

# Asymptotic Safety

(Martin Reuter)

(I) The Effective Average Action  
approach to quantum gravity  
and Asymptotic Safety

(II) The importance of "Background Independence"  
for Asymptotic Safety

(or: What is the physical meaning of  
a coarse graining scale when the  
metric is quantized? )

# Standard quantization of gravity $\hat{=}$

degrees of freedom

carried by :

$$g_{\mu\nu}(x)$$

bare action:

$$\int d^4x \sqrt{-g} R$$

calculational method:

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{8\pi G} h_{\mu\nu},$$

perturbative quantization, renormalization

What should be given up in order to arrive at a "fundamental" or "microscopic" quantum theory of gravity?

String Theory: d.o.f., action, calc. meth.

Loop Quantum Gravity: d.o.f., calc. meth.

Asymptotic Safety: calc. meth., action

## Asymptotic Safety Approach:

- ↪ degrees of freedom carried by  $\mathcal{G}_{\mu\nu}$
- ↪ quantization/renormalization is non-perturbative in an essential way
- ↪ bare action  $\Gamma_*$  is not an ad hoc assumption, but a prediction:

$$\Gamma_* \sim \int d^4x \sqrt{-g} \mathcal{R} + \text{"more"} \quad \text{is a}$$

non-Gaussian fixed point of the

( $\infty$ -dimensional, non-pert.) Wilsonian

renormalization group flow

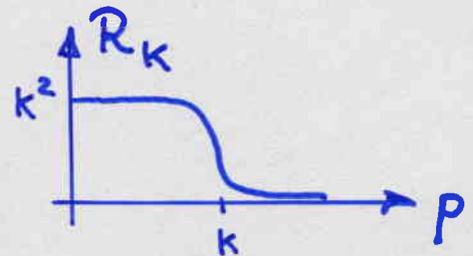
- ↪ fixed point "controls" UV divergences

# The Effective Average Action $\Gamma_k [g_{\mu\nu}, \dots]$

M.R. '96

- Scale-dependent (coarse grained) effective action functional for the metric
- Defines family of effective field theories:  
 $\{\Gamma_k \mid 0 \leq k < \infty\}$
- Built-in IR cutoff: Only metric fluctuations with cov. momentum  $p > k$  are integrated out fully.  
Modes with  $p < k$  are suppressed by "mass" term added to the bare action:

$$(\text{mass})^2 = R_k(p^2)$$



- $\Gamma_{k \rightarrow \infty} = S = \text{bare action}$
- $\Gamma_{k \rightarrow 0} = \Gamma = \text{standard eff. action}$
- $\Gamma_k$  satisfies a FRGE; symbolically:  
$$k \partial_k \Gamma_k = \frac{1}{2} \text{STr} \left[ (\Gamma_k^{(2)} + R_k)^{-1} k \partial_k R_k \right]$$
- Natural (nonperturbative) approximation scheme:  
project RG flow onto truncated theory space

# Construction of $\Gamma_k$ for Gravity

- starting point:  $\int \mathcal{D}\gamma_{\mu\nu} e^{-S[\gamma_{\mu\nu}]}$
- decompose  $\gamma_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu}$   
arbitrary  
backgrd. metric
- add background gauge fixing  $S_{gf}[h; \bar{g}]$  + ghost terms
- expand  $h_{\mu\nu}$  in  $\bar{D}^2$ -eigenmodes, and introduce IR cutoff  $k^2$ : only modes with generalized momenta ( $\bar{D}^2$ -eigenvalues)  $> k$  are integrated out.
- add sources: generating fctl.  $W_k[\text{sources}; \bar{g}]$

Legendre transf. ↓

$$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle$$

$$\Gamma_k[g_{\mu\nu}, \bar{g}_{\mu\nu}, \text{ghosts}]$$

- derive exact RG equation from path integral:

$$k \frac{\partial}{\partial k} \Gamma_k[g, \bar{g}, \dots] = \text{Tr}(\dots)$$

- "Ordinary" diffeomorphism invariant action:

$$\Gamma_k[g] = \Gamma_k[g, \bar{g}=g, \text{ghosts}=0]$$

## Taking the UV-limit in QEG

If there exists a non-Gaussian Fixed Point  $\Gamma_*$ ,  
 $\beta_i(\Gamma_*) = 0$ , Quantum Einstein Gravity is  
nonperturbatively renormalizable ("asymptotically safe").

