Loop Quantum Gravity

Reduced Phase Space Approach

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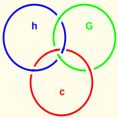
Bad Honnef 2008



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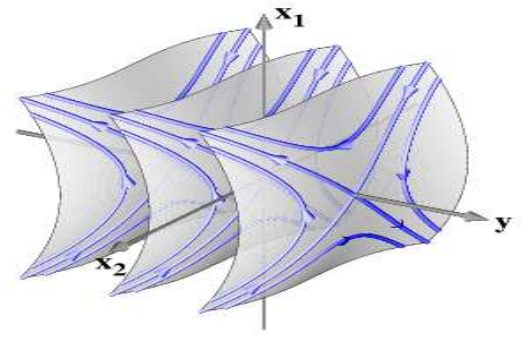
Classical Canonical Formulation

Canonical formulation: $M \cong R \times \sigma$



Starting Point:

- Well posed (causal) initial value formulation for geometry and matter
- $\Rightarrow$ Globally hyperbolic spactimes (M, g)
- $\Rightarrow$ Topological restriction: $\mathrm{M} \cong \mathbb{R} \times \sigma$ [Geroch, 60's]
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Consequences: [ADM 60's]

- Consider arbitrary foliations $Y: \mathbb{R} \times \sigma \rightarrow M$
- Require spacelike leaves of foliation $\Sigma_t := Y(t, \sigma)$
- Pull all fields on M back to $\mathbb{R} \times \sigma$
- Obtain velocity phase space of spatial fields (e.g. 3 – metric q_{ab} and extrinsic curvature $K_{ab} \propto \partial q_{ab}/\partial t$)
- Legendre transform $K_{ab} \mapsto p^{ab}$ singular (due to Diff(M) invariance): Spatial diffeomorphism and Hamiltonian constraints $c_a, \; c$
- Canonical Hamiltonian

$$H_{canon} = \int_\sigma d^3x \; n \; c + v^a \; c_a =: c(n) + \vec{c}(v)$$

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Remarks:

- Algebraic structure of $c,\ c_a$ Foliation independent
- Symplectic structure of geometry and matter fields Foliation independent
- Foliation dependence encoded in lapse, shift $n,\ v^a$
- Foliation independence (Diff(M) invariance) $\Rightarrow H_{canon} \approx 0$
- 10 Einstein Equations equivalent to

$$\partial_t q_{ab} = \{H_{canon}, q_{ab}\},\quad \partial_t p^{ab} = \{H_{canon}, p^{ab}\},\quad c = 0,\ c_a = 0$$

- In particular, building $g_{\mu\nu},\ n^\mu$ from $q_{ab},\ n,\ v^a$ one obtains $q_{\mu\nu} = g_{\mu\nu} + n_\mu n_\nu$ and

$$\{H_{canon}, q_{\mu\nu}\} = [\mathcal{L}_u q]_{\mu\nu},\quad u^\mu = nn^\mu + (\vec{v})^\mu$$

recovery of Diff(M) (on shell).

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- First class (Dirac) hypersurface deformation algebra $\mathfrak{D}$

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\{\vec{c}(v), \vec{c}(v')\} &= -\vec{c}([v, v']) \\
\{\vec{c}(v), c(n)\} &= -c(v[n]) \\
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- Universality: purely geometric origin, independent of matter content
  [Hojman, Kuchař, Teitelboim 70's]
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- $\mathfrak{D}$ no Lie algebra (structure functions)

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Problem of Time

Interpretation:

- H_{canon} constrained to vanish, no true Hamiltonian
- H_{canon} generates gauge transformations, not physical evolution
- $q_{ab},\ p^{ab}, \ldots$ not gauge invariant, not observable
- $\{H_{canon}, O\} = 0$ for observable, gauge invariant O
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Solution: Deparametrisation [Brown & Kuchař 90's]

- In GR, gauge invariant definition of curvature etc. only relative to geodesic test observers [Wald 90's]
- Test observers = mathematical idealisation
- Brown – Kuchař dust action: 4 scalar fields T, S^J minimially coupled however: geometry backreaction taken seriously
- Natural: Superposition of ∞ # of point particle actions
- EL Equations: Dust particles move on unit geodesics, $T(x) =$ proper time along geodesic trough x, $S^J(x)$ labels geodesic
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- Deparametrisation:

$$c := c^D + c^{ND}, \quad c_a = c_a^D + c_a^{ND} \quad \Rightarrow \tilde{c} = P + h, \quad h = \sqrt{[c^{ND}]^2 - q^{ab} c_a^{ND} c_b^{ND}}$$

