Loop Quantum Cosmology: A Status Report

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Based on work with Corichi, Pawlowski, Singh, Vandersloot, Wilson-Ewing

PRL and PRD (06,07,08)

(Short pedagogical review: AA gr-qc/0702030)

Understanding emerged bfrom the work of:

AA, Bentevigna, Bojowald, Corichi, Chiou, Kaminski, Lewandowski, Mena,

Pawlowski, Singh, Szulc, Taveras, Vandersloot, Velhinho, Willis, ...

Bad-Honef Workshop; April 14th, 2008

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- In general relativity, the gravitational field encoded in the very geometry of space-time $\Rightarrow$ space-time itself ends at singularities. General expectation: theory is pushed beyond its domain of applicability. Must incorporate quantum physics. Singularities are our gateways to physics beyond Einstein.

- But straightforward incorporation of quantum physics a la traditional WDW quantum cosmology did not succeed.

- Situation very different in LQG: Physics does not stop at these singularities. Quantum Geometry extends its life. Goal of the talk: Present highlights, emphasizing recent developments.

- Organization:

1. Conceptual Setting
2. k=0 Models
3. LQC Vs WDW Theory
4. k=1 models
5. Extensions and Summary .

1. Conceptual Setting

Some Long-Standing Questions expected to be answered by Quantum Gravity Theories from first principles:

⋆ How close to the big-bang does a smooth space-time of GR make sense? (Onset of inflation?)

⋆ Is the Big-Bang singularity naturally resolved by quantum gravity?
(answer is ‘No’ in the Wheeler-DeWitt theory)

⋆ Is a new principle/ boundary condition at the Big Bang essential?
(e.g. The Hartle-Hawking ‘no-boundary proposal’.)

⋆ Is the quantum evolution across the ‘singularity’ deterministic?
(answer ‘No’ e.g. in the Pre-Big-Bang and Ekpyrotic scenarios)

⋆ What is on the ‘other side’? A quantum foam? Another large, classical universe? ...

Some Long Standing Questions (contd)

⋆ How does one extract physics from solutions to the Hamiltonian constraint (e.g. WDW equation)? dynamics from the frozen formalism? Dirac observables? Emergent time? (Scale factor —natural candidate in the Misner parametrization— not single-valued in closed models.)

⋆ Can one have a deterministic evolution across the singularity and agreement with GR at low curvatures, e.g., recollpase in the closed models? (Background dependent perturbative approaches have difficulty with the first while background independent approaches, with second (Green and Unruh))

In LQC, these issues have been resolved for several minisuperspaces. (Massless scalar field as internal/emergent time; Physical Hilbert space, Dirac observables, semi-classical states, detailed dynamics.)

Emerging Scenario: Physical sector of the theory can be constructed in detail. Continuum a good approximation till curvature attains Planck scale. In simplest models: Vast classical regions bridged deterministically by quantum geometry. No new principle needed.

Merits and Limitations of Quantum Cosmology

One's first reaction: Symmetry reduction gives only toy models! Full theory much richer and much more complicated. But examples can be powerful.

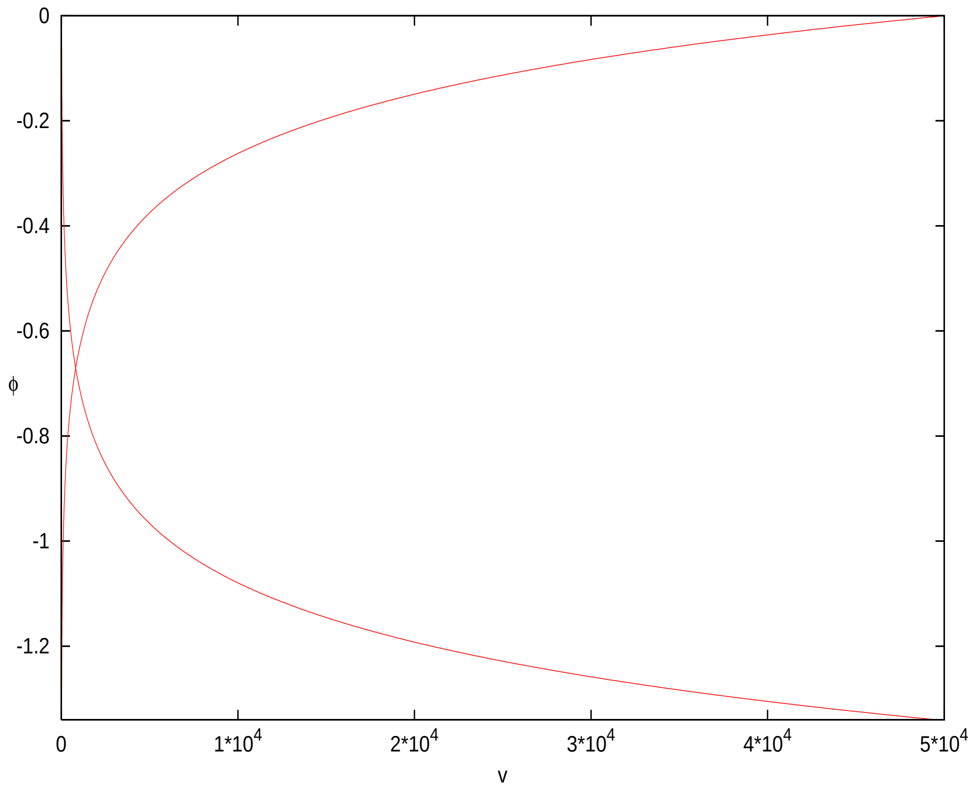
- Full QED versus Dirac's hydrogen atom.
- Singularity Theorems versus first discoveries in simple models.
- BKL behavior: homogeneous Bianchi models. (Henneaux's talk)

Do *not* imply that behavior found in examples is necessarily generic. Rather, they can reveal important aspects of the full theory and should not be dismissed a priori.

One can work one's way up by considering more and more complicated cases. (e.g. recent work of the Madrid group on Gowdy models which have infinite degrees of freedom). At each step, models provide important physical checks well beyond formal mathematics. Can have strong lessons for the full theory. For example, LQC has taught us that loopy techniques do capture sectors with good semi-classical behavior but only if the Hamiltonian constraint is quantized in a certain way.