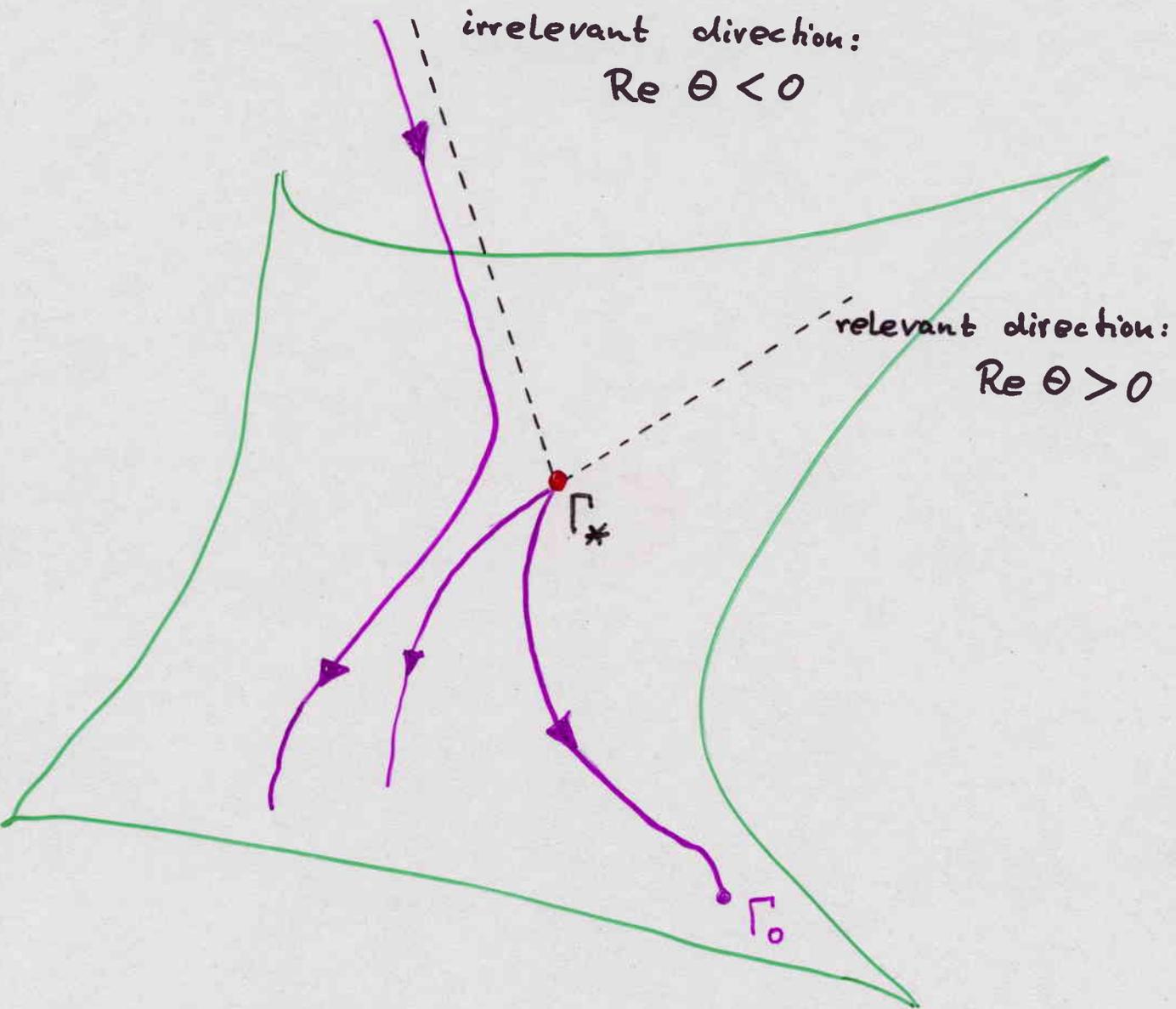
Weinberg 1979

Quantum theory is defined by a RG trajectory  
running inside the UV-critical hypersurface of  
the FP, with

initial point =  $\Gamma_{k \rightarrow \infty} \equiv S$  = action infinitesimally close  
to  $\Gamma_*$

end point =  $\Gamma_0 \equiv \Gamma$

# The UV-critical hypersurface $\mathcal{F}_{UV}$ :



$\Delta_{UV} \equiv \dim \mathcal{F}_{UV} = \# \text{ relevant directions}$   
 $= \# \text{ free parameters in the a.s. quantum field theory}$

UV  $\longrightarrow$  IR

$\Theta$ : critical exponent (neg. eigenvalue of lin. flow)

# Properties of QEG

- Background-independent quantization scheme:  
No special metric plays any distinguished role!

The background field method:

- a) Fix arbitrary  $\bar{g}_{\mu\nu}$
- b) Quantize (nonlinear) fluctuations  $h_{\mu\nu} \equiv \gamma_{\mu\nu} - \bar{g}_{\mu\nu}$   
in the backgrd. of  $\bar{g}_{\mu\nu}$
- c) Adjust  $\bar{g}_{\mu\nu}$  such that  $\langle h_{\mu\nu} \rangle = 0$   
 $\leadsto g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \bar{g}_{\mu\nu}$

- Fundamental action  $S \approx \Gamma_*$  is a prediction:  
No special action plays any distinguished role!

The only input: field contents + symmetries  
 $\hat{=}$  theory space

The output:  $\Gamma_* = S_{\text{Einstein-Hilbert}} + \text{"more"}$

Einstein-Hilbert action is often a reliable approximation,  
but not distinguished conceptually.

- Combination average action + background method successfully tested in QED and Yang-Mills theory.
- QEG reproduces successes of classical General Relativity:  $\exists$  trajectories with long classical regime ( $G = \text{const}$ ,  $\Lambda = \text{const}$ )
- QEG reproduces results of "QFT in curved spacetimes" in the classical regime:  
Hawking radiation, cosmological particle creation, ...
- Coexistence Asymptotic Safety  $\leftrightarrow$  perturbative non-renormalizability well understood;  
A.S. tested in models (Gross-Neveu, ...)
- Consistent quantization of gravity seems not to require "fine tuning" of matter system, special symmetries (SUSY, etc.), or unification with the other fundamental forces of Nature.

# The Einstein - Hilbert Truncation

(M.R., 1996)

ansatz:

$$\Lambda_k \equiv \bar{\lambda}_k$$

$$\Gamma_k = - \frac{1}{16\pi G_k} \int d^d x \sqrt{g} \{ R - 2\Lambda_k \}$$

+ classical gauge fixing and ghost terms

two running parameters:

Newton constant  $G_k$ , dimensionless:  $g(k) = k^{d-2} G_k$

cosmological constant  $\Lambda_k$ , dimensionless:  $\lambda(k) = \Lambda_k / k^2$

