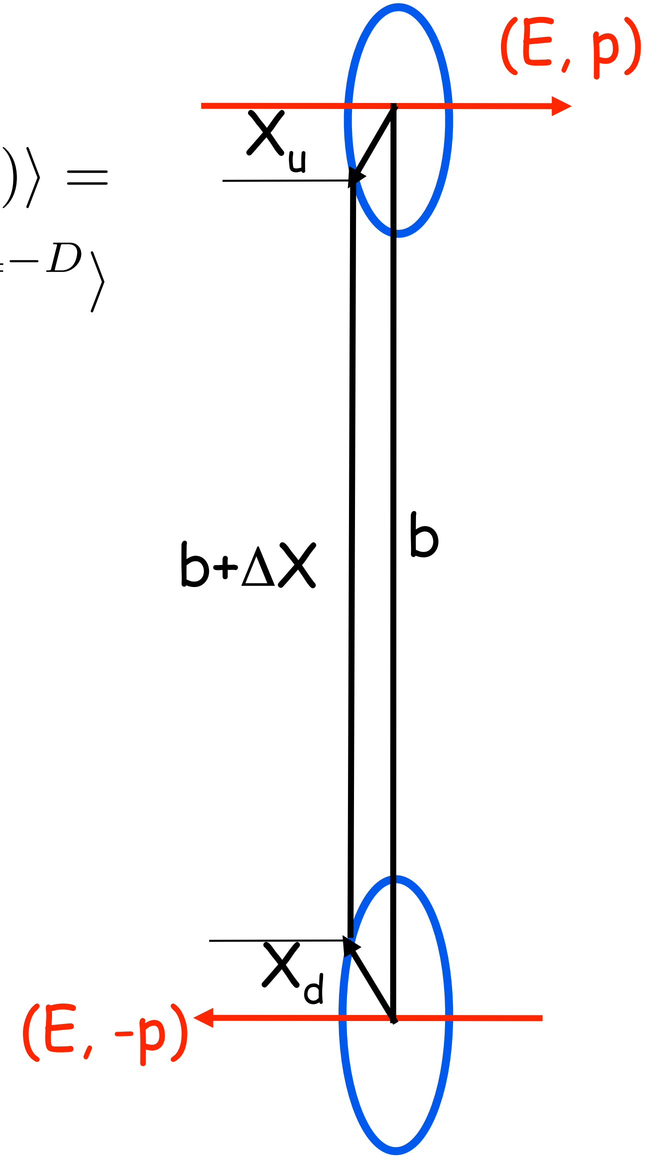
$$S(E, M, b) \sim \exp(i\frac{\mathcal{A}_{cl}}{\hbar}) = \exp\left(i\frac{4GEM}{\hbar}c_D b^{4-D}\right) \equiv e^{2i\delta(E,M,b)}$$

$$c_D = \Omega_{D-4}^{-1} \equiv \frac{\Gamma(\frac{D-4}{2})}{2\pi^{\frac{D-4}{2}}}$$

Check of deflection angle @ saddle point

$$\theta = \frac{8\pi GM}{\Omega_{D-2}b^{D-3}} \sim \left(\frac{R}{b}\right)^{D-3} \ll 1 \;\; ; \;\; (GM)^{\frac{1}{D-3}} \sim R \ll b$$

$$\delta(E,M,b) \to \hat{\delta}(E,M,b) = \langle \delta(b + \hat{X}_H - \hat{X}_L) \rangle = 2GEM\hbar^{-1} c_D \langle (b + \hat{X}_H - \hat{X}_L)^{4-D} \rangle$$



Adding tidal excitation a la ACV-DDRV

$$\langle (b + \hat{X}_H - \hat{X}_L)^{4-D} \rangle \equiv \int_0^{2\pi} \frac{d\sigma_L}{2\pi} \int_0^{2\pi} \frac{d\sigma_H}{2\pi} : \left(b + \hat{X}_H(\sigma_H, 0) - \hat{X}_L(\sigma_L, 0) \right)^{4-D} :$$

Expansion of phase shift operator in l_s/b:

$$2(\hat{\delta} - \delta) = \frac{2\pi G E M (D-2)}{\hbar \Omega_{D-2} b^{D-2}} \langle Q_H^{ij} + Q_L^{ij} \rangle \hat{b}_i \hat{b}_j$$

$$Q_H^{ij} = \hat{X}_H^i \hat{X}_H^j - \frac{\delta_{ij}}{D-2} \sum_{i=1}^{D-2} \hat{X}_H^i \hat{X}_H^i$$

also:

$$Q_H^{ij} \hat{b}_i \hat{b}_j = \hat{X}_H^i \hat{X}_H^j \left(\hat{b}_i \hat{b}_j - \frac{\delta_{ij}}{D-2} \right) \equiv \Pi_{ij} \hat{X}_H^i \hat{X}_H^j$$

b-projection of Lorentz-contracted quadrupole operator!

