Consistent truncations and the M-theory Matrix model

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[G. B, F. Ciceri, G. Inverso, A. Kleinschmidt, 2209.02729]
[G. B, F. Ciceri, G. Inverso, A. Kleinschmidt, 2309.07232]
[G. B, F. Ciceri, G. Inverso, A. Kleinschmidt, 2309.07233]

Motivations

Consistent truncations are powerful tools to compute solutions in higher dimensions.

- ⋆ Find AdS vacua with less symmetry
- ⋆ Solutions with fixed asymptotics,
  - ↪ e.g. perturbations of AdS solutions
- ⋆ Kaluza–Klein spectrum [Samtleben–Malek]
  - ↪ complete stability in higher dimensions
  - ↪ higher point functions

First example

↪ $SO(8)$-gauged supergravity in four dimensions [de-Wit–Nicolai]
$\subset$ 11D supergravity on $\mathrm{AdS}_4 \times S^7$ [de-Wit–Nicolai–Pilch–Godazgars]

Modern technique

↪ Generalised Scherk–Schwarz reduction [Hohm–Samtleben]

Motivations

- Generalisation to two dimensions

  ↪ Involves affine Kac–Moody E_9 [Julia–Nicolai]

- Conformal $\mathrm{AdS}_2 \times S^8$ / M-theory Matrix quantum mechanics

- A large variety of other AdS_2 vacua

  ↪ Different phases of the M-theory matrix model

  ↪ Other matrix models, BMN mass deformation, ...

Outline

- M-theory Matrix model
- Generalised Scherk–Schwarz for affine
- $SO(9)$ gauged supergravity to eleven dimensions
- BMN thermodynamics

Super-Membrane on S^2

The super-membrane Lagrangian [de Wit–Hoppe–Nicolai]

$$\mathcal{L} = -\sqrt{-\det \eta_{\mu\nu}(\nabla X^\mu + \bar{\theta}\gamma^\mu \nabla\theta)(\nabla X^\nu + \bar{\theta}\gamma^\nu \nabla\theta)} \\ - \frac{1}{2}\Big(dX^\mu (dX^\nu + \bar{\theta}\gamma^\nu d\theta) + \tfrac{1}{6}\bar{\theta}\gamma^\mu d\theta \bar{\theta}\gamma^\nu d\theta \Big) \bar{\theta}\gamma_{\mu\nu} d\theta$$

in light-cone gauge $\partial_i X^+ = 0$, $\partial_0 X^+ = 1$ and $\gamma_+ \theta = 0$

$$\mathcal{L} = \tfrac{1}{2} e (D_0 X)^2 + e\bar{\theta} D_0 \theta - \frac{1}{4e} \big| dX \wedge dX \big|^2 + \bar{\theta} dX \wedge d\theta$$

with $D_0 = \partial_0 - \frac{1}{e} dA \wedge d$ and $d = d\sigma^i \partial_i$ on Σ.

Decomposed in spherical harmonics on $\Sigma = S^2$

$$X^a(t, \sigma) = \sum_{\ell, m} X^a_{\ell,m}(t) Y_{\ell,m}(\sigma)$$

becomes $\lim_{N\to\infty} U(N)$ matrix quantum mechanics.

BFSS matrix Model

The $U(N)$ matrix quantum mechanics with $Spin(9)$ symmetry [Banks, Fischler, Shenker, Susskind]

$$L = \mathrm{Tr}\Big(\frac{1}{R}\dot{X}^2 + \bar{\theta}\dot{\theta} - \frac{R}{2\ell^6}\big|[X,X]\big|^2 + \frac{2R}{\ell^3}\bar{\theta}[\not{X},\theta]\Big)$$

where the M-theory radius

$$R = e^{\frac{2\phi}{3}}\ell = e^{\phi}\sqrt{\alpha'} \ ,$$

describes gravitons at large N in eleven dimensions

$$p_{10} = \frac{N}{R} \ , \qquad E = \frac{N}{R} + \frac{Rp_\perp^2}{2N}$$

Holographic description

Dual to the pp-wave background

$$ds^2_{\mathrm{11D}} = 2dtd\psi + N\left(\frac{\ell}{r}\right)^7 d\psi^2 + dr^2 + r^2 d\Omega_8^2$$

for

$$\text{Planck scale} \sim N^{\frac{1}{9}}\ell \ll r \ll N^{\frac{1}{3}}\ell \sim \text{String scale} \rightsquigarrow \text{ Matrix .}$$

Can be realised in type IIA supergravity for $r \gg N^{\frac{1}{7}}\ell$

$$ds^2_{\mathrm{10D}} = \sqrt{\frac{N\ell^7}{r^3}}\left(-\frac{r^5}{N\ell^7}dt^2 + \frac{dr^2}{r^2} + d\Omega_8^2\right) ,$$

$\hookrightarrow$ Conformal AdS / QFT correspondance [Boonstra, Skenderis, Townsend]

Holographic description

The black hole solution

$$ds^2_{11\text{D}} = \left(1-\left(\frac{\tau}{r}\right)^7\right)dt\left(2d\psi-\frac{\tau}{N\ell^2}dt\right)+N\left(\frac{\ell}{r}\right)^7 d\psi^2+\frac{dr^2}{1-\left(\frac{\tau}{r}\right)^7}+r^2 d\Omega_8^2$$

describes the M-theory matrix model at finite temperature T with

$$\tau = \ell(NT^2\ell^2)^{\frac{1}{5}} \ .$$

Can trigger the BMN deformation [Berenstein, Maldacena, Nastase]

$$L = \mathrm{Tr}\left(\frac{1}{R}\dot{X}^2+\bar{\theta}\dot{\theta}-\frac{R}{2\ell^6}\left|[X,X]\right|^2+\frac{2R}{\ell^3}\bar{\theta}[\not{X},\theta] -\frac{\mu^2}{9R}X_\parallel^2-\frac{\mu^2}{36R}X_\perp^2-\frac{\mu}{4}\bar{\theta}\gamma^{123}\theta-\frac{2i\mu}{3\ell^3}\varepsilon_{ijk}X_\parallel^i X_\parallel^j X_\parallel^k\right)$$

with $C = \mu dt \wedge r^3\zeta^3 d\Omega_2$. [Costa, Greenspan, Penedones, Santos]