insert ansatz into flow equation, expand

$$\text{Tr} [\dots] = (\dots) \int \sqrt{g} + (\dots) \int \sqrt{g} R + \dots$$

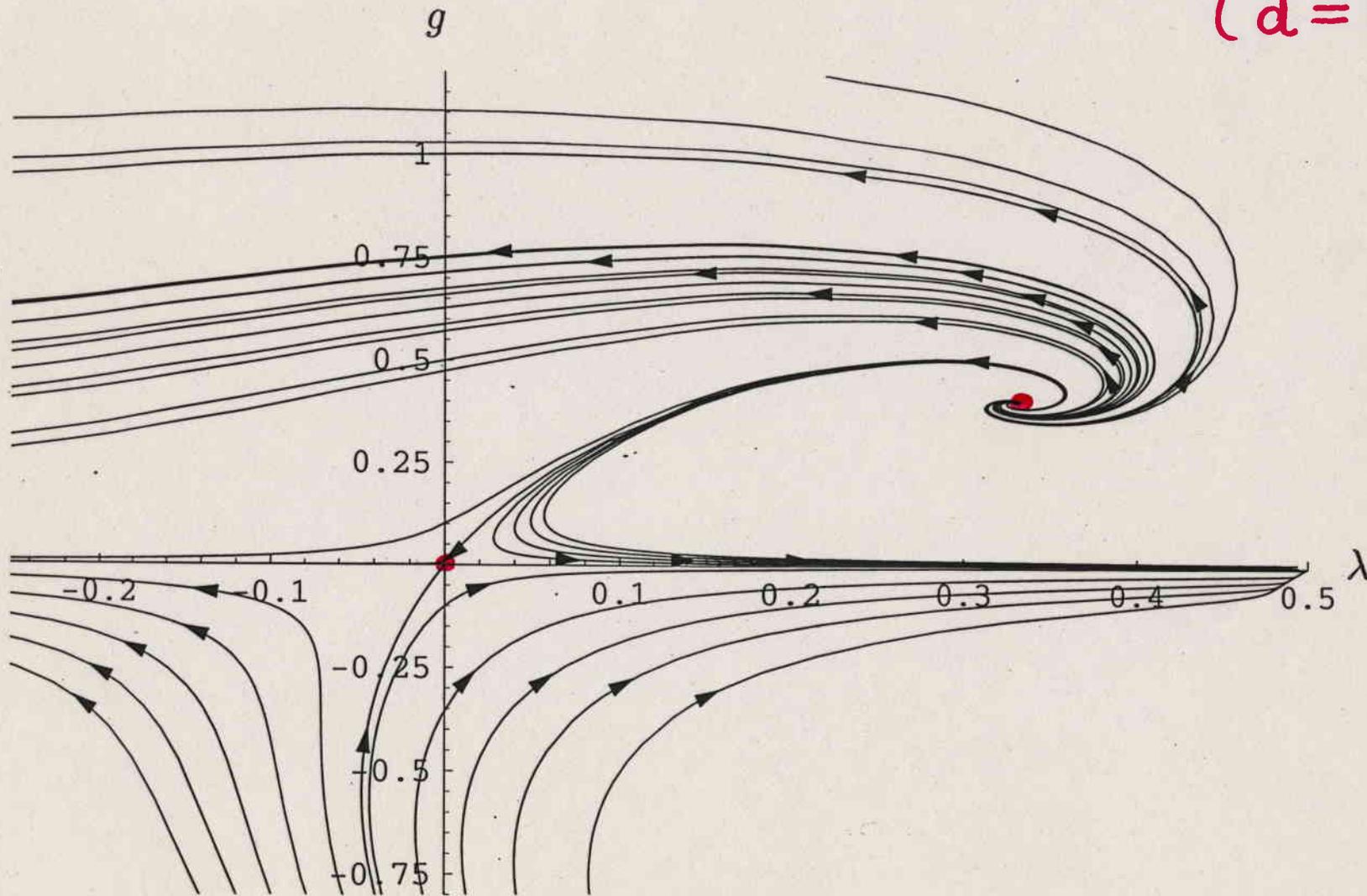


$$k \partial_k g(k) = \beta_g(g, \lambda)$$

$$k \partial_k \lambda(k) = \beta_\lambda(g, \lambda)$$

# RG-Flow in the Einstein-Hilbert Truncation

( $d=4$ )



# Conformally Reduced QEG

(M.R., H. Weyer, 2008)

- Simplified version of full QEG:  
only the conformal factor is quantized
- The same approach as in full QEG is used:  
effective average action,  
background field method
- Disentangles conceptual / technical problems
- Illustrates importance of "background independence" for the RG flow:  
scalar-like theory, but with RG behavior  
very different from that of a scalar matter  
field on a rigid spacetime
- Disentangles role of backgrd. field method  
for gauge invariance / "backgrd. independence"
- Has the same qualitative features as full QEG  
→ play ground for gaining new  
conceptual insights

# Conformally Reduced QEG

- quantize only the conformal factor:

$$\underbrace{\gamma_{\mu\nu}}_{\text{integration variable}} = \chi^2 \underbrace{\hat{g}_{\mu\nu}}_{\text{class. reference metric, } \neq \text{ backgrd. metric!}}$$

- treat scalar-like theory  $\int \mathcal{D}\chi e^{-S[\chi]}$  in the same way as full QEG:  
effective average action  $\oplus$  backgrd. field method

- introduce background conf. factor:

$$\bar{g}_{\mu\nu} = \chi_B^2 \hat{g}_{\mu\nu}$$

- decompose quantum field:

$$\chi = \chi_B + \underbrace{f}_{\text{"fluctuation"}}$$

- expectation values:

$$\phi \equiv \langle \chi \rangle = \chi_B + \bar{f}, \quad \bar{f} \equiv \langle f \rangle$$

$$g_{\mu\nu} \equiv \langle \gamma_{\mu\nu} \rangle = \langle \chi^2 \rangle \hat{g}_{\mu\nu} = \langle (\chi_B + f)^2 \rangle \hat{g}_{\mu\nu}$$

## The innocent first steps:

- define, formally,  $e^{W_k[J; \chi_B]} =$   
$$= \int \mathcal{D}f e^{-S[\chi_B + f] - \Delta_k S[f; \chi_B] + \int d^4x \sqrt{\hat{g}} J f}$$
  
with  $\Delta_k S[f; \chi_B] = \frac{1}{2} \int d^4x \sqrt{\hat{g}} f(x) \mathcal{R}_k[\chi_B] f(x)$
- define  $\bar{f} \equiv \langle f \rangle_k = \frac{1}{\sqrt{\hat{g}}} \frac{\delta W_k}{\delta J}$   
 $\rightsquigarrow J = \mathcal{J}_k[\bar{f}; \chi_B]$
- define effective average action:

$$\begin{aligned} \Gamma_k[\bar{f}; \chi_B] &= \int d^4x \sqrt{\hat{g}} \bar{f} \mathcal{J}_k - W_k[\mathcal{J}_k; \chi_B] - \Delta_k S[\bar{f}; \chi_B] \\ &\equiv \Gamma_k[\Phi, \chi_B] \quad \Phi \equiv \chi_B + \bar{f} \end{aligned}$$

- derive FRGE:

$$\begin{aligned} &k \partial_k \Gamma_k[\bar{f}; \chi_B] \\ &= \frac{1}{2} \text{Tr} \left[ \left( \Gamma_k^{(2)}[\bar{f}; \chi_B] + \mathcal{R}_k[\chi_B] \right)^{-1} k \partial_k \mathcal{R}_k[\chi_B] \right] \end{aligned}$$

## Truncations employed :

- Conformally Reduced Einstein-Hilbert ("CREH") truncation:

$$\begin{aligned}\Gamma_k[\bar{f}; \chi_B] &\equiv \Gamma_k[\phi, \chi_B] \\ &= -\frac{1}{16\pi G_k} \int d^4x \sqrt{g} (R(g) - 2\Lambda_k) \Big|_{g_{\mu\nu} = \phi^2 \hat{g}_{\mu\nu}} \\ &= \frac{3}{4\pi G_k} \int d^4x \sqrt{\hat{g}} \left\{ \frac{1}{2} \phi \hat{\square} \phi - \frac{1}{12} \hat{R} \phi^2 + \frac{1}{6} \Lambda_k \phi^4 \right\}\end{aligned}$$

depends only on the combination  $\phi \equiv \chi_B + \bar{f}$  ;

2-dimensional theory space:  $\{g, \lambda\}$

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- Local Potential Approximation (LPA):

$$\Gamma_k[\phi, \chi_B] = \frac{3}{4\pi G_k} \int d^4x \sqrt{\hat{g}} \left\{ \frac{1}{2} \phi \hat{\square} \phi - \mathcal{F}_k(\phi) \right\}$$

infinite dimensional theory space:

$$\{G, \mathcal{F}(\cdot)\} \sim \{g, \Upsilon(\cdot)\}$$

## Project flow on truncated theory space:

$$k \partial_k \Gamma_k[\bar{f}; \chi_B]$$

$$= \frac{3}{4\pi} \int d^4x \sqrt{\hat{g}} \left\{ \frac{1}{2} k \partial_k \left( \frac{1}{G_k} \right) \phi \hat{\square} \phi - k \partial_k \left( \frac{F_k(\phi)}{G_k} \right) \right\}$$

$\phi = \chi_B + \bar{f}$

$$= \frac{1}{2} \text{Tr} \left[ \left( \dots \frac{\delta^2}{\delta \bar{f}^2} \Gamma_k[\bar{f}; \chi_B] + \dots \right)^{-1} \dots \right]$$

- perform  $\bar{f}$ -derivatives ( $\bar{f}$  arb.)
- set  $\bar{f} = 0$ ,  $\chi_B = \phi$
- equate coeffs. of equal terms with 0 and 2 deriv.'s

$$\Rightarrow \beta_G, \beta_F$$

$$\Gamma_k^{(2)} \Big|_{\bar{f}=0} = - \frac{3}{4\pi G_k} \left[ - \hat{\square} + F_k''(\phi) \right]$$

## Constructing $\mathcal{R}_k$ :

"backgrd. independence" vs. rigid backgrd.