Higher multipoles appear at higher orders.

We can rewrite the S-matrix in the form

$$S(E, M, b) = \exp(2i\delta)\ \Sigma_L\ \Sigma_H\ ;\ \Sigma_{L,H} = \exp\left(i(D-2)\Delta\ \tilde{Q}^{ij}_{L,H}\ \hat{b}_i\hat{b}_j\right)$$

where $\Delta = \dfrac{2\pi GEMl_s^2}{\hbar\Omega_{D-2}b^{D-2}}$; $\tilde{Q} = l_s^{-2}Q$ $\qquad \Pi_{ij} \equiv \hat{b}_i\hat{b}_j - \dfrac{\delta_{ij}}{D-2}$

$$\tilde{Q}^{ij}\ \hat{b}_i\hat{b}_j = \Pi_{ij}\ \sum_{n=1}^{\infty}\frac{1}{n}\left(a_n^{\dagger i}a_n^j + \tilde{a}_n^{\dagger i}\tilde{a}_n^j + a_n^i\tilde{a}_n^j + a_n^{\dagger i}\tilde{a}_n^{\dagger j}\right)$$

Using standard techniques we can get a normal ordered Σ (useful between coherent states) as:

$$\begin{aligned}
\Sigma_H &= \Sigma^{(univ)}\ \Sigma^{(hair)}\ ;\ \Sigma^{(univ)} = \Gamma(1+i\Delta)^{D-3}\ \Gamma(1-i(D-3)\Delta) \\
\Sigma^{(hair)} &= :\exp\left(\sum_{n=1}^{\infty}(a_n^{\dagger i}+\tilde{a}_n^i)(a_n^j+\tilde{a}_n^{\dagger j})\left[C_n(\Delta)(\delta_{ij}-\hat{b}_i\hat{b}_j)+\tilde{C}_n(\Delta)\hat{b}_i\hat{b}_j\right]\right): \\
C_n(\Delta) &= -\frac{i\Delta}{n+i\Delta}\ ;\ \tilde{C}_n(\Delta) = C_n(-(D-3)\Delta)\,.
\end{aligned}$$

We finally take the heavy string to be a "stringhole" the idea being to interpret the result now in terms of BH properties (unfortunately we are presently unable to make reliable calculation much above the SH mass scale). Then:

$$\Delta = \frac{GEMl_s^2}{\hbar\, b^{D-2}} \to \frac{El_s}{\hbar} \left(\frac{l_s}{b}\right)^{D-2}$$

with

$$1 \ll \frac{El_s}{\hbar} \ll g_s^{-2} \qquad \left(\frac{l_s}{b}\right)^{D-2} \sim \theta^{\frac{D-2}{D-3}} \ll 1$$

in our kinematical region $\theta^{\frac{D-2}{D-3}} \ll \Delta \ll g_s^{-2}\theta^{\frac{D-2}{D-3}}$

and there is a lot of parameter space for Δ to be large

The resulting S-matrix has many universal (i.e. no-hair) factors but it also has terms that probe the quadrupole (and also other multipoles) of the SH. At leading order in Δ/n:

$$\Sigma^{(hair)} =: \exp\left(-i(D-2)\Delta\sum_{n=1}^{\infty}\frac{1}{n}(a_n^{\dagger i}+\tilde{a}_n^{i})(a_n^{j}+\tilde{a}_n^{\dagger j})\Pi_{ij}\right):$$

This is the quantum hair of the SH as "seen" by the probe string via our thought experiment.
It turns out to be relatively large, possibly only a power of g_s^2 smaller than the no-hair terms.
If we apply the S-BH correspondence idea, we would conclude that also BHs should have such a large amount of quantum hair in agreement with Dvali-Gomez's recent papers, but:
Q_1: Are SHs good representatives of BH?
Q_2: Can the situation suddenly change above the CC?

Summarizing last part

- The string-black hole correspondence (and stringholes) can be useful tools for testing quantum-string gravity ideas in a regime still under control.

- Definite conclusions on the information puzzle will have to wait for a better understanding of how the correspondence works particularly much above the correspondence curve.

Thank you!