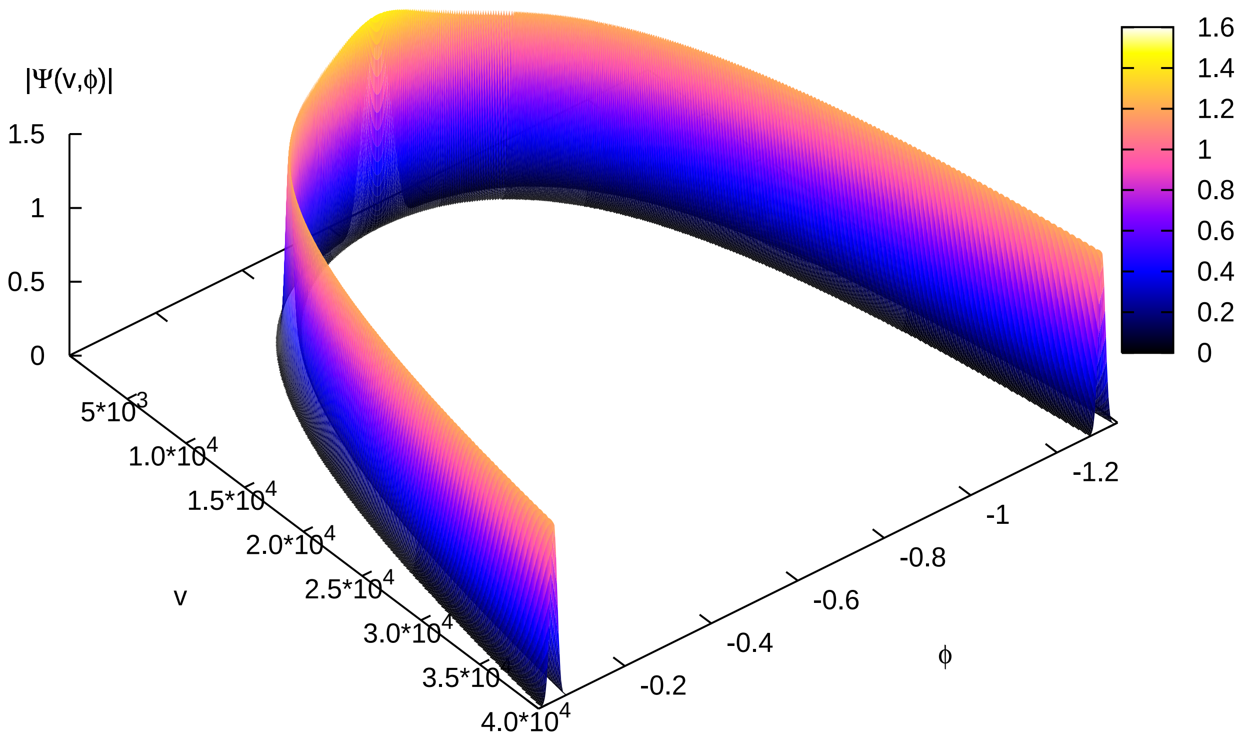
2. The k=0 FLRW Model

FRW, k=0 Model coupled to a massless scalar field ϕ. Instructive because every classical solution is singular. Provides a foundation for more complicated models.



Classical trajectories

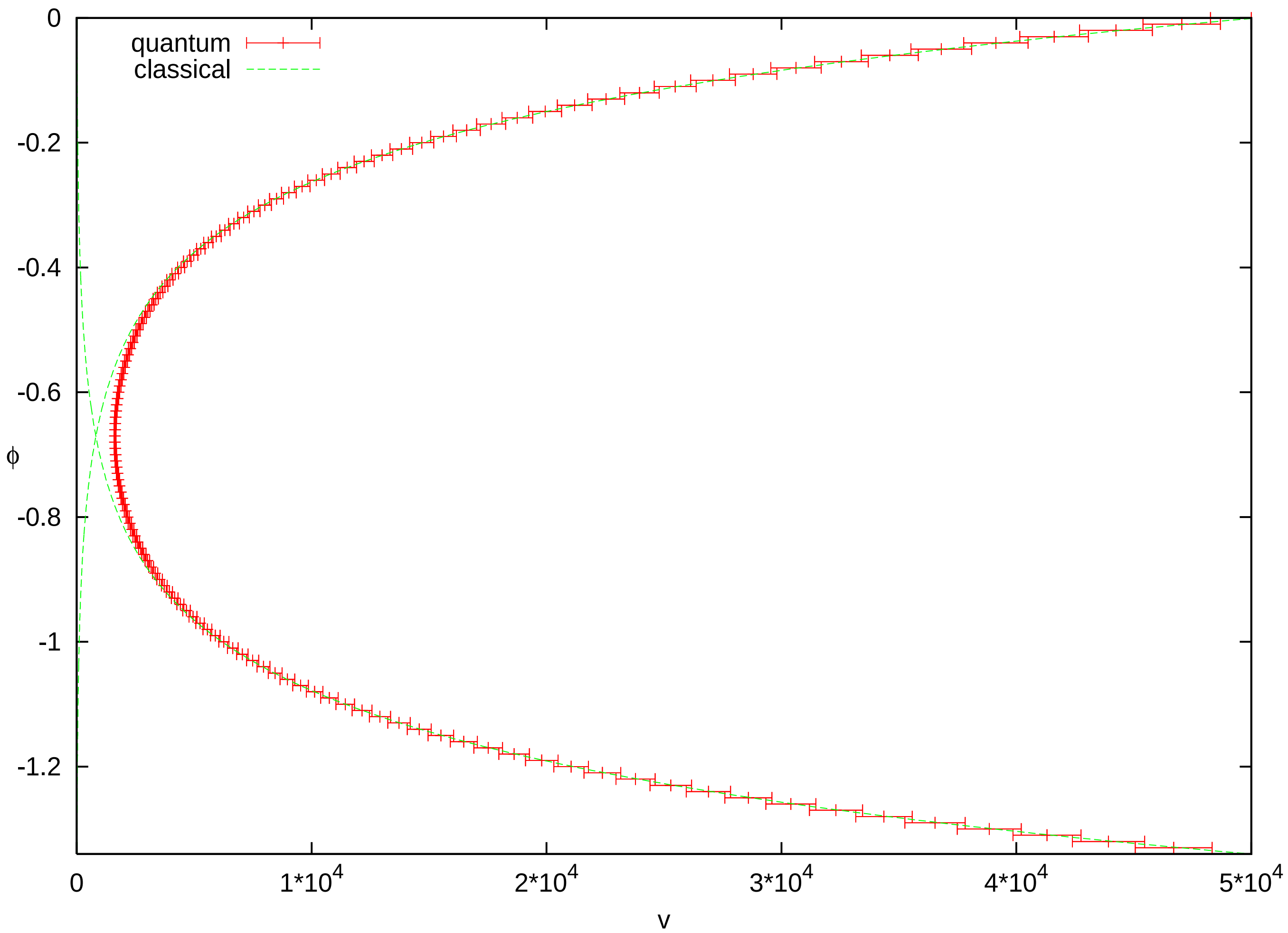
k=0 LQC



Absolute value of the physical state $\Psi(v, \phi)$

(AA, Pawlowski, Singh)

k=0 LQC



Expectations values and dispersions of $\hat{V}|_{\phi}$ & classical trajectories.