Supergravity solutions

Solutions with given pp-wave asymptotic at $r \gg \tau$

- $SO(3) \times SO(6)$ symmetry [Costa, Greenspan, Penedones, Santos]

$$d\Omega_8^2 = \frac{d\zeta^2}{1-\zeta^2} + \zeta^2 d\Omega_2^2 + (1-\zeta^2) d\Omega_5^2$$

$$\rightsquigarrow \; ds_8^2 = T_1(r,\zeta)\frac{d\zeta^2}{1-\zeta^2} + T_2(r,\zeta)\zeta^2 d\Omega_2^2 + T_3(r,\zeta)(1-\zeta^2) d\Omega_5^2$$

- Consistent truncation to maximal supergravity

[Anabalón, Ortiz, Samtleben]

$$ds_8^2 = \left(\zeta^2 e^{\phi} + (1-\zeta^2)e^{-2\phi}\right)\frac{d\zeta^2}{1-\zeta^2} + e^{-2\phi}\zeta^2 d\Omega_2^2 + e^{\phi}(1-\zeta^2) d\Omega_5^2$$

for a single $\phi(t,r)$.

E_9 exceptional field theory

The scalar fields in $K(E_9)\backslash E_9$

$$\mathcal{V}^{\mathrm{A}}{}_M(x,y) \sim \mathcal{V} = \rho^{-L_0} V \prod_{n=1}^{\infty} \exp(-Y_A^n T^A{}_{-n})$$

The vector fields in the basic module $R(\Lambda_0)$

$$A^M(x,y) \sim |A\rangle$$

and the derivative in the conjugate module

$$\frac{\partial}{\partial y^M} \sim \langle\partial|$$

satisfying

$$\eta_{\alpha\beta}\langle\partial F|T^\alpha \otimes \langle\partial G|T^\beta = \langle\partial G| \otimes \langle\partial F| - \langle\partial F| \otimes \langle\partial G|$$

Affine extension E_9

- One defines the affine Kac–Moody algebra

$$
\begin{aligned}
[T^A_n, T^B_m] &= f^{AB}{}_C T^C_{n+m} + n\,\delta_{n+m}\eta^{AB} \\
[L_n, T^A_m] &= -m T^A_{n+m} \\
[L_n, L_m] &= (n-m)L_{n+m} + \frac{2}{3}n(n^2-1)\delta_{n+m} \,.
\end{aligned}
$$

- The basic module $R(\Lambda_0)$ generated from the vacuum

$$
T^A_n|0\rangle = 0 \,, \quad \forall n \leq 0 \,, \qquad L_n|0\rangle = 0 \,, \quad \forall n \leq 1 \,.
$$

Affine extension E_9

- One defines the affine Kac–Moody algebra

$$
\begin{aligned}
[T_n^A, T_m^B] &= f^{AB}{}_C T^C_{n+m} + n\,\delta_{n+m}\eta^{AB} \\
[L_n, T_m^A] &= -m T^A_{n+m} \\
[L_n, L_m] &= (n-m)L_{n+m} + \frac{2}{3}n(n^2-1)\delta_{n+m}\ .
\end{aligned}
$$

- The basic module $R(\Lambda_0)$ generated from the vacuum

$$
T_n^A|0\rangle = 0\ , \quad \forall n \leq 0\ , \qquad L_n|0\rangle = 0\ , \quad \forall n \leq 1\ .
$$

- The bilinear forms

$$
\eta_{k\,\alpha\beta} T^\alpha \otimes T^\beta = \eta_{AB} \sum_n T_n^A \otimes T^B_{k-n} - L_k \otimes 1 - 1 \otimes L_k
$$

are invariant under $\widehat{E_8}$ and L_k. We write $\eta_{\alpha\beta} = \eta_{0\,\alpha\beta}$.

Spectral flow

One further decomposes the module according to both L_0 and $T^{k}_{0\,k} \in \mathfrak{gl}_8 \subset \mathfrak{e}_8$

$$
\begin{array}{c}
\overline{\mathbf{8}}^{2}_{-1} \oplus \mathbf{28}^{2}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{2}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^{2}_{0} \oplus \mathbf{56}^{2}_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^{2}_{\frac{2}{3}} \oplus \mathbf{8}^{2}_{1} \\
\overline{\mathbf{8}}^{1}_{-1} \oplus \mathbf{28}^{1}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{1}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^{1}_{0} \oplus \mathbf{56}^{1}_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^{1}_{\frac{2}{3}} \oplus \mathbf{8}^{1}_{1} \\
\color{red}{\overline{\mathbf{8}}^{0}_{-1}} \oplus \color{purple}{\mathbf{28}^{0}_{-\frac{2}{3}}} \oplus \color{green}{\overline{\mathbf{56}}^{0}_{-\frac{1}{3}}} \oplus \color{red}{(\mathfrak{gl}_8)^{0}_{0}} \oplus \color{purple}{\mathbf{56}^{0}_{\frac{1}{3}}} \oplus \color{green}{\overline{\mathbf{28}}^{0}_{\frac{2}{3}}} \oplus \color{red}{\mathbf{8}^{0}_{1}} \\
\overline{\mathbf{8}}^{-1}_{-1} \oplus \mathbf{28}^{-1}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{-1}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^{-1}_{0} \oplus \mathbf{56}^{-1}_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^{-1}_{\frac{2}{3}} \oplus \mathbf{8}^{-1}_{1} \\
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\end{array}
$$

with $\mathfrak{sl}_9$ recomposition

$$\mathfrak{e}_9 = \bigoplus_n \left(\color{red}{\mathfrak{sl}_9^n} \oplus \color{purple}{\mathbf{84}^n} \oplus \color{green}{\overline{\mathbf{84}}^n} \right) \oplus K \oplus L_0^{(1)}$$