- require coarse graining scale  $k^{-1}$  of  $\Gamma_k[\bar{g}]$  to be a proper (rather than coordinate) length
- "proper" w.r.t. which metric?
- "backgrd. independence"  $\Rightarrow k^{-1}$  can be proper only w.r.t. metric given by the arguments  $[\bar{g}]$ , but not w.r.t. any rigid metric (such as  $\hat{g}_{\mu\nu}$  in CR-QEG)
- Our choice:  $k^{-1}$  is proper w.r.t.  $\bar{g}_{\mu\nu}$

more precisely:

$-k^2$  is a cutoff in the spectrum of

$$\bar{\square} \equiv (\mathcal{D}^\mu \mathcal{D}_\mu)(\bar{g})$$

$\Rightarrow$  Typical structures (periods, ...) of  $\bar{\square}$ -eigenfunction with eigenvalue  $-k^2$  have  $\bar{g}$ -proper size of the order  $k^{-1}$ .

$\Rightarrow \Gamma_k \hat{=} \text{"effective field theory valid near } k \text{"}$

Cf. rigid backgrd.:  $-k^2$  cutoff in  $\hat{\square}$ -spectrum

## Implementation (LPA):

- $\mathcal{R}_k$  must be such that  $\Gamma_k^{(2)} \rightarrow \Gamma_k^{(2)} + \mathcal{R}_k$  entails the replacement

$$(-\bar{\square}) \rightarrow (-\bar{\square}) + k^2 \mathcal{R}^{(0)} \left( \frac{-\bar{\square}}{k^2} \right)$$

$\underbrace{\hspace{10em}}_{= \begin{cases} 0 & \text{if } -\bar{\square} \gg k^2 \\ 1 & \text{if } -\bar{\square} \ll k^2 \end{cases}}$

- Since  $\bar{\square} = \chi_B^{-2} \hat{\square}$  when  $\bar{g}_{\mu\nu} = \chi_B^2 \hat{g}_{\mu\nu}$  with  $\chi_B = \text{const}$ , this is equivalent to:

$$(-\hat{\square}) \rightarrow (-\hat{\square}) + \chi_B^2 k^2 \mathcal{R}^{(0)} \left( \frac{-\hat{\square}}{\chi_B^2 k^2} \right)$$

absent when  $k$  is proper w.r.t.  $\hat{g}_{\mu\nu}$  !

- "Backgrd. independent" choice of  $\mathcal{R}_k$  contains additional factors of  $\chi_B$  compared to standard quantization of scalar matter field on rigid backgrd.:

$$\mathcal{R}_k = -\frac{3}{4\pi G_k} \chi_B^2 k^2 \mathcal{R}^{(0)} \left( \frac{-\hat{\square}}{\chi_B^2 k^2} \right)$$

# Flow equations and $\beta$ -functions

$$Y_k(\varphi) \equiv k^2 F_k\left(\frac{\varphi}{k}\right), \quad \varphi \equiv k\phi \quad \text{dim. less}$$

$$g_k \equiv k^2 G_k, \quad \lambda_k \equiv \Lambda_k / k^2$$

$$k \partial_k g_k = [2 + \eta_N(g_k, [Y_k])] g_k$$

$$k \partial_k Y_k(\varphi) = (2 + \eta_N) Y_k - \varphi Y_k'$$

$$- \frac{g_k}{24\pi} \left(1 - \frac{1}{6} \eta_N\right) \frac{\varphi^6}{\varphi^2 + Y_k''(\varphi)}$$

anomalous dimension:

$$\eta_N(g, [Y]) = - \frac{g}{24\pi} \frac{[\varphi_i^3 Y'''(\varphi_i)]^2}{[\varphi_i^2 + Y''(\varphi_i)]^4}$$

( $\mathbb{R}^4$  topology)

$\varphi_i$ : normalization point ( $\varphi_i \rightarrow \infty$ ).

Compare to standard scalar on rigid backgrd.:

$$\frac{\phi^6 k^6}{\phi^2 k^2 + F_k''(\phi)} \longrightarrow \frac{k^6}{k^2 + F_k''(\phi)}$$

$\beta_Y$  for any topology:

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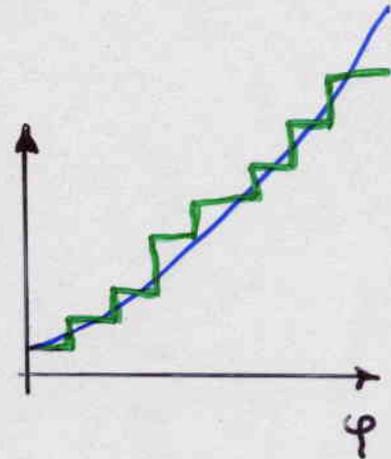
$$k \partial_k Y_k(\varphi) = (2 + \gamma_N) Y_k - \varphi Y_k'$$

$$- \frac{g_k}{4\pi} \frac{(2 - \gamma_N) \varphi^2 \mathcal{S}(\varphi) + \gamma_N \tilde{\mathcal{S}}(\varphi)}{\varphi^2 + Y_k''(\varphi)}$$

$$\begin{cases} \mathcal{S}(\varphi) \equiv \text{Tr} [\Theta(\varphi^2 + \hat{\alpha})] \\ \tilde{\mathcal{S}}(\varphi) \equiv \text{Tr} [(-\hat{\alpha}) \Theta(\varphi^2 + \hat{\alpha})] \end{cases}$$