(AA, Pawlowski, Singh)

k=0 Results

Assume that the quantum state is semi-classical at a late time and evolve backwards and forward. Then: (AA, Pawlowski, Singh)

• The state remains semi-classical till *very* early and *very* late times, i.e., till $R \approx 1/lp^2$ or $\rho \approx 0.02\rho_{\rm Pl}$. $\Rightarrow$ We know 'from first principles' that space-time can be taken to be classical during the inflationary era.

• In the deep Planck regime, semi-classicality fails. But quantum evolution is well-defined through the Planck regime, and remains deterministic unlike in other approaches. No new principle needed.

• The situation is the same if we include a cosmological constant (AA, Bentevigna, Pawlowski) or an inflationary potential (AA, Pawlowski, Singh). In all cases, the quantum space-time is vastly larger than what general relativity had us believe.

k=0 Results

- Big bang replaced by a quantum bounce. The Friedmann equation replaced by
$(\dot{a}/a)^2 = (8\pi G\rho/3)[1 - \rho/\rho_{\rm crit}]$ where $\rho_{\rm crit} \sim 0.82\rho_{\rm Pl}$.

- The matter density operator $\hat{\rho} = \frac{1}{2}\,(\hat{V}_\phi)^{-1}\,\hat{p}^2_{(\phi)}\,(\hat{V}_\phi)^{-1}$ has an absolute upper bound on the physical Hilbert space (AA, Cirichi, Singh):

$$\rho_{\rm sup} = \sqrt{3}/16\pi^2\gamma^3 G^2\hbar \approx 0.82\rho_{\rm Pl}!$$

Provides a precise sense in which the singularity is resolved.
(Brunnemann & Thiemann)

- Bounce universal: for any physical state Ψ we have:
$(\Psi,\, \hat{V}_\phi \Psi)_{\rm Phy} = V_+ e^{\sqrt{12\pi G}\phi} + V_- e^{-\sqrt{12\pi G}\phi}$
where $V_\pm$ are determined by the 'initial data' $\Psi(v, \phi_o)$ at any ϕ_o.
$V_{\rm min} = \sqrt{(V_- V_+)}$

- Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Physics does not end at singularities.

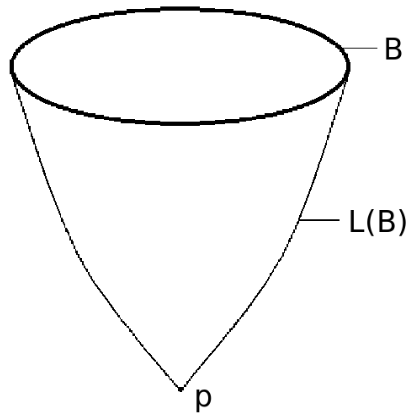
Bousso's Covariant Entropy Bound

- Conjecture (Simplest Version): The matter entropy flux across $\mathcal{L}(\mathcal{B})$ is bounded by

$$S := \int_{\mathcal{L}(\mathcal{B})} S^a dA_a \leq A_{\mathcal{B}}/4\ell_{\rm Pl}^2.$$

- Curious features:

i) Requires a notion of entropy current;
ii) Refers to quantum gravity;
iii) Requires a classical geometry.
Consequently, quite difficult to test in practice!



- In classical GR:

If we consider k=0 FRW models filled with radiation,

$$\frac{S}{A_{\mathcal{B}}} = \frac{\ell_{\rm Pl}^2}{6} \left(\frac{2}{45\pi}\right)^{1/4} \frac{\sqrt{\ell_{\rm Pl}}}{\sqrt{\tau_f}} \left(1 - \sqrt{\frac{\tau_i}{\tau_f}}\right)$$

For round $\mathcal{B}$, the bound holds if $\tau_f > 0.1\ell_{\rm Pl}$ but arbitrarily large violations near the singularity.

- LQC provides an ideal arena:

i) Singularity is resolved by quantum gravity;
ii) The wave function is sharply peaked about a mean metric, a smooth field (although coefficients involve $\hbar$).

- Answer: $\frac{S}{A_{\mathcal{B}}} < 0.244/\ell_{\rm Pl}^2$ (AA, Wilson-Ewing)

The bound is satisfied in LQC!

3. Contrasting LQC and WDW Quantum Cosmology

• Why was LQC able to resolve the Big Bang singularity when the WDW theory had failed in these models?

• In the WDW quantum cosmology, one did not have guidance from a full quantum gravity theory. Therefore, in quantum cosmology, one just followed standard QM and constructed the Schrödinger representation of the fundamental Weyl algebra.

• By contrast, quantum kinematics of LQG has been rigorously developed. Background independence $\Rightarrow$ unique representation of the kinematic algebra (Lewandowski, Okolow, Sahlmann, Thiemann; Fleishhack) Provides the arena to formulate quantum Einstein equations.

• In LQC we could mimic this framework step by step. One of the assumptions of the von Neumann uniqueness theorem for quantum mechanics is bypassed. In LQC we are led to a new presentation of the Weyl algebra, i.e., new quantum mechanics. WDW theory and LQC are distinct already kinematically!

Contrasting LQC and WDW Quantum Cosmology

• The LQC kinematics cannot support the WDW dynamics. The LQC dynamics is based on quantum geometry. The WDW differential equation is replaced by a difference equation.

$$C^+(v)\,\Psi(v+4,\,\phi)+C^o(v)\,\Psi(v,\phi)+C^-(v)\Psi(v-4,\,\phi)\;=\;\gamma\ell_P^2\;\hat{H}_\phi\Psi(v,\phi)\quad(\star)$$

• In quantum geometry, basic geometrical observables such as areas of physically defined surfaces and volumes of physically defined regions are quantized. The area operator has a smallest eigenvalue, the area gap Δ.