Spectral flow

One chooses a spectral flowed Virasoro

$$L_0^{(p)} = L_0 + p\, T^k_{0\,k} + \frac{4p^2}{9}$$

for $p = 1$

$$\begin{array}{l}
\overline{\mathbf{8}}^{2}_{-1} \oplus \mathbf{28}^{2}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{2}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^2_0 \oplus \mathbf{56}^2_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^2_{\frac{2}{3}} \oplus \mathbf{8}^2_1 \\
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\end{array}$$

with $\mathfrak{sl}_9$ recomposition

$$\mathfrak{e}_9 = \bigoplus_n \left(\mathfrak{sl}_9^n \oplus \mathbf{84}^{n+\frac{1}{3}} \oplus \overline{\mathbf{84}}^{n+\frac{2}{3}} \right) \oplus K \oplus L_0^{(1)}$$

Spectral flow

One has then the algebra

$$
\begin{aligned}
\left[T^{I}_{m\,J}, T^{K}_{n\;L}\right] &= \delta^K_J\, T^I_{m+n\,L} - \delta^I_L\, T^K_{m+n\,L} + m\delta_{m+n}(\delta^I_J\delta^K_L - \tfrac{1}{9}\delta^i_L\delta^K_j) \\
\left[T^{I_1 I_2 I_3}_{m-p/3}, T_{n+p/3\; J_1 J_2 J_3}\right] &= 18\,\delta^{[I_1 I_2}_{[J_1 J_2}\, T^{I_3]}_{m+n\; J_3]} + 6\,(m - \tfrac{p}{3})\delta_{m+n}\delta^{I_1 I_2 I_3}_{J_1 J_2 J_3} \\
\left[T^{I_1 I_2 I_3}_{m-p/3}, T^{I_4 I_5 I_6}_{n-p/3}\right] &= -\frac{1}{6}\varepsilon^{I_1\ldots I_9}\, T_{m+n-2p/3\; I_7 I_8 I_9} \\
\left[T_{m+p/3\; I_1 I_2 I_3}, T_{n+p/3\; I_4 I_5 I_6}\right] &= \frac{1}{6}\varepsilon_{I_1\ldots I_9}\, T^{I_7 I_8 I_9}_{m+n+2p/3}
\end{aligned}
$$

The $\widehat{\mathfrak{sl}}_9$ subalgebras are conjugate for all

⋆ $p = 0 \bmod 3$ $\rightsquigarrow$ $SL(9) \subset E_8$ with $\mathfrak{e}_8 = \mathfrak{sl}_9 \oplus \mathbf{84} \oplus \overline{\mathbf{84}}$

⋆ $p = \pm 1 \bmod 3$.

$\hookrightarrow$ $p = 1$ is the gravity line $SL(9)$ decomposition associated to eleven dimensions

Basic module

Generalised vector field in the $SL(9)$ ($p = 1$) decomposition

$$|\Lambda\rangle = \left(\xi^I + \tfrac{1}{2}\lambda_{JK} T_{1/3}^{IJK} + \tfrac{1}{6}\lambda^{*IJKL} T_{2/3\,JKL} + \xi^{*IJ}{}_K T_1^K{}_J + \dots\right)|0\rangle_I$$

defines ordinary diffeomorphism and gauge transformations

$$\delta G_{IJ} = \nabla_I \xi_J + \nabla_J \xi_I \ , \quad \delta A_{IJK} = \xi^L F_{LIJK} + 3\partial_{[I}\lambda_{JK]} \ ,$$

E_9 Generalised diffeomorphisms

$$\mathcal{L}_\Lambda \quad |V\rangle = \langle \partial_V|\Lambda\rangle|V\rangle - \eta_{\alpha\beta}\langle\partial_\Lambda|T^\alpha|\Lambda\rangle T^\beta|V\rangle - \langle\partial_\Lambda|\Lambda\rangle|V\rangle$$

In the $SL(9)$ $(p=1)$ decomposition

$$|\Lambda\rangle = \big(\xi^I + \tfrac{1}{2}\lambda_{JK}T^{IJK}_{1/3} + \tfrac{1}{6}\lambda^{*IJKL}T_{2/3\,JKL} + \xi^{*IJ}{}_K T^K_1{}_J + \ldots\big)|0\rangle_I$$

The derivative on section

$$\langle\partial| = \langle 0|^I\partial_I$$

and

$$\begin{aligned}\mathcal{L}_{\Lambda,\Sigma}|V\rangle &= \xi^I\partial_I|V\rangle - \partial_I\xi^J T^I_0{}_J|V\rangle + \partial_I\xi^I(L_0 - \tfrac{5}{9})|V\rangle \\ &\quad -\frac{1}{2}\partial_I\lambda_{JK}T^{IJK}_{1/3}|V\rangle - \frac{1}{6}\partial_I\lambda^{*IJKL}T_{2/3\,JKL}|V\rangle \\ &\quad - \partial_K\xi^{*KJ}{}_I T^I_1{}_J|V\rangle + \ldots\end{aligned}$$