Example:  $S^4$

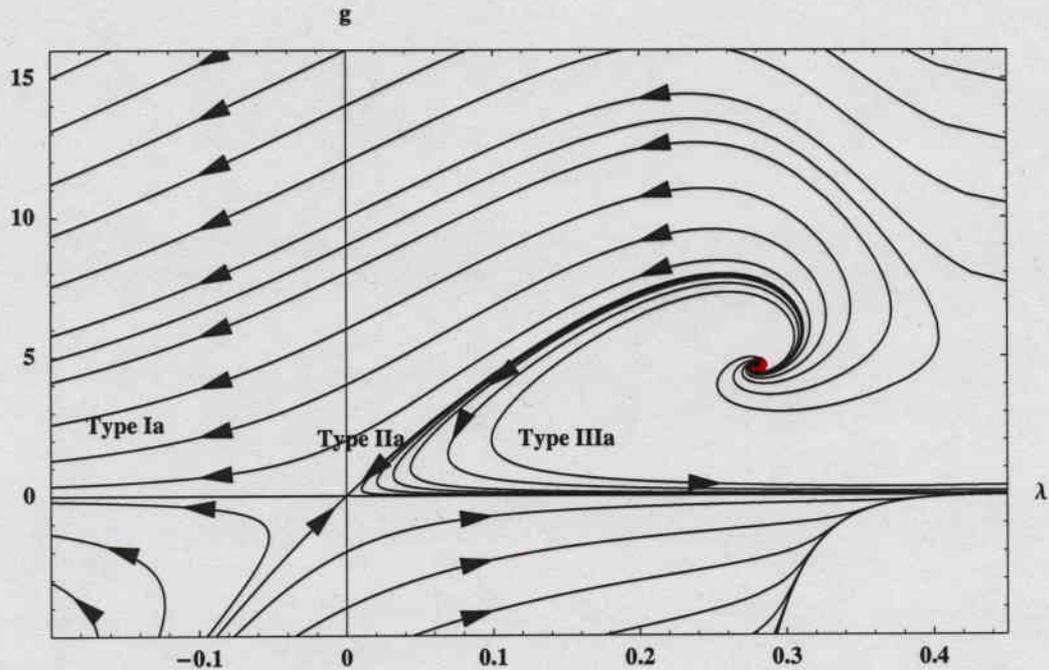
$$\begin{cases} \mathcal{S}(\varphi) = \sum_{n=0}^{\infty} D_n \Theta(\varphi^2 - \mathcal{E}_n) \\ \tilde{\mathcal{S}}(\varphi) = \sum_{n=0}^{\infty} \mathcal{E}_n D_n \Theta(\varphi^2 - \mathcal{E}_n) \end{cases}$$



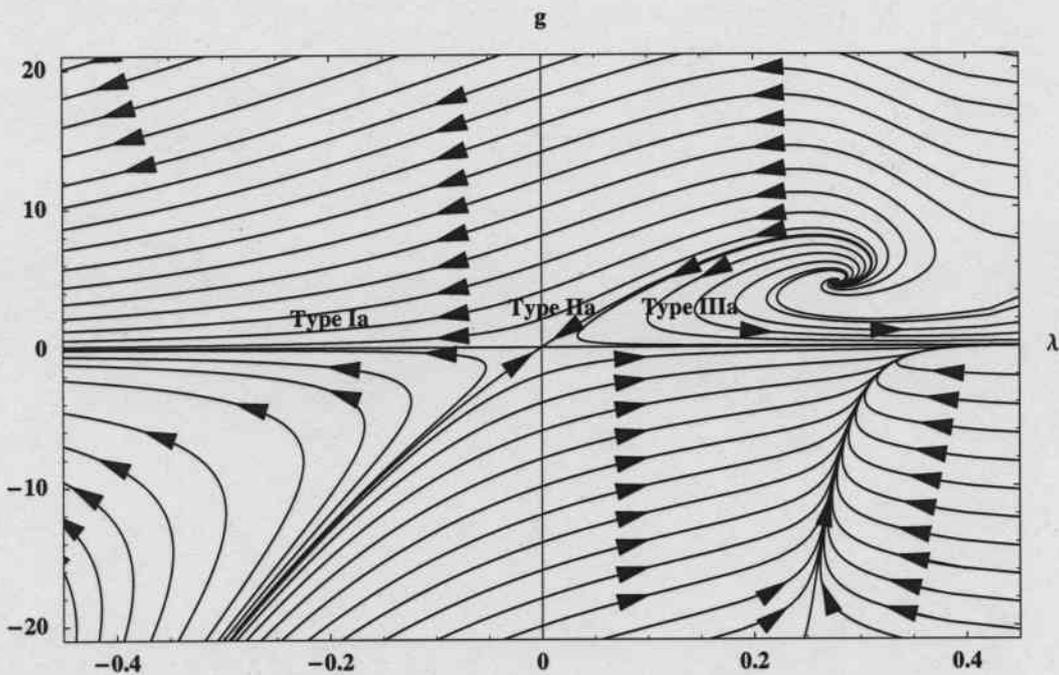
$$\mathcal{E}_n \equiv n(n+3)$$

$$D_n \equiv \frac{1}{6} (n+1)(n+2)(2n+3)$$

The CREH flow:  $Y_k(\varphi) = -\frac{1}{6} \lambda_k \varphi^4$



(a)



(b)

Figure 1: The figures show the RG flow on the  $(g, \lambda)$ -plane which is obtained from the CREH truncation with  $\eta_N^{(\text{kin})}$ . The arrows point in the direction of decreasing  $k$ .

M.R., H.Weyer, arXiv: 0801.3287

Example: Running of the cosmological constant near the GFP

Obtains from interaction term  $\Lambda_k \phi^4$

● rigid background quantization:

standard  $\phi^4$ -theory  $\Rightarrow$   $\Lambda_k \sim \log(k)$

Polyakov (2001)

Jackiw et al. (2005)

● "background independent" quantization:

$$\Lambda_k \sim G_0 k^4$$

consistent with full QEG

and sum over zero-point energy approach

# Gaussian Fixed Point (LPA)

$$g_*^{\text{GFP}} = 0, \quad Y_*^{\text{GFP}}(\varphi) = c \varphi^2$$

Linearization:

$$\delta g_k \sim y_g k^{-\Theta}$$

$$\delta Y_k(\varphi) \sim r(\varphi) k^{-\Theta}$$

Scaling fields  $(y_g, r(\cdot))$  and dimensions  $\Theta$ :

(i)  $\exists$  scaling field  $(y_g \neq 0, r = \hat{r})$  with  $\Theta = -2$  ;

$$\hat{r} = -\frac{y_g}{96\pi} \varphi^4 + o(\varphi^3)$$

(ii)  $\forall \Theta \in \mathbb{R} \exists$  scaling field  $(y_g = 0, r = \varphi^{2+\Theta})$  ,

i.e.  $\varphi^n$  has dimension  $\Theta = n - 2$  !