• It turns out that the step size in $(\star)$ is governed by the smallest eigenvalue of the area operator in LQG. Good agreement with the WDW equation at low curvatures but drastic departures in the Planck regime precisely because the WDW theory ignores quantum geometry.

Precise relation between LQC and the WDW Theory

Question analyzed in detail for the k=0 model. (Corichi, Singh, AA). Expect the answer to be the same for others.

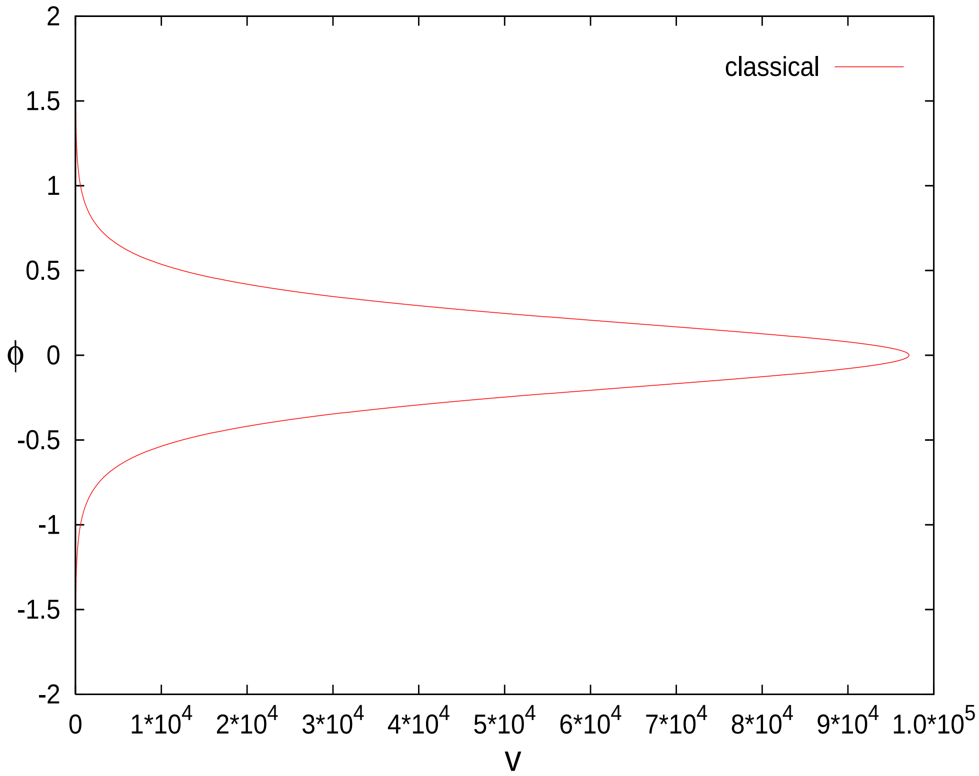
Start with the 'same physical state at time $\phi = \phi_o$' and evolve using LQC or WDW theory. Then:

- Certain predictions of LQC approach those of the WDW theory as the area gap λ goes to zero:
  Given a semi-infinite 'time' interval $\Delta\phi$ and $\epsilon > 0$, there exists a $\delta > 0$ such that $\forall\lambda < \delta$, 'physical predictions of the two theories are within ϵ of each other.'

- However, approximation is *not* uniform. The WDW theory is *not* the limit of sLQC:
  Given $N > 0$ however large, there exists a ϕ such that
  $\langle\hat{V}_\phi\rangle_{\rm sLQC} - \langle\hat{V}_\phi\rangle_{\rm WDW} > N$.
  LQC is *fundamentally* discrete.