- For close to flat geometry $h \approx c^{ND} \approx h^{SM}$ hard to achieve!
- Remarkably $\{\tilde{c}(n), \tilde{c}(n')\} = 0$ [Brown & Kuchař 90's]
  $\Rightarrow$ Explicit relational solution [Bergmann 60's, Rovelli 90's, Dittrich 00's]
- First symplectic reduction wrt c_a [Kuchař 90's] e.g.

$$q_{ab}(x) \to q_{JK}(s) := [q_a b(x) S^a_J(x) S^b_K(x)]_{S^J(x)=s^J}, \quad S^a_J S^J_{,b} = \delta^a_b, \quad S^a_J S^K_{,a} = \delta^K_J$$

- For any spatially diffeo inv., dust indep. f get observable

$$O_f(\tau) := \exp(\{H_\tau, .\}) \cdot f, \quad H_\tau := \int_\sigma d^3x \, (\tau - T(x)) \, h^{ND}(x)$$

- Physical time evolution

$$\frac{d}{d\tau} O_f(\tau) = \{H_{phys}, O_f(\tau)\}, \quad H_{phys} := \int_\sigma d^3x \, h^{ND}(x)$$

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- Closed observable algebra due to automorphism property of Hamiltonian flow

$$\{O_f(\tau), O_{f'}(\tau)\} = O_{\{f,f'\}}(\tau)$$

- Reduced phase space Q'ion conceivable since e.g.

$$Q_{JK}(s) := O_{q_{JK}(s)}(0), \quad P^{JK}(s) := O_{p^{JK}(s)}(0)$$

$$\Rightarrow \{P^{JK}(s), Q_{LM}(s')\} = \delta_L^{(J}\delta_M^{K)}\delta(s,s')$$

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- Reduced phase space Q'ion conceivable since e.g.

$$Q_{JK}(s) := O_{q_{JK}(s)}(0), \quad P^{JK}(s) := O_{p^{JK}(s)}(0)$$

$$\Rightarrow \{P^{JK}(s), Q_{LM}(s')\} = \delta_L^{(J}\delta_M^{K)}\delta(s,s')$$

Physics of the Dust:

- Dust = Gravitational Higgs, Non-Dust = Gravitational Goldstone Bosons
- Conservative Hamiltonian system w/o constraints but true Hamiltonian
- Hamiltonian EOM wrt H_{phys} of physical Non-Dust dof agree with Gauge Transformations wrt H_{canon} of unphysical Non-Dust dof under proper field substitutions, e.g. $q_{ab}(x) \leftrightarrow Q^{jk}(s)$
- No constraints but energy – momentum current conservation law

$$\{H_{phys}, O_{h^{ND}(s)}\} = 0, \ \{H_{phys}, O_{c_j^{ND}(s)}\} = 0,$$

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Canonical Quantisation Strategies

- Objective: Irreducible representation of the $^*-$algebra (or C*) $\mathfrak{A}_{\text{phys}}$ of Dirac observables supporting $\widehat{\text{H}}_{\text{phys}}$
- Strategy 1: Constraint Q'ion (CQ) = Q'ion before reduction
- Strategy 2: Reduced phase space Q'ion (RQ) = Q'ion after reduction
- Complementary Advantages and Disadvantages

  CQ+: Reps. of $\mathfrak{A}_{\text{kin}}$ easy to find
  CQ-: Phys. HS = Kernel(constraints) construction complicated (group averaging)
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Algebra of Kinematical Functions

Gauge Theory Formulation:

- Due to fermionic dof need to start with Palatini/Holst action [Ashtekar 80's], [Barbero, Holst, Immirzi 90's]
- After solving 2nd class (simplicity) constraints obtain

$$\{E^a_j(x), A^k_b(y)\} = \kappa\delta^a_b\delta^k_j\delta(x,y)$$

- Non-dust, gravitational contributions to the constraints

$$\begin{aligned}
c^{geo}_j &= \mathcal{D}_a E^a_j \\
c^{geo}_a &= \mathrm{Tr}\left(F_{ab}E^b\right) \\
c^{geo} &= \frac{\mathrm{Tr}\left(F_{ab}[E^a, E^b]\right)}{\sqrt{|\det(E)|}} + \ldots
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Algebra of Physical Observables