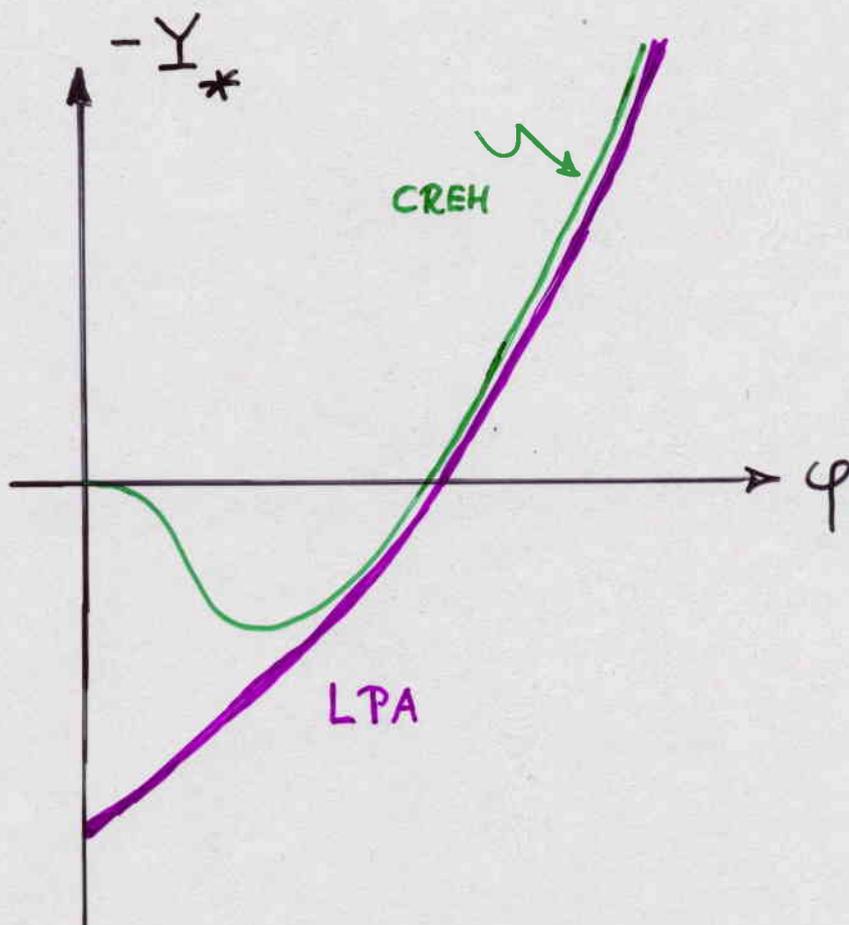
cf. standard scalar:  $\Theta = n - 4$

$\Theta$	...	-3	-2	-1	0	+1	+2	+3	...
		irrelevant			marg.	relevant			
$y_g = 0, r =$	...	$\varphi^{-1}$	$\varphi^0$	$\varphi^1$	$\varphi^2$	$\varphi^3$	$\varphi^4$	$\varphi^5$	...
$y_g \neq 0, r =$			$\hat{r}$						

# Non - Gaussian Fixed Point (LPA)

$$\left\{ \begin{array}{l} g_*^{\text{NGFP}} = g_*^{\text{CREH}} \\ \Upsilon_*^{\text{NGFP}}(\varphi) = y_* - \frac{1}{6} \lambda_*^{\text{CREH}} \varphi^4 \quad (\mathbb{R}^4) \end{array} \right.$$

Numerical solution for  $S^4$  topology :



Corresponds to non-trivial fixed points of infinitely many couplings !

## Scaling fields / dimensions ( $\mathbb{R}^4$ ) :

(i)  $\exists$  two scaling fields ( $g_g \neq 0, \gamma = \varphi^4$ )  
with  $\Theta = \Theta^{CREH}$ .

(ii)  $\exists$  infinitely many scaling fields ( $g_g = 0, \gamma$ ) :

- $\gamma = \varphi^n, n \in \mathbb{C}, \Theta(n) = (1+d)n - \alpha n^2 \in \mathbb{C}$

- $\gamma = \varphi^{\frac{1+d}{2}}, \varphi^{\frac{1+d}{2}} \ln \varphi, \Theta = (1+d)^2 / 4\alpha \quad \alpha \equiv \frac{2^{1/3}}{3}$

Eigenvalue problem well defined if  $\text{Re } n \leq 4$

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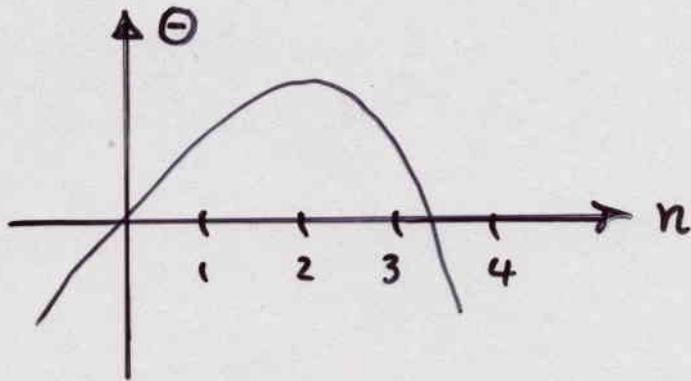
- Number of relevant / irrelevant directions depends on the precise definition of theory space, i.e. of  $\{\Upsilon(\cdot)\}$  and its tangent space at the N&FP,  $\{\Upsilon'(\cdot)\}$ .

- Choice of  $\{\Upsilon(\cdot)\}$  determines set of allowed

$$n \equiv n' + i n'' \longleftrightarrow \Theta \equiv \Theta' + i \Theta''$$

- Example:  $n \in \mathbb{Z}$  or  $n \in \mathbb{N}$

$\leadsto \Theta$  real



$\varphi, \varphi^2, \varphi^3$  relevant,  $\varphi^0$  marginal,  
all other  $\varphi^n$  irrelevant

- Example:  $n \in \mathbb{C}$

infinitely many relevant and irrelevant directions;  
generic scaling field is non-polynomial oscillatory  
function of  $\varphi$ :

$$\text{Re } \delta Y_k(\varphi) \sim \varphi^{n'} e^{-\Theta' t} \cos(n'' \ln \varphi - \Theta'' t)$$

Similar to Halpern-Huang directions of  
standard scalar at its GFP.