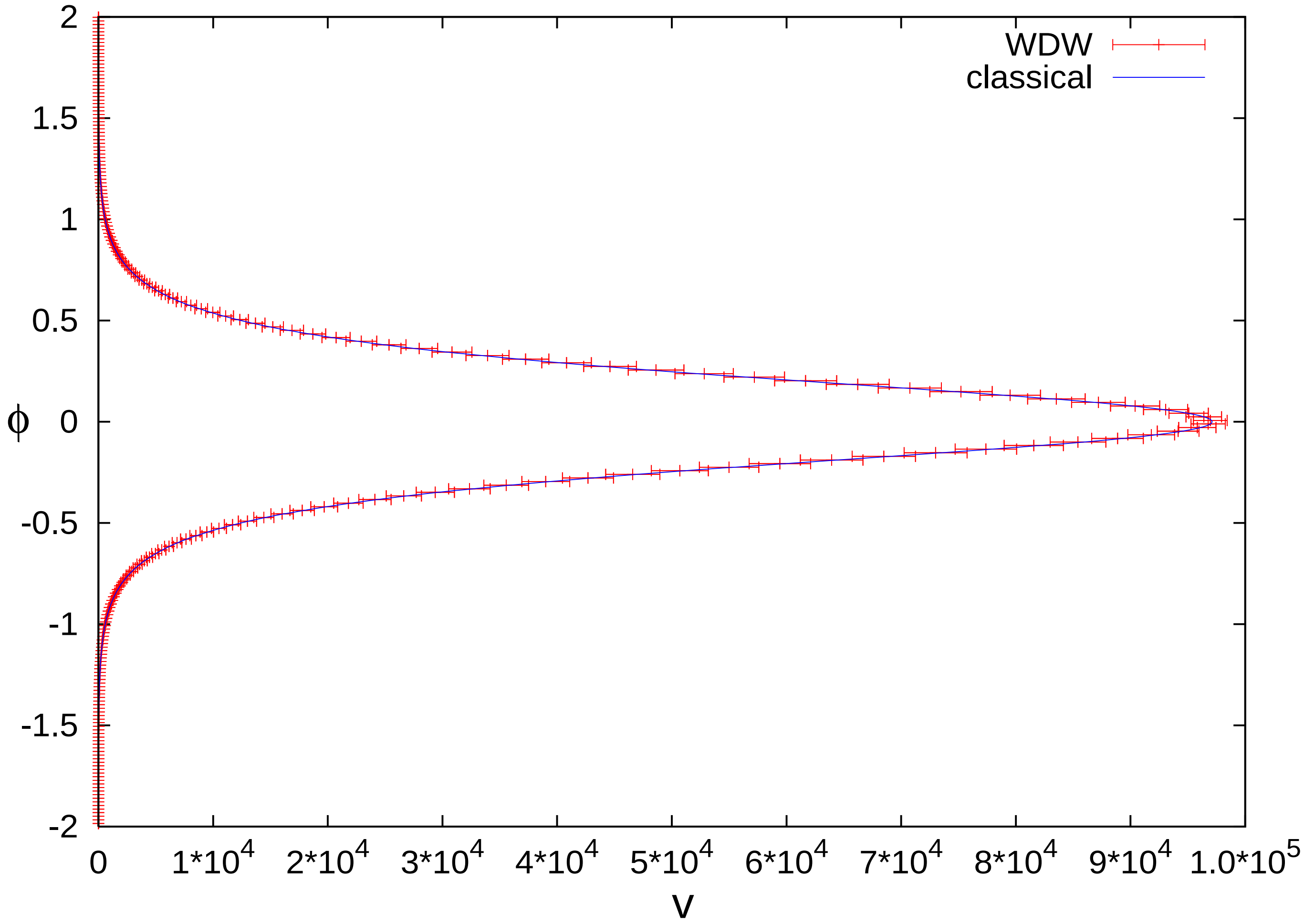
4. The k=1 Model

Another Example: $k = 1$ FRW model with a massless scalar field ϕ. Instructive because again every classical solution is singular; scale factor not a good global clock; More stringent tests because of the classical re-collapse. Provides a foundation for more complicated models.



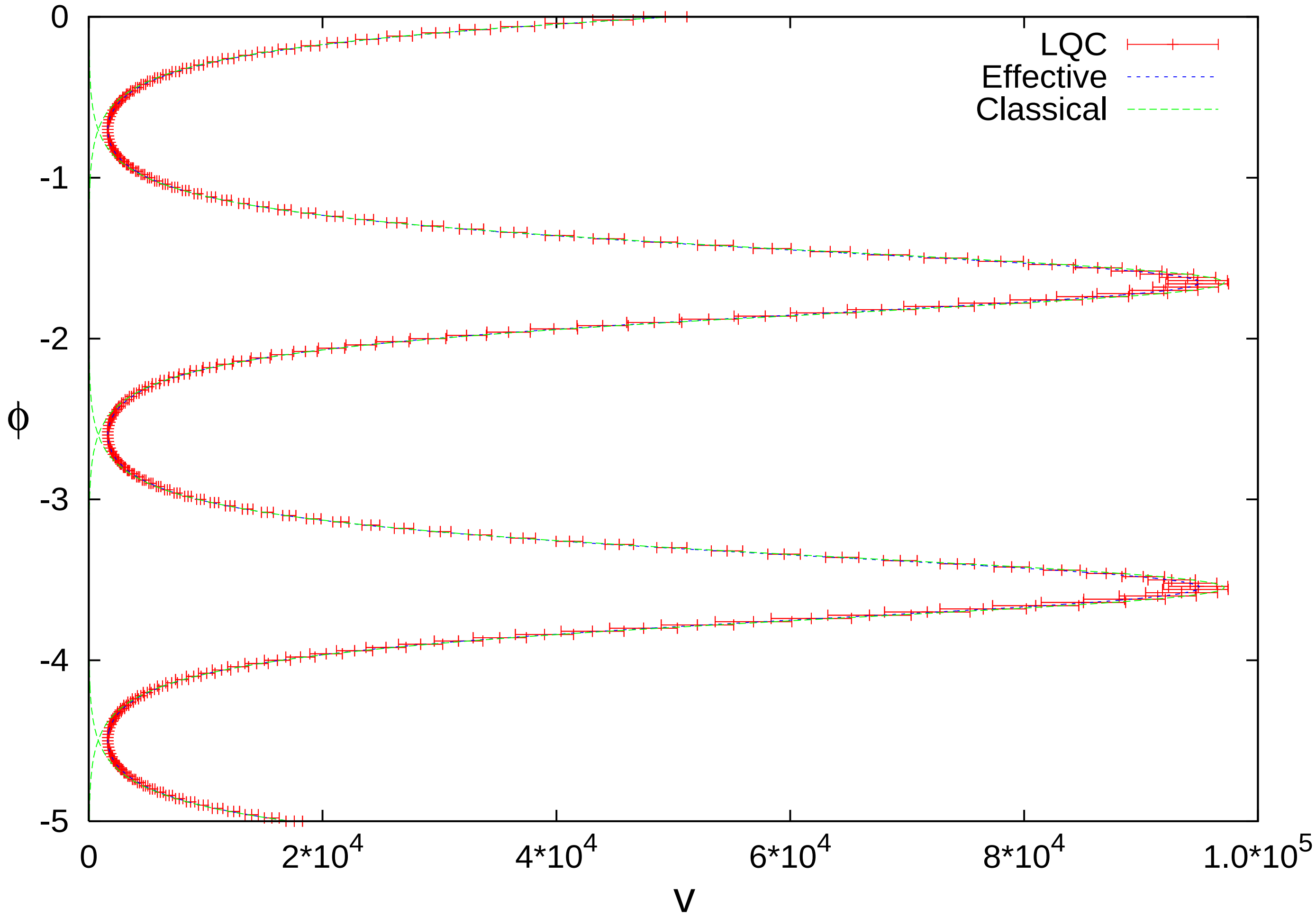
Classical trajectories

k=1 Model: WDW Theory



Expectations values and dispersions of $\hat{V}|_{\phi}$.

k=1 Model: LQC



Expectations values and dispersions of $\hat{V}|_{\phi}$ & classical trajectories.

(AA, Pawlowski, Singh, Vandersloot)

k=1: Domain of validity of classical GR

(AA, Pawlowski, Singh, Vandersloot)

- Classical Re-collapse: The infra-red issue.

$\rho_{\rm min} = (3/8\pi G a^2_{\rm max}) \left(1 + O(\ell^4_{\rm Pl}/a^4_{\rm max})\right)$

So, even for a very small universe, $a_{\rm max} \approx 23\ell_{\rm Pl}$, (i.e. $p_{(\phi)} = 5 \times 10^3\hbar$), agreement with the classical Friedmann formula to one part in 10^5. Classical GR an excellent approximation between $a \sim 8\ell_{\rm Pl}$ and $a \sim 23\ell_{\rm Pl}$. For macroscopic universes, LQC prediction on recollapse indistinguishable from the classical Friedmann formula.

- Quantum Bounces: The ultra-violet issue

For a universe which attains $v_{\rm max} \approx 1\,{\rm Mpc}^3$, $v_{\rm min} \approx 6 \times 10^{16}{\rm cm}^3 \approx 10^{115}\ell^3_{\rm Pl}$! What matters is curvature which enters Planck regime at this volume.