- Simply define (similar for $E^l_j(s)$)

$$A^j_l(s) := O_{a^j_l(s)}(0),\ a^j_l(s) := [A^j_a S^a_l](x)_{S(x)=s},$$

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- No constraints but phys. Hamiltonian ($\Sigma = S(\sigma)$)

$$H = \int_\Sigma \sqrt{|-\eta^{\mu\nu}\ \mathrm{Tr}(\tau_\mu\ F \wedge \{A,V\})\ \mathrm{Tr}(\tau_\nu\ F \wedge \{A,V\})|} =: \int d^3s\ H(s)$$

- Physical total volume

$$V = \int_\Sigma \sqrt{|\det(E)|}$$

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Physical Hilbert Space

Lattice – inspired canon. gauge theory variables [Gambini & Trias 81], [Jacobson, Rovelli, Smolin 88]

- Magnet. dof.: Holonomy (Wilson – Loop)

$$A(e) = \mathcal{P}\ \exp(\int_e A)$$

- Electr. dof: flux

$$E_j(S) = \int_S \epsilon_{abc}\, E_j^a\, dx^b \wedge dx^c$$

- Poisson – brackets:

$$\{E_j(S), A(e)\} = G\, A(e_1)\, \tau_j\, A(e_2); \quad e = e_1 \circ e_2,\ e_1 \cap e_2 = e \cap S$$

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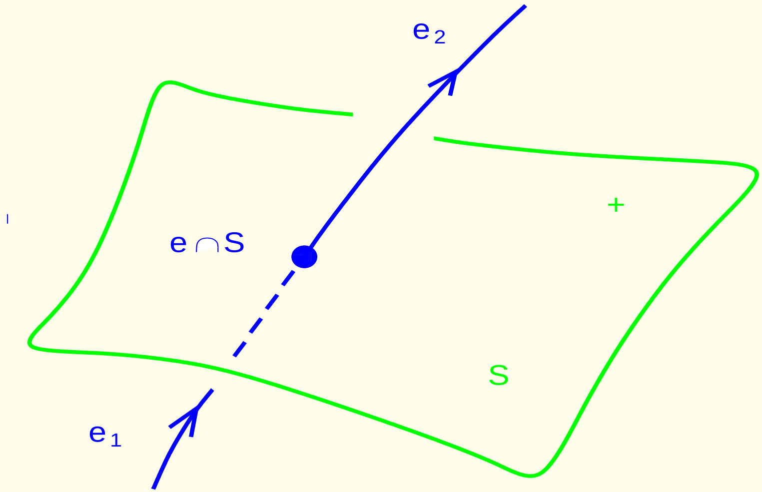
e 2
e ∩ S
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- **HS Reps.:** In QFT no Stone – von Neumann Theorem!!!

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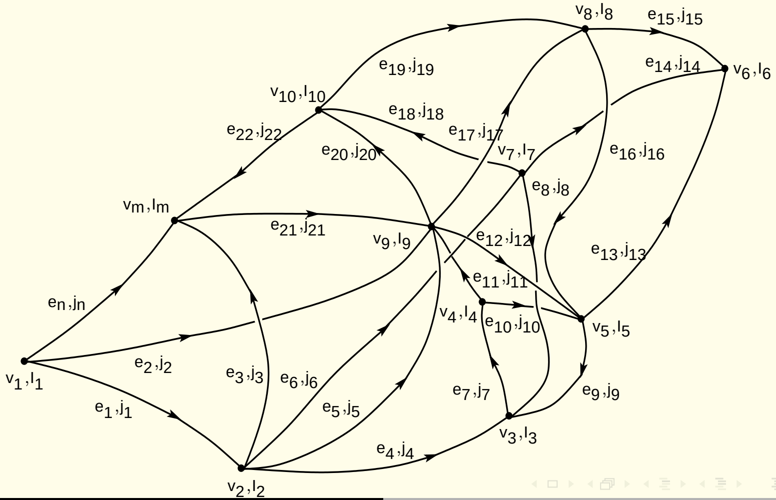
Spin Network ONB $T_{\gamma,j,l}$



Does rep. support $\widehat{\mathrm{H}}$ with correct semiclassical limit?