E_9 Generalised diffeomorphisms

$$\mathcal{L}_{\Lambda,\Sigma}|V\rangle = \langle\partial_V|\Lambda\rangle|V\rangle - \eta_{\alpha\beta}\langle\partial_\Lambda|T^\alpha|\Lambda\rangle T^\beta|V\rangle - \langle\partial_\Lambda|\Lambda\rangle|V\rangle \\ - \eta_{-1\alpha\beta}\mathrm{Tr}[T^\alpha\Sigma]T^\beta|V\rangle$$

In the $SL(9)$ ($p=1$) decomposition

$$|\Lambda\rangle = \big(\xi^I + \tfrac{1}{2}\lambda_{JK}T_{1/3}^{IJK} + \tfrac{1}{6}\lambda^{*IJKL}T_{2/3\,JKL} + \xi^{*IJ}{}_K T_1^K{}_J + \dots\big)|0\rangle_I$$

The derivative and constrained ancillary parameter on section

$$\langle\partial| = \langle 0|^I\partial_I \qquad \Sigma = \big(\Sigma^{*J}{}_I|0\rangle_J + \dots\big)\langle 0|^I$$

and

$$\begin{aligned}\mathcal{L}_{\Lambda,\Sigma}|V\rangle &= \xi^I\partial_I|V\rangle - \partial_I\xi^J T_0^I{}_J|V\rangle + \partial_I\xi^I(L_0 - \tfrac{5}{9})|V\rangle \\ &\quad -\frac{1}{2}\partial_I\lambda_{JK}T_{1/3}^{IJK}|V\rangle - \frac{1}{6}\partial_I\lambda^{*IJKL}T_{2/3\,JKL}|V\rangle \\ &\quad -(\partial_K\xi^{*KJ}{}_I + \Sigma^{*J}{}_I)T_1^I{}_J|V\rangle + \dots\end{aligned}$$

Generalised Scherk–Schwarz

Generalises to $E_d/K(E_d)$ symmetric matrix as

$$\mathcal{M}_{MN}(x,y) = \mathcal{U}^{\underline{P}}{}_M(y)\mathcal{M}_{\underline{P}\underline{Q}}(x)\mathcal{U}^{\underline{Q}}{}_N(y)$$

with [Berman–Musaev–Thompson, Hohm–Samtleben]

$$r^{-1}\left(\mathcal{U}^{-1M}{}_{\underline{P}}\mathcal{U}^{-1N}{}_{\underline{Q}}\partial_M\mathcal{U}^{\underline{R}}{}_N\right)\Big|_{R(\Lambda_{9-d})} = \Theta_{\underline{P}\alpha}T^{\alpha\underline{R}}{}_{\underline{Q}}$$

with $E_{\underline{P}}{}^M = r^{-1}\mathcal{U}^{-1M}{}_{\underline{P}}$ one has

$$\mathcal{L}_{E_{\underline{P}}}E_{\underline{Q}} = -\Theta_{\underline{P}\alpha}T^{\alpha\underline{R}}{}_{\underline{P}}E_{\underline{R}}$$

Generalised Scherk–Schwarz

We take

$$\mathcal{V}(x,Y) = V(x)\mathcal{U}(Y)\,, \quad \rho(x,Y) = r(Y)\varrho(x)$$

and define the $\mathfrak{e}_9$-valued Weitzenböck connection

$$\langle \mathcal{W}_\alpha | \otimes T^\alpha = r^{-1} \langle e^M | \mathcal{U}^{-1} \otimes \partial_M \mathcal{U}\,\mathcal{U}^{-1}$$

with

$$\begin{aligned} \langle \mathcal{W}_\alpha | \otimes T^\alpha &= \sum_n \langle W_A^n | \otimes T_n^A + \langle W_0 | \otimes L_0 + \langle W_K | \otimes 1 \\ \langle \mathcal{W}_\alpha^\pm | \otimes T^\alpha &= \sum_n \langle W_A^n | \otimes T_{n\pm1}^A + \langle W_0 | \otimes L_{\pm1} + \langle w^\pm | \otimes 1 \end{aligned}$$

Gauged algebra

The gauge parameters

$$|\Lambda\rangle = r^{-1}\mathcal{U}^{-1}|\lambda\rangle\,, \quad \Sigma = r\,\mathcal{U}^{-1}\,T^{\alpha}|\lambda\rangle\langle\mathcal{W}^{+}_{\alpha}|\,\mathcal{U}\,,$$

satisfy

$$r\,\mathcal{U}\mathcal{L}_{|\Lambda\rangle,\Sigma}\left(\,r^{-1}\mathcal{U}^{-1}|v\rangle\,\right) = \eta_{-1\,\alpha\beta}\langle\theta|\,T^{\alpha}|\lambda\rangle\,T^{\beta}|v\rangle$$

with the embedding tensor

$$\langle\theta| = -\langle\mathcal{W}^{+}_{\alpha}|\,T^{\alpha}\;.$$

$\hookrightarrow$ implies quadratic constraint [Samtleben–Weidner]

$$\eta_{-1\,\alpha\beta}\langle\theta|\,T^{\alpha}\otimes\langle\theta|\,T^{\beta} = 0$$

Eleven-dimensional supergravity basis

To obtain compactifications from eleven-dimensional supergravity

$\hookrightarrow$ Identifies the coordinate module $\mathbf{9} \subset R(\Lambda_0)$

$\hookrightarrow$ The structure group of the tangent space $SO(9) \subset GL(9)$.