5. Summary

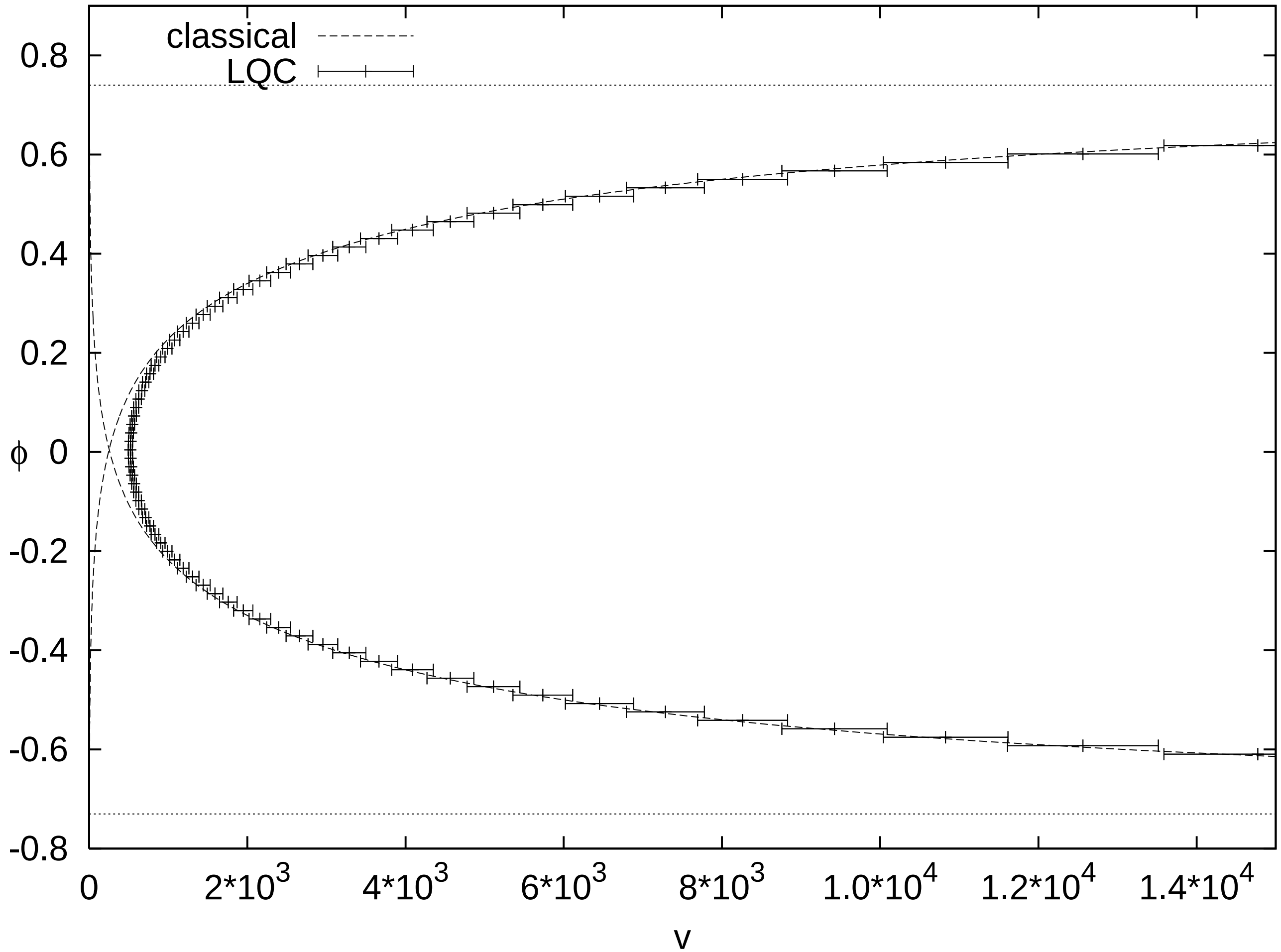
• Quantum geometry creates a brand new repulsive force in the Planck regime, replacing the big-bang by a quantum bounce. Repulsive force rises and dies *very* quickly but makes dramatic changes to classical dynamics. (Origin: Planck scale non-locality of quantum Einstein's equations.) Physics does not end at singularities.

• In $k = 1$ and $k = 0$ FRW models with or without Λ, complete control on the physical sector of the theory. LQC evolution deterministic across the classical big bang and big crunch for all quantum states. For the $k = 0$ model, $\hat{\rho}$ bounded above on the physical Hilbert space and $\rho_{\rm sup}$ attained arbitrarily closely by $\rho_{\rm boun}$ in semi-classical states.

• In Bianchi I models (Recall BKL!) numerics not as detailed. But main features the same, and again $\rho_{\rm sup}$. But there is a 'bounce' whenever a curvature invariant enters the Planck regime (Chiou, Vandersloot, AA).

• Challenge to background independent theories: Detailed recovery of classical GR at low curvatures/densities (Green and Unruh). Met in cosmological models.