- Correct / optimal choice of theory space  
unknown:

should mimic full QEG as well as possible

# Phase Transitions to a new phase

of gravity: Unbroken Diffeomorphism Invariance

- Solve full nonlinear PDE for  $\Upsilon_k$  numerically, search for trajectories inside  $\mathcal{L}_{UV}$ .
- Global minimum  $\phi_0(k) \equiv k \varphi_0(k)$  of  $F_k(\phi) \sim -\Upsilon_k(\varphi)$  determines expectation value

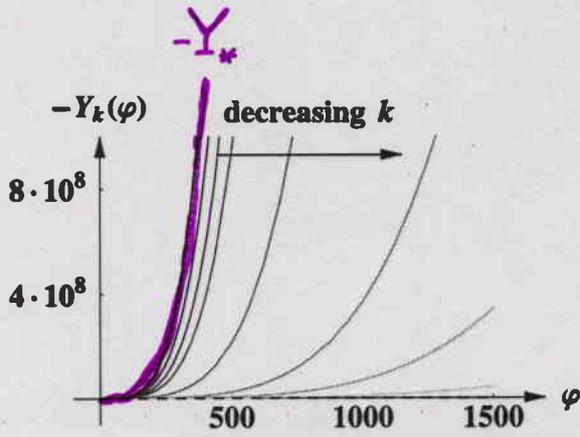
$$\langle \gamma_{\mu\nu} \rangle \equiv \langle g_{\mu\nu} \rangle_k = \phi_0^2(k) \hat{g}_{\mu\nu}$$

- $\phi_0 = 0$ : phase with vanishing exp. val. of the metric (vielbein)
- $\phi_0 \neq 0$ : exp. val.  $\neq 0$ , spontaneously breaks group of diffeo.'s to stability group of  $\langle g_{\mu\nu} \rangle_k$

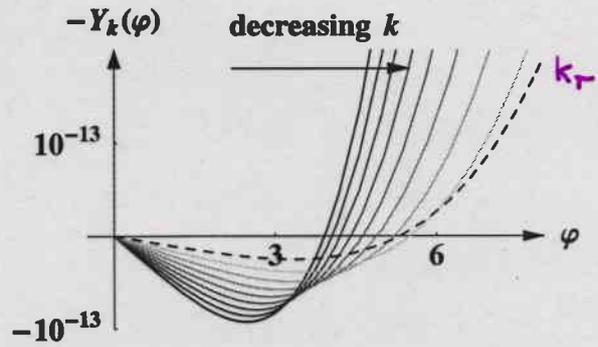
## Forms of phase transitions (w.r.t. scale $k$ ):

- "1<sup>st</sup> order"  $\longleftrightarrow$  "2<sup>nd</sup> order"  
( $\varphi_0$  discontinuous) ( $\varphi_0$  continuous)
- at  $k = \infty$   $\longleftrightarrow$  at  $k = k_c < \infty$

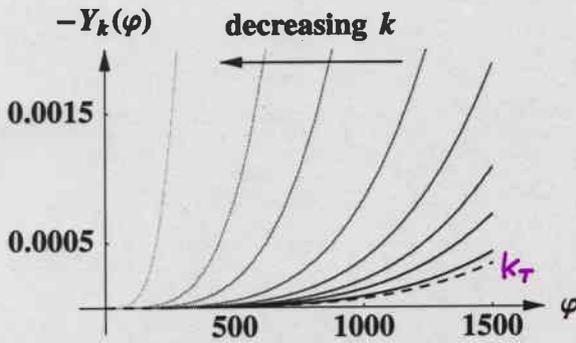
2<sup>nd</sup> order transition at  $k = \infty$  ( $R^4$ )



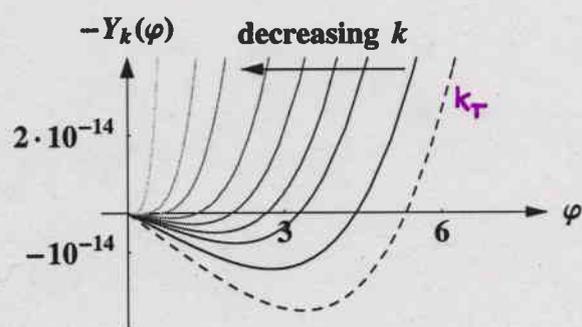
(a)



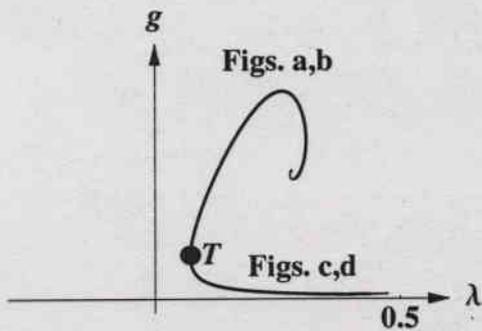
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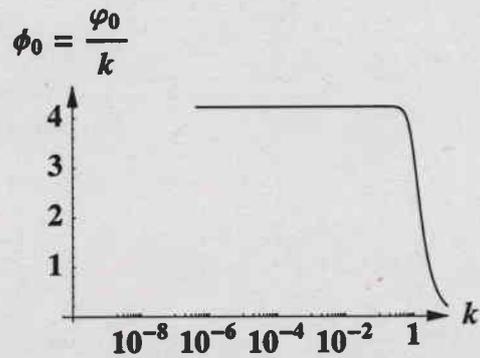
(c)



(d)



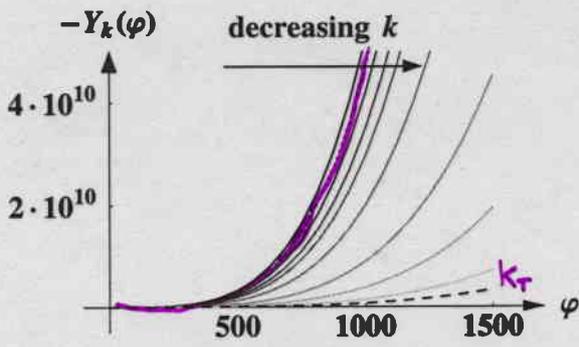
(e)



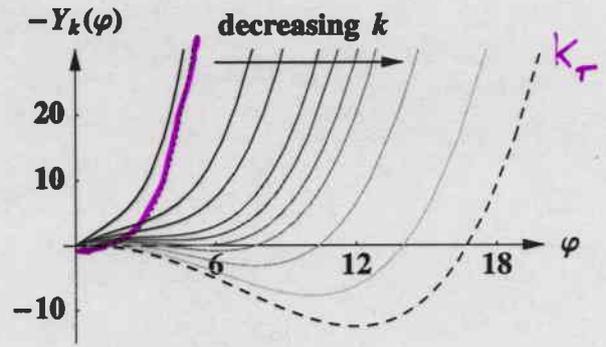
(f)

classical spacetime emerges:  
 $\phi_0 \approx \text{const} \neq 0$

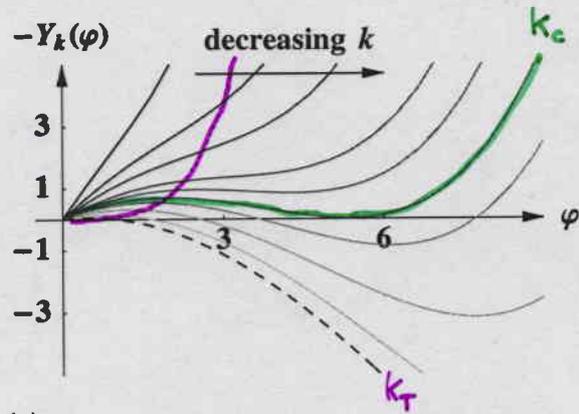
1<sup>st</sup> order transition at  $k_c < \infty$  ( $\mathbb{R}^4$ )