To exhibit the symmetries of S^8

$\hookrightarrow$ The isometry group of the sphere $SO(9) \subset GL(9)$.

Two conjugate $SL(9)$ decompositions of E_9 to identify.

Spectral flow

One chooses a spectral flowed Virasoro

$$L_0^{(p)} = L_0 + p\, T^k_{0\,k} + \frac{4p^2}{9}$$

for $p = 1$

$$\begin{array}{l}
\overline{\mathbf{8}}^{2}_{-1} \oplus \mathbf{28}^{2}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{2}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^2_0 \oplus \mathbf{56}^2_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^2_{\frac{2}{3}} \oplus \mathbf{8}^2_1 \\
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\end{array}$$

with

$$\mathfrak{e}_9 = \bigoplus_n \left(\mathfrak{sl}_9^n \oplus \mathbf{84}^{n+\frac{1}{3}} \oplus \overline{\mathbf{84}}^{n+\frac{2}{3}}\right) \oplus K \oplus L_0^{(1)}$$

Spectral flow

One chooses a spectral flowed Virasoro

$$L_0^{(p)} = L_0 + p\, T^k_{0\,k} + \frac{4p^2}{9}$$

for $p = 2$

$$\begin{array}{l}
\overline{\mathbf{8}}^{2}_{-1} \oplus \mathbf{28}^{2}_{-\frac{2}{3}} \oplus \overline{\mathbf{56}}^{2}_{-\frac{1}{3}} \oplus (\mathfrak{gl}_8)^{2}_{0} \oplus \mathbf{56}^{2}_{\frac{1}{3}} \oplus \overline{\mathbf{28}}^{2}_{\frac{2}{3}} \oplus \mathbf{8}^{2}_{1} \\
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$$\mathfrak{e}_9 = \bigoplus_n \left(\mathfrak{sl}_9^n \oplus \overline{\mathbf{84}}^{n+\frac{1}{3}} \oplus \mathbf{84}^{n+\frac{2}{3}} \right) \oplus K \oplus L_0^{(2)}$$

Spectral flow

One has then the algebra

$$\begin{aligned}
\left[T^{I}_{m\,J},\, T^{K}_{n\ \,L}\right] &= \delta^{K}_{J}\, T^{I}_{m+n\,L} - \delta^{I}_{L}\, T^{K}_{m+n\,L} + m\delta_{m+n}(\delta^{I}_{J}\delta^{K}_{L} - \tfrac{1}{9}\delta^{i}_{L}\delta^{K}_{j}) \\
\left[T^{I_1 I_2 I_3}_{m-p/3},\, T_{n+p/3\; J_1 J_2 J_3}\right] &= 18\,\delta^{[I_1 I_2}_{[J_1 J_2}\, T^{I_3]}_{m+n\; J_3]} + 6\,(m - \tfrac{p}{3})\delta_{m+n}\delta^{I_1 I_2 I_3}_{J_1 J_2 J_3} \\
\left[T^{I_1 I_2 I_3}_{m-p/3},\, T^{I_4 I_5 I_6}_{n-p/3}\right] &= -\frac{1}{6}\varepsilon^{I_1 \ldots I_9}\, T_{m+n-2p/3\; I_7 I_8 I_9} \\
\left[T_{m+p/3\; I_1 I_2 I_3},\, T_{n+p/3\; I_4 I_5 I_6}\right] &= \frac{1}{6}\varepsilon_{I_1 \ldots I_9}\, T^{I_7 I_8 I_9}_{m+n+2p/3}
\end{aligned}$$

- ⋆ $p = 1$ arises in dimensional reduction on T^9.
- ⋆ $p = 2$ appears for the S^8 isometries.

$SO(9)$ supergravity in two dimensions

Supergravity in Misner frame obtained directly by reduction on T^9

$$\mathcal{L} = \varrho\sqrt{-g}R + \frac{1}{4}\varrho DM_{IJ} \star DM^{IJ} - \frac{1}{12}\varrho^{\frac{1}{3}} M^{IL}M^{JP}M^{KQ}Da_{IJK} \star Da_{LPQ} - \frac{1}{6^4}\varepsilon^{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} a_{I_1 I_2 I_3} Da_{J_1 J_2 J_3} \wedge Da_{K_1 K_2 K_3}$$

with

$$G_{IJ} = \varrho^{\frac{2}{3}} M_{IJ} \ , \qquad a_{IJK} \in \overline{\mathbf{84}} \ .$$

$SO(9)$ supergravity in two dimensions

In the $p = 2$ frame the $SL(9)$ representations are conjugate

$$\mathcal{L} = 2d\varrho \star d\sigma + \frac{1}{4}\varrho DM_{IJ} \star DM^{IJ} - \frac{1}{12}\varrho^{\frac{1}{3}} M_{IL} M_{JP} M_{KQ} Da^{IJK} \star Da^{LPQ} \\ - \frac{1}{6^4}\varepsilon_{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} a^{I_1 I_2 I_3} Da^{J_1 J_2 J_3} \wedge Da^{K_1 K_2 K_3}$$

with

$$M_{IJ} \in SL(9)/SO(9) \; , \qquad a^{IJK} \in \mathbf{84} \; .$$

$\hookrightarrow$ $44 + 84$ degree of freedom.