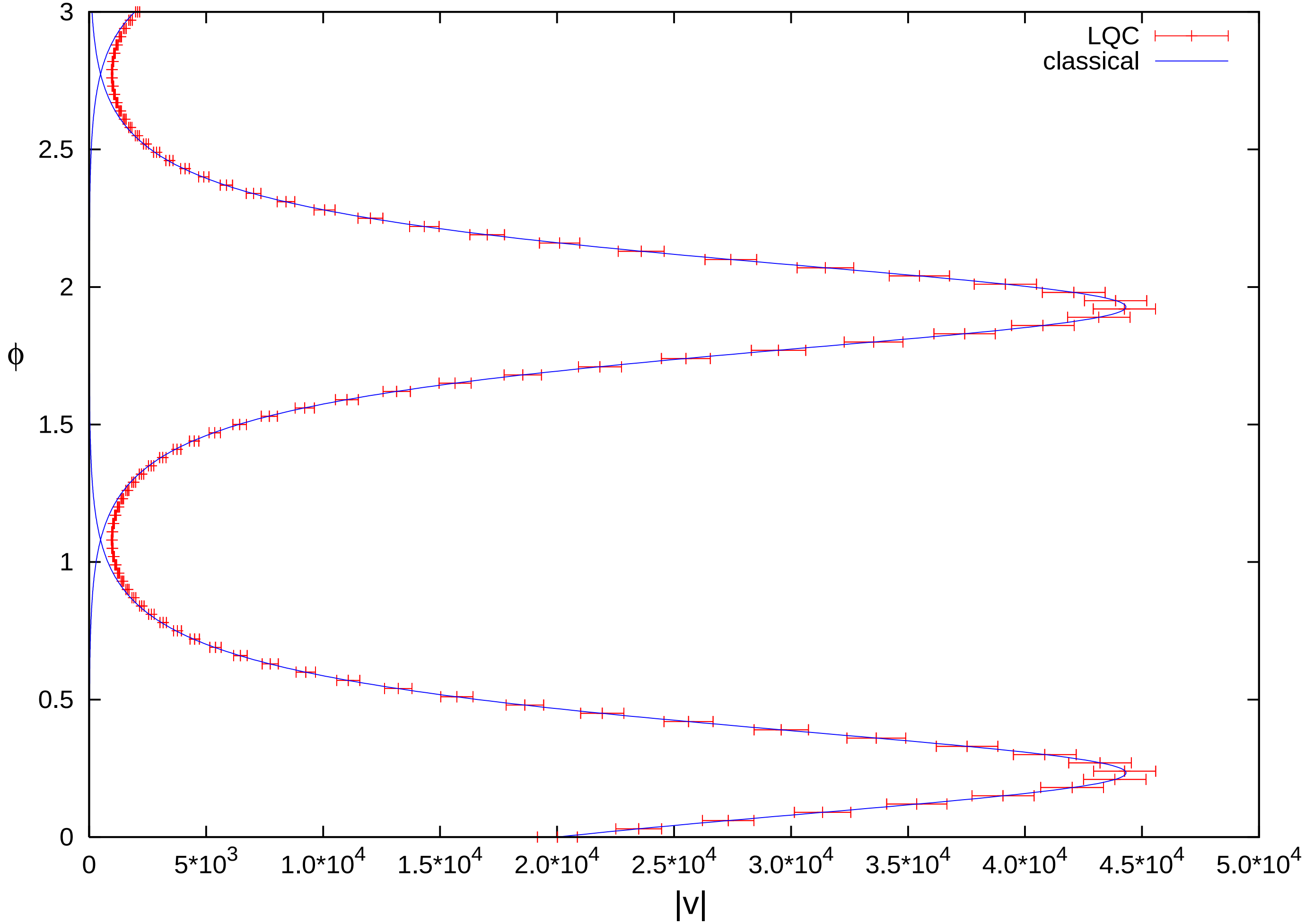
k=0 Model with Positive Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(AA, Pawlowski)

k=0 Model with Negative Λ



Expectations values and dispersions of $\hat{V}|_\phi$ & classical trajectories.

(Bentevigna, Pawlowski)

SUPPLEMENT: LQC Kinematics

- Recall the canonically conjugate variables of LQG:

A_a^i, SU(2) gravitational connections and, E_i^a, orthonormal triads.
Spatial homogeneity and isotropy implies

$\star$ $$A_a = c\underbrace{\mathring{\omega}_a^i \sigma_i}_{\text{fixed}}, \quad E^a = p\underbrace{\mathring{e}_i^a \sigma^i}_{\text{fixed}}$$

$\star$ $c \sim \dot{a}$

$\star$ holonomy: $h_e(c) = \cos\mu c\, \mathbf{1} + \sin\mu c\; \dot{e}^a \mathring{\omega}_a^i \sigma_i$
(Almost periodic in c)

$\star$ $|p| = a^2$.

$\star$ $p \to -p$ changes only the orientation of the triad.
Large gauge transformation; leaves physics invariant.

$\star$ Canonically conjugate pairs:
c, p for gravity ϕ, p_ϕ for matter

- Loop quantum cosmology:

Key strategy:
Do not naively set $\mathcal{H} = L^2(R, \mathrm{d}c)$ and $\hat{c}\Psi(c) = c\Psi(c)$; $\hat{p}\Psi(c) = -i\hbar\frac{\mathrm{d}\Psi}{\mathrm{d}c}$.
Rather, Follow full theory. $\hat{a}\Psi(a,\phi) = a\Psi(a,\phi)$ etc.

SUPPLEMENT: New Quantum Mechanics

- States: $\Psi(p) = \sum_i \Psi_i \, |p_i\rangle \qquad ||\Psi||^2 = \sum_i |\Psi_i|^2$

Note: $< p_i | p_j >= \delta_{ij}$ (Kronecker delta, not Dirac!)
Hilbert space: $\mathcal{H} = L^2(\bar{\mathcal{R}}_{\rm Bohr}, d\mu_{\rm Bohr}) \neq L^2(\mathcal{R}, dp)$

- In full LQG, Quantum configuration space is larger than the classical configuration space: $\mathcal{A} \longrightarrow \bar{\mathcal{A}}$.

Trickles down to the symmetry reduced sector: $\mathcal{R} \longrightarrow \bar{\mathcal{R}}_{\rm Bohr}$.

- Operators: $\hat{p}\Psi(p) = p\Psi(p)$ (self-adjoint);

$$\hat{h}_\mu \Psi(p) \equiv \widehat{\exp i\mu c}\, \Psi(p) = \Psi(p + \mu) \text{ (unitary)}$$

But no connection operator $\hat{c}$! Reason: $\hat{h}_\mu$ fails to be continuous in μ.

- Von-Neumann theorem bypassed. New Quantum Mechanics possible. Representation indeed inequivalent to Schrödinger's, i.e. to the WDW theory already kinematically. This kinematic structure mimics that of the full LQG.

SUPPLEMENT

How is the Hamiltonian constraint handled in LQC?

- Form of the constraint $C_{\rm H} \;\sim\; \underbrace{(\epsilon^{ij}{}_k E^a_i E^b_j/\sqrt{q})}_{\rm Thiemann} \quad \underbrace{F^k_{ab}}_{\rm holonomy}$

- Classically: $F^k_{ab} = -2 \lim_{{\rm Ar}_\Box \to 0} \left({\rm Tr}(h_{\Box_{ab}} - 1)\tau^k / {\rm Ar}_\Box \right)$

Quantum Theory: Limit does not exist because there is no local operator corresponding to the connection or curvature. Different from full LQG: Diff constraint handled by gauge fixing.