- Gauss constraint solved by restriction of intertwiners I
- $\mathcal{H}_{\mathrm{phys}}$ not separable

$$\mathcal{H}_{\mathrm{phys}} = \oplus_\gamma \mathcal{H}_\gamma, \quad \mathcal{H}_\gamma = \overline{\mathrm{span}\{\mathrm{T}_{\gamma,\mathrm{j},\mathrm{I}};\ \mathrm{j} \neq 0, \mathrm{I}\}}$$

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- Choose cell complex γ^*, dual graph γ s.t. $e \leftrightarrow S_e$
- Choose classical field configuration $(A_0(x), E_0(x))$, compute
$$g_e := \exp(i\tau_j E_0^j(S_e))\, A_0(e) \in G^{\mathbb{C}}$$
- Define [Hall 90's], [Sahlmann, T.T., Winkler 00's]
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- Minimal uncertainty states, that is, $\forall\, e \in E(\gamma)$
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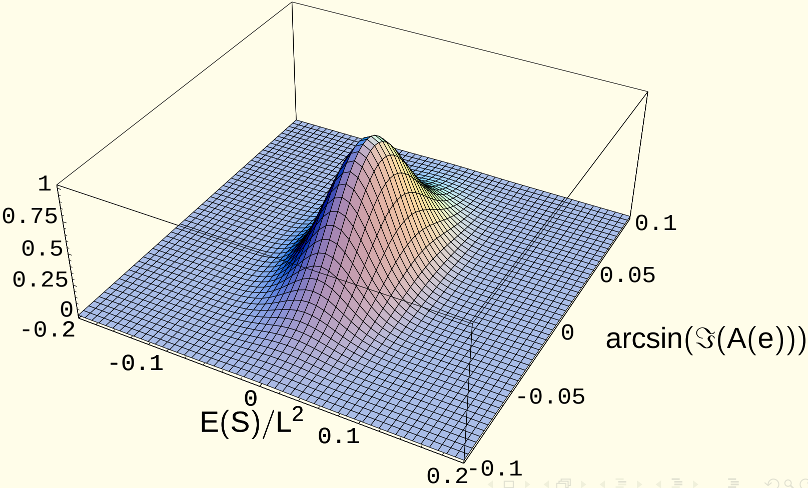
Overlap Function



Remarks:

- Notice: Σ just differential manifold, no Riemannian space!
- No a priori meaning to how densely γ embedded
- In particular, final operator $\widehat{\mathrm{H}}$ cannot depend on short distance regulator used at intermediate stages of construction
- Notice: Operator family $(\widehat{\mathrm{H}}_\gamma)$ defines Continuum operator
- Expect that good semiclassical states depend on graphs which are very densely embedded wrt background metric to be approximated
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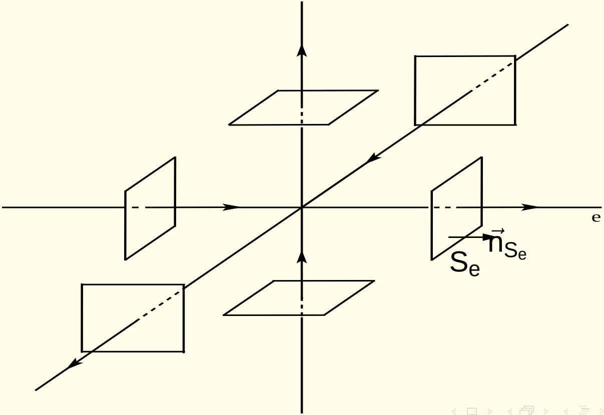
Physical Hamiltonian

Example: Cubic graph



Comparison with YM theory on cubic lattice

- Yang – Mills on (R^4, η) [Kogut & Susskind 74]

$$H_\gamma = \frac{\hbar}{2\, g^2\, \epsilon} \sum_{v \in V(\gamma)} \sum_{a=1}^{3} \mathrm{Tr}\left(E(S_v^a)^2 + [2 - A(\alpha_v^a) - A(\alpha_v^a)^{-1}] \right)$$

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Theorem [Giesel & T.T. 06] For any (A_0, E_0), suff. large γ

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If LQG/AQG pass consistency tests then:

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