Compactification on $S^8 \times S^1$

Conjugate such that $\langle\partial|$ reads in the (p=2) $SL(9)$

$$\langle\partial| = \frac{1}{7}\langle 0|_J\, T^{0iJ}_{1/3}\partial_i = \langle \tfrac{1}{3}|^{0i}\partial_i$$

We take

$$\mathcal{U}^{-1} = r^{L_0} e^s\, U^{-1}$$

the embedding tensor

$$\begin{aligned}\langle\theta| &= r^{-\frac{2}{9}} e^s U^{-1\,0}{}_K U^{-1\,i}{}_L \partial_i U^S{}_I U^{-1\,I}{}_R \left(\langle\tfrac{1}{3}|^{[KL}\, T^{R]}_{1\,S} - \frac{2}{7}\langle\tfrac{1}{3}|^{Q[K}\, T^L_{1\,Q}\delta^{R]}_S\right)\\
&+ \frac{1}{8} r^{-\frac{2}{9}} e^s \left(U^{-1\,0}{}_K \partial_i U^{-1\,i}{}_L - U^{-1\,i}{}_K \partial_i U^{-1\,0}{}_L + w_9^+\, U^{-1\,0}{}_K U^{-1\,0}{}_L\right)\langle\tfrac{1}{3}|^{P(K}\, T^{L)}_{1\,P}\\
&+ \frac{9}{14} r^{-\frac{16}{9}} e^s \partial_i\left(r^{\frac{14}{9}} U^{-1\,0}{}_K U^{-1\,i}{}_L\right)\langle\tfrac{1}{3}|^{KL} L_1 = -\frac{1}{8}\Theta_{IJ}\langle\tfrac{1}{3}|^{K(I}\, T^{J)}_{1\,K} = -\Theta_{IJ}\langle\tfrac{4}{3}|^{IJ}\end{aligned}$$

Scherk–Schwarz solution

Introduce $SO(9)$ embedding coordinates with metric δ_{IJ} for $\Theta_{IJ} = \lambda\delta_{IJ}$

$$\delta_{IJ}\mathcal{Y}^I\mathcal{Y}^J = 1$$

The $SL(9)$ twist matrix components are taken to be

$$U^{-1\,i}{}_I = |\mathrm{det} g|^{\frac{1}{9}} \left(g^{ij}\partial_j\mathcal{Y}_I + c^i\mathcal{Y}_I\right) \ ,$$
$$U^{-1\,0}{}_I = |\mathrm{det} g|^{-\frac{7}{18}}\mathcal{Y}_I \, ,$$

and

$$r = |\mathrm{det} g|^{\frac{1}{2}} \ , \quad e^s = \lambda|\mathrm{det} g|^{\frac{7}{18}} \, .$$

where $g_{ij} = \delta^{IJ}\partial_i\mathcal{Y}_I\partial_j\mathcal{Y}_J$ and c^i the 7-form potential

$$g^{ij}\partial_i\mathcal{Y}_I\partial_j\mathcal{Y}_J = \delta_{IJ} - \mathcal{Y}_I\mathcal{Y}_J \ , \quad |\mathrm{det} g|^{-\frac{1}{2}}\partial_i\left(|\mathrm{det} g|^{\frac{1}{2}}g^{ij}\partial_j\mathcal{Y}_I\right) = -8\mathcal{Y}_I$$

and

$$|\mathrm{det} g|^{-\frac{1}{2}}\partial_i\left(|\mathrm{det} g|^{\frac{1}{2}}c^i\right) + |\mathrm{det} g|^{\frac{1}{2}}w_9^+ = 7$$

Scherk–Schwarz solution

With the embedding tensor

$$\langle\theta| = -\Theta_{IJ}\langle 4/3|^{IJ} ,$$

and the coset component

$$\mathcal{V}^{-1} = \cdots\ e^{h^I{}_J T^J_{-1\,I}}\, e^{\frac{1}{6}a^{IJK}\,T_{-1/3\,IJK}}\,\varrho^{L_0}\, e^{\sigma}\, V^{-1}$$

$$\begin{aligned}\langle\theta|\mathcal{V}^{-1} = \frac{1}{56}\Theta_{IJ}e^{\sigma}\Big(& V^{-1\,I}{}_A V^{-1\,J}{}_B \langle 0|_C\, T^{CDA}_{1/3}\, T^{B}_{1\,D} - 8\varrho^{-1} h^I{}_K V^{-1\,J}{}_A V^{-1\,K}{}_B \langle 0|_C\, T^{ABC}_{1/3} \\ & + 28\varrho^{-1/3} a^{IKL} V^{-1\,J}{}_A V^B{}_K V^C{}_L \langle 0|_B\, T^A_{1\,C} + 56\varrho^{-4/3} a^{IKL} h^J{}_L V^A{}_K \langle 0|_A \\ & - 7\varrho^{-2/3} a^{IKL} a^{JPQ} V^A{}_K V^B{}_L V^C{}_P V^D{}_Q \langle 0|_A\, T_{2/3\,BCD} \\ & - \frac{1}{36}\varrho^{-1} a^{IK_1K_2} a^{JK_3K_4} a^{K_5K_6K_7} \varepsilon_{K_1\ldots K_7RS} V^{-1\,R}{}_A V^{-1\,S}{}_B \langle 0|_C\, T^{ABC}_{1/3} \\ & - \frac{7}{144}\varrho^{-4/3} a^{IK_1K_2} a^{JK_3K_4} a^{K_5K_6K_7} a^{K_8K_9L} \varepsilon_{K_1\ldots K_9} V^A{}_L \langle 0|_A \Big) ,\end{aligned}$$

$\hookrightarrow$ Components fit the Yukawa couplings [Ortiz–Samtleben]