- LQC View (Bojowald, Lewandowski, AA): Quantum geometry $\Rightarrow$ should not shrink the loop to zero but only till the area enclosed ${\rm Ar}\Box$ w.r.t. the fiducial metric equals the lowest eigenvalue $\Delta = 2\sqrt{3}\pi\gamma\ell_{\rm Pl}^2$ of the area operator. So, the fundamental operator has Planck scale non-locality; Familiar local expression emerges only in the classical limit. (μ_o-Scheme)

- Singularity resolved. But the resulting quantum Hamiltonian constraint had a serious limitation: Predicted deviations from the classical theory even in certain ‘tame’ situations. (More later). Physically motivated, improved constraint remedies this drawback while retaining all desirable features.

SUPPLEMENT

- New idea (Pawlowski, Singh, AA): Do this with Physical area of □ (which is state dependent). The resulting operator mimics certain features of the full theory. Idea subtle to implement but important physical consequences: Overcomes problems with the older LQC dynamics. ($\bar{\mu}$-Scheme). (more later)

- Hamiltonian constraint: Use a representation in which geometry (i.e. $\hat{V} \sim \hat{a}^3$) and matter field (i.e., $\hat{\phi}$) are diagonal : $\Psi(v, \phi)$

Then the Wheeler DeWitt equation is replaced by a difference equation:

$$C^+(v)\,\Psi(v+4,\,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\Psi(v-4,\,\phi) \;=\; \hat{H}_\phi \Psi(v,\phi)$$

Fundamentally, a constraint equation. Selects physical states. However, this equation also dictates quantum dynamics.

- The 'lattice' has uniform spacing in $v \sim a^3$ (not p or μ which $\sim a^2$). Dynamics cannot be supported by a Vehlino type quantum kinematics.

SUPPLEMENT

How do you extract dynamics/physics from the 'frozen formalism'?

To extract physics, we need to:

- Isolate 'time' to give meaning to 'evolution'.
- Solutions to the constraint: Physical states. Introduce a physical inner product and suitable Dirac observables.
- Construct states which represent the actual universe at late time. 'Evolve back' towards the big bang.
- Is the classical singularity 'resolved'? In what sense? (Brunnemann and Thiemann) 'Wave function vanishes at the singularity' not enough; Physical inner product may be non-local. Need to analyze the behavior of the Dirac observables.
- What is on the 'other side' of the classical big-bang? (Quantum foam?? Another classical universe??)

SUPPLEMENT

- The quantum Hamiltonian constraint takes the form:

$$-\Theta\,\Psi(v,\phi) = \partial_\phi^2 \Psi(v,\phi) \qquad (\star)$$

where Θ is a positive, self-adjoint difference operator independent of ϕ :
$\Theta\,\Psi(v,\phi) = \; C^+(v)\,\Psi(v+4,\,\phi) + C^o(v)\,\Psi(v,\phi) + C^-(v)\,\Psi(v-4,\,\phi).$
Suggests ϕ could be used as 'emergent time' also in the quantum theory. Relational dynamics.

- Physical states: solutions to ($\star$), invariant under $v \to -v$. Observables: $\hat{p_\phi}$ and $\hat{V}|_{\phi=\phi_o}$. Inner product: Makes these self-adjoint or, equivalently, use group averaging. Analogy with KG equation in a static space-time. Semi-classical states: Generalized coherent states.

- Physical states:

$\Psi(v,\phi)$ satisfying $\quad -i\hbar\partial_\phi\Psi(v,\phi) = \sqrt{\Theta}\,\Psi(v,\phi)$

Dirac observables:

$$\hat{p}_{(\phi)}\Psi(v,\phi) = -i\hbar\partial_\phi\,\Psi(v,\phi) \equiv \sqrt{\Theta}\Psi(v,\phi)$$

$\hat{V}|_\phi\,\Psi(v,\phi) = e^{i\sqrt{\Theta}(\phi-\phi_o)}\,|v|\Psi(v,\phi_o)$. Similarly $\hat{\rho}|_\phi$.

SUPPLEMENT

What are the differences between the older, μ_o evolution of (Bojowald, Lewandowski, AA) and the $\bar{\mu}$ framework (Pawlowski, Singh, AA) in these models?

Differences are very significant with lessons for full LQG.

- In the k=0 model on $\mathbb{R}^3$, scale factor a refers to a fiducial metric: $q_{ab} = a^2(t)\, q^o_{ab}$. If $q^o_{ab} \to \alpha^2\, q^o_{ab}$, $a \to \alpha^{-1} a$. Physics should not depend on q^o_{ab} or the value of $a(t)$. (So, claims such as quantum effects are important for $a < a^\star$ in the older literature (based on the spectrum of $\widehat{1/V}$) are physically unsound.).

- Further, in this case *every* quantization requires an additional structure: An elementary Cell $\mathcal{C}$. We absorb factors of the volume V_o of $\mathcal{C}$ w.r.t. q^o_{ab} in the definition of canonical variables c, p so that the symplectic structure is independent of the q^o_{ab} choice. So, the classical Hamiltonian theory depends only $\mathcal{C}$ and not on q^o_{ab}. Same is true of quantum kinematics. Thus, e.g., $p^{3/2}$ is the physical volume of $\mathcal{C}$.

- i) In μ_o quantization, the Hamiltonian constraint operator depends on q_o^{ab} again. In the $\bar{\mu}$ quantization, it does not.

SUPPLEMENT

- ii) For each choice of $\mathcal{C}$ we get a quantum theory. In the μ_o evolution, the density at the bounce point goes as: $\rho_b \propto 1/p_\phi$. So, a Gaussian peaked at a classical phase space point can bounce with $\rho_b =$ density of water! Major departures from the classical theory also away from the bounce: in presence of a cosmological constant, large deviations occur when $\Lambda a^2 \geq 1$ although the space-time curvature is low. In $\bar{\mu}$ evolution, $\rho_b \approx 0.82\rho_{\rm pl}$ always. No departures from GR at low curvatures.

- iii) *Physical results* should be independent of the choice of $\mathcal{C}$. In $\bar{\mu}$ evolution they are. Not in the μ_o scheme. Ex: Given a classical solution $(a(t), \phi(t))$ when do quantum effects become important? Answer in the μ_o scheme depends on the choice of the cell! Answer not 'gauge invariant'. In the $\bar{\mu}$ scheme it is.

- Lessons:

a) LQC: Although it seems natural at first, detailed considerations show that the μ_o quantization of the Hamiltonian constraint is physically incorrect;
b) LQG: Whether a quantization of the Hamiltonian constraint has a 'good infra-red behavior' is likely to be very subtle.