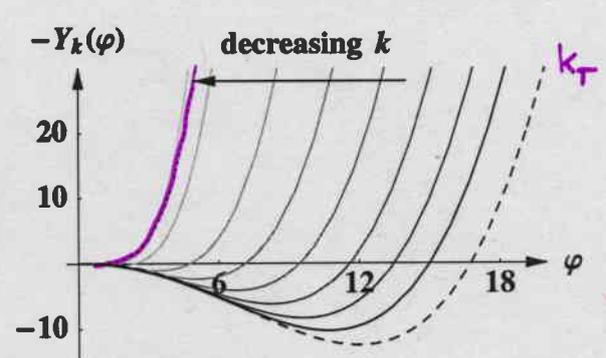
(a)



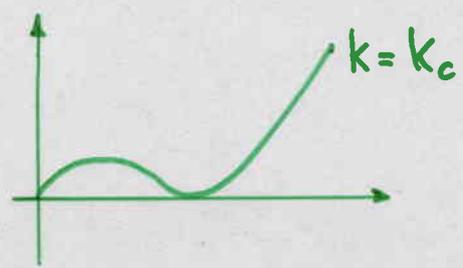
(b)



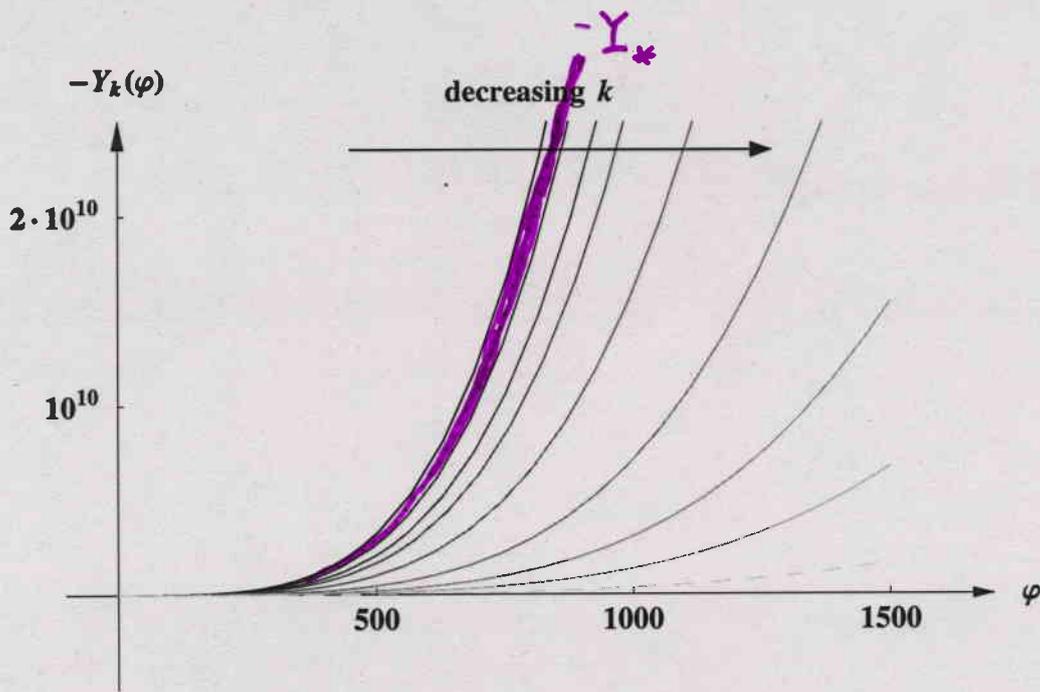
(c)



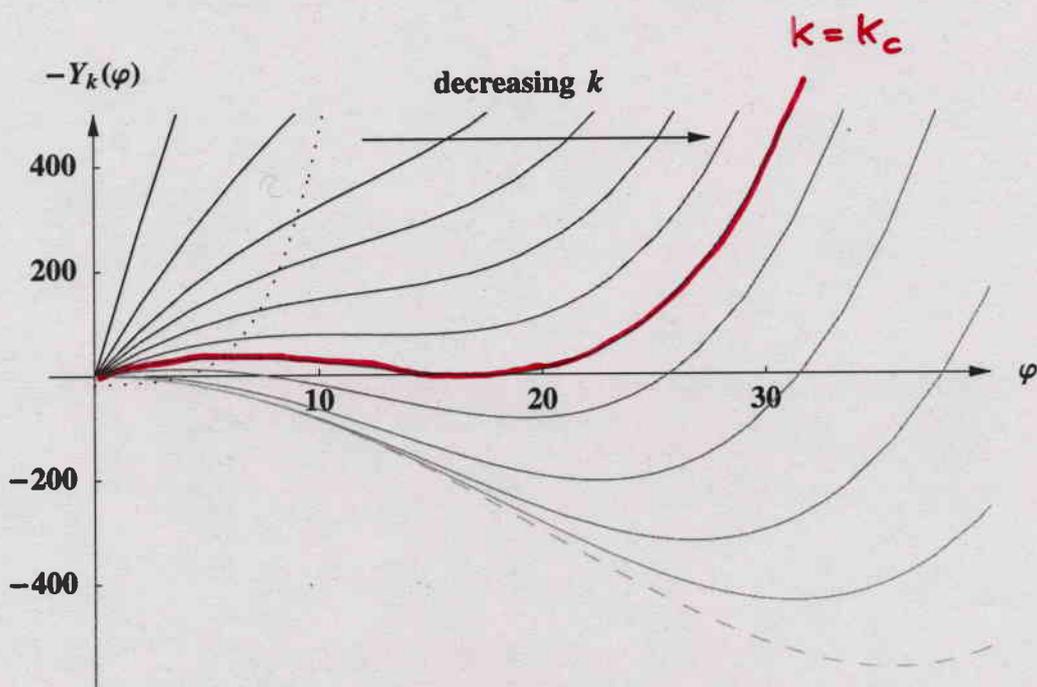
(d)



1<sup>st</sup> order transition at  $k_c < \infty$  ( $S^4$ )



(a)



(b)

# Summary

- "Backgrd. independence" has a crucial impact on the RG flow of the eff. average action:

■ rigid backgrd: standard  $\phi^4$  theory  
(asymptotically free: Symanzik '73)

■ "backgrd. indep.": NGFP forms  $\Rightarrow$  A.S.

- RG flow due to the conformal factor is typical of the full set of metric degrees of freedom:

"backgrd. indep." seems to be more important to A.S. than spin-2 excitations, their complicated self-interactions, etc. !