$SO(9)$ gauged supergravity in two dimensions

$$\mathcal{L} = 2d\varrho \star d\sigma + \frac{1}{4}\varrho DM_{IJ} \star DM^{IJ} - \frac{1}{12}\varrho^{\frac{1}{3}} M_{IL} M_{JP} M_{KQ} Da^{IJK} \star Da^{LPQ}$$
$$- \frac{1}{6^4}\varepsilon_{I_1 I_2 I_3 J_1 J_2 J_3 K_1 K_2 K_3} a^{I_1 I_2 I_3} Da^{J_1 J_2 J_3} \wedge Da^{K_1 K_2 K_3} + h^K{}_I \Theta_{JK} F^{IJ} - V_{\text{gauged}}$$

with

$$\begin{aligned}
V_{\text{gauged}} = \frac{e^{2\sigma}}{2}\varrho^{\frac{5}{9}}\Theta_{IJ}\Theta_{KL} \Bigg(& \left(2M^{IK}M^{JL} - M^{IJ}M^{KL}\right) + \frac{1}{2}\varrho^{-2/3}\left(a^{IPQ}a^{KRS}M^{JL}M_{PR}M_{QS} - 2a^{IKP}a^{JLQ}M_{PQ}\right) \\
& + 2\varrho^{-2} h^I{}_P h^K{}_Q M^{Q[P}M^{J]L} + \varrho^{-8/3} a^{IPR} h^J{}_P a^{KQS} h^L{}_Q M_{RS} \\
& + \frac{\varrho^{-2}}{72} h^J{}_P a^{KQ_1Q_2} a^{LQ_3Q_4} a^{Q_5Q_6Q_7} \varepsilon_{Q_1\ldots Q_9} M^{IQ_8} M^{PQ_9} \\
& + \frac{3}{8}\varrho^{-4/3} a^{I[M_1M_2} a^{M_3M_4]J} a^{K[N_1N_2} a^{N_3N_4]L} M_{M_1N_1} M_{M_2N_2} M_{M_3N_3} M_{M_4N_4} \\
& + \frac{\varrho^{-2}}{2\cdot 144^2} a^{IN_1N_2} a^{JN_3N_4} a^{N_5N_6N_7} \varepsilon_{N_1\ldots N_9} a^{KP_1P_2} a^{LP_3P_4} a^{P_5P_6P_7} \varepsilon_{P_1\ldots P_9} M^{N_8P_8} M^{N_9P_9} \\
& + \frac{\varrho^{-8/3}}{576} a^{IRP} h^J{}_R a^{KN_1N_2} a^{LN_3N_4} a^{N_5N_6N_7} a^{N_8N_9Q} \varepsilon_{N_1\ldots N_9} M_{PQ} \\
& + \frac{\varrho^{-8/3}}{1152^2} a^{IN_1N_2} a^{JN_3N_4} a^{N_5N_6N_7} a^{N_8N_9Q} \varepsilon_{N_1\ldots N_9} a^{KP_1P_2} a^{LP_3P_4} a^{P_5P_6P_7} a^{P_8P_9S} \varepsilon_{P_1\ldots P_9} M_{QS} \Bigg)
\end{aligned}$$

Uplift formula

The metric in eleven dimensions reads

$$ds^2_{11\mathrm{D}} = \rho^{-\frac{8}{9}} e^{2\varsigma}(-dt^2 + dz^2) + \rho^{\frac{2}{9}} G_{\tilde{I}\tilde{J}}(\mathrm{d}y^{\tilde{I}} + \mathcal{A}^{\tilde{I}})(\mathrm{d}y^{\tilde{J}} + \mathcal{A}^{\tilde{J}})\,.$$

with the dilaton

$$\rho(x,y) = (\mathrm{det} g)^{\frac{1}{2}}\varrho(x)\,,$$

The internal metric

$$G_{\tilde{I}\tilde{J}}\mathrm{d}y^{\tilde{I}}\mathrm{d}y^{\tilde{J}} = G_{ij}\mathrm{d}y^i\mathrm{d}y^j + (\det G_{ij})^{-1}\,(\mathrm{d}\psi + K_i\mathrm{d}y^i)^2$$

satisfies

$$e^{2\varsigma}G^{ij} = \lambda^2\varrho^{\frac{2}{3}}e^{2\sigma}(\mathrm{det} g)^{\frac{5}{9}}\,\mathcal{Y}_I\, g^{ik}\partial_k\mathcal{Y}_J\,\mathcal{Y}_K\, g^{jl}\partial_l\mathcal{Y}_L\big(2M^{K[I}M^{J]L} + \varrho^{-2/3}M_{PQ}a^{IJP}a^{KLQ}\big)\,.$$

and $A_{tij} = \partial_i\mathcal{Y}^I\partial_j\mathcal{Y}^J\alpha_{IJ}(x,\mathcal{Y})$ satisfies

$$\mathcal{Y}_L\mathcal{Y}_K\big(2M^{K[P}M^{J]L} + \varrho^{-2/3}M_{RQ}a^{PJR}a^{KLQ}\big)\alpha_{IJ}(x,\mathcal{Y}) = \varrho^{-2/3}\mathcal{Y}_K a^{KLP}M_{IP}\,.$$

The $SO(3) \times SO(6)$ invariant truncation

$$\mathcal{L} = 2d\varrho \star d\sigma - \frac{1}{2}\varrho d\phi \star d\phi - \frac{1}{2}\varrho^{\frac{1}{3}} e^{-6\phi} da \star da + \frac{3}{2}\varrho^{\frac{5}{2}}\left(e^{4\phi} + 12e^{\phi} + 8e^{-2\phi} + \varrho^{-2/3}e^{-2\phi}a^2\right)$$

with explicit uplift [Anabalón–Ortiz–Samtleben for $a = 0$]

$$ds^2 = e^{2\sigma}\Delta(1+f)^{\frac{1}{3}}(-dt^2 + dz^2) + \varrho^{-\frac{14}{9}}\frac{\Delta}{(1+f)^{\frac{2}{3}}}(d\psi + \zeta^2 A_1 + (1-\zeta^2)A_2)^2 + \varrho^{\frac{4}{9}}(1+f)^{\frac{1}{3}}\left(e^{-\phi}\Delta\frac{d\zeta^2}{1-\zeta^2} + \frac{e^{-2\phi}}{(1+f)}\zeta^2 d\Omega_2^2 + e^{\phi}(1-\zeta^2)d\Omega_5^2\right)$$

where

$$\Delta = \zeta^2 e^{2\phi} + (1-\zeta^2)e^{-\phi} > 0\,, \qquad f = \frac{\zeta^2 e^{-4\phi}\varrho^{-\frac{2}{3}}a^2}{\Delta} \geq 0\,,$$

and

$$\begin{aligned} dA_1 &= e^{2\sigma}\varrho^{\frac{5}{9}}\left(e^{4\phi} + 6e^{\phi} + e^{-2\phi}\varrho^{-\frac{2}{3}}a^2\right)dt \wedge dz \\ dA_2 &= e^{2\sigma}\varrho^{\frac{5}{9}}\left(4e^{-2\phi} + 3e^{\phi}\right)dt \wedge dz \end{aligned}$$

Asymptotic pp-wave solutions

With $x = \tau/r$ and $\ell = 1$ we define $z(x)$ by $\frac{\partial x}{\partial z} = \frac{1-x^7}{x^{3/2}}$ and the pp-wave black hole solution

$$e^{2\sigma} = \frac{1-x^7}{x^7} , \quad \varrho = x^{-\frac{9}{2}} .$$

The linearised solutions with $\hat{\mu} \sim \frac{\mu}{2\pi T}$

$$\begin{aligned} a(x) &= -\frac{3}{4}\left(\hat{\mu} x \, {}_2F_1(\tfrac{1}{7}, \tfrac{4}{7}; \tfrac{5}{7}; x^7) + \alpha x^3 \, {}_2F_1(\tfrac{3}{7}, \tfrac{6}{7}; \tfrac{9}{7}; x^7)\right) + \mathcal{O}(x^5) , \\ \phi(x) &= \nu x^2 \, {}_2F_1(\tfrac{2}{7}, \tfrac{2}{7}; \tfrac{4}{7}; x^7) + \beta x^5 \, {}_2F_1(\tfrac{5}{7}, \tfrac{5}{7}; \tfrac{10}{7}; x^7) + \mathcal{O}(x^4) , \end{aligned}$$

- ⋆ $\hat{\mu}$ and ν non-normalisable modes:
  - $\hookrightarrow$ Sources trigger deformations of the matrix model.
- ⋆ α and β normalisable modes:
  - $\hookrightarrow$ Expectation values determined by a regular horizon.

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$$\begin{aligned} a(x) \;&=\; -\frac{\hat{\mu}}{4}\Big(3x\,{}_2F_1(\tfrac{1}{7},\tfrac{4}{7};\tfrac{5}{7};x^7) - \tfrac{\Gamma(\frac{5}{7})\Gamma(\frac{6}{7})\Gamma(\frac{10}{7})}{\Gamma(\frac{4}{7})\Gamma(\frac{8}{7})\Gamma(\frac{9}{7})}x^3\,{}_2F_1(\tfrac{3}{7},\tfrac{6}{7};\tfrac{9}{7};x^7)\Big) + \mathcal{O}(x^5) \,, \\ \phi(x) \;&=\; \nu\Big(x^2\,{}_2F_1(\tfrac{2}{7},\tfrac{2}{7};\tfrac{4}{7};x^7) - \tfrac{2\Gamma(\frac{11}{7})\Gamma(\frac{12}{7})^2}{5\Gamma(\frac{17}{7})\Gamma(\frac{9}{7})^2}x^5\,{}_2F_1(\tfrac{5}{7},\tfrac{5}{7};\tfrac{10}{7};x^7)\Big) + \mathcal{O}(x^4) \,, \end{aligned}$$

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triggers the expansion

$$e^{2\sigma} = \frac{1-x^7}{x^7}\Big(1 - \frac{14\nu^2}{13}x^4 + \mathcal{O}(x^5)\Big) \ , \quad \varrho = x^{-\frac{9}{2}}\Big(1 - \frac{9\nu^2}{13}x^4 + \mathcal{O}(x^5)\Big) \ ,$$

and

$$A^{\text{11D}} = \Big[\frac{1}{x^3}\Big(\hat{\mu} + \frac{3(\alpha - 3\nu\hat{\mu})}{2}x^2 + \mathcal{O}(x^4)\Big)dt - \frac{3}{4}\hat{\mu}x^4(1 + \mathcal{O}(x^2))d\psi\Big] \wedge \zeta^3 d\Omega_2 \ .$$

BMN thermodynamics

For $\hat{\mu} = \frac{\mu}{2\pi T}$ small enough [Hadizadeh, Ramadanovic, Semenoff, Young]
$\hookrightarrow$ "deconfined phase" $F \sim N^2$.

For $\hat{\mu} \geq 0.1$ the degeneracy $p(N)$ of vacua gives $F \sim 1$.

Study at strong coupling [Costa, Greenspan, Penedones, Santos]
$\hookrightarrow$ Can the consistent truncation gets analytic?

Conclusions

- E_9 exceptional field theory Scherk–Schwarz reduction
- Uplift formula for $SO(9)$ supergravity
- Application to M-theory Matrix Model
- Consistent truncations for BMN vacua?