

2d maximal supergravities from higher dimensions

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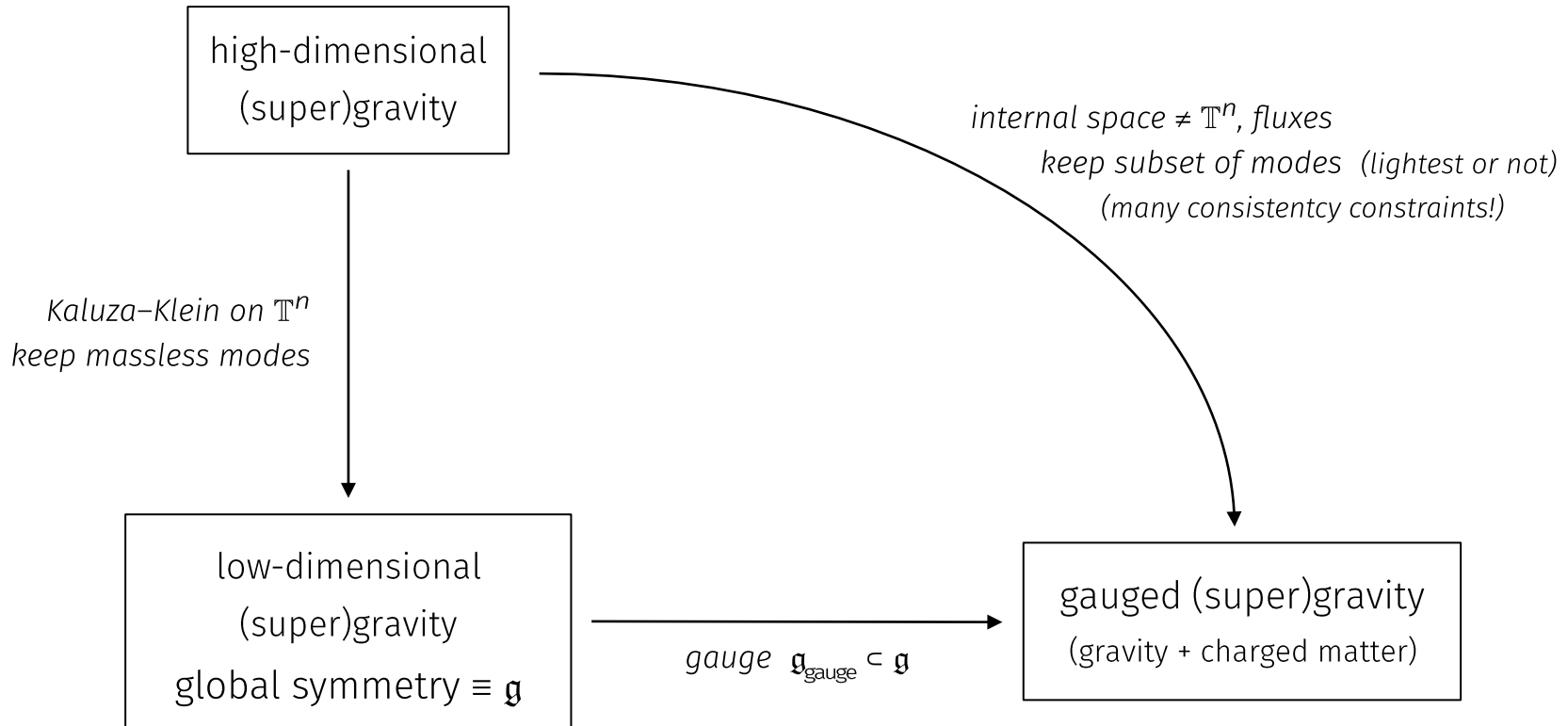
Supergravity techniques & the CFT bootstrap
AEI Potsdam, 09/11/23

based on works with:

Guillaume Bossard, Franz Ciceri, Axel Kleinschmidt 2209.02729, 2309.07232, 2309.07233

above + Henning Samtleben: 1811.04088, 2103.12118,

2d supergravity & KK reductions



2d supergravity & KK reductions

Prototype: 'simple' AdS solutions \iff $SO(N)$ gauged (maximal) supergravities

$$D3 : \text{AdS}_5 \times S^5 \iff SO(6) \mathcal{N} = 8, D = 5 \text{ gauged sugra}$$

$$M2 : \text{AdS}_4 \times S^7 \iff SO(8) \mathcal{N} = 8, D = 4 \text{ gauged sugra}$$

$$M5 : \text{AdS}_7 \times S^4 \iff SO(5) \mathcal{N} = 2, D = 7 \text{ gauged sugra}$$

▷ list grows much bigger with other vacua/internal spaces/less susy

▷ 'bottom-up': found interesting gauged sugra first, then *uplifted*

▷ recent maximal examples:

- mIIA on S^6 \iff $ISO(7)$, $D = 4$
- IIB on $S^5 \times S^1$ S-folds \iff $SO(6) \ltimes \dots$, $D = 4$

[Guarino Jafferis Varela]
[.....]

[Gi Samtleben Trigiante]
[Assel Tomasiello] [.....]

2d supergravity & KK reductions

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▷ list grows much bigger with other vacua/internal spaces/less susy (we focus here on maximal)

▷ use gauged sugra for

- SOLUTION GENERATION

- relevant/marginal deformations \leftrightarrow vacua in gauged sugra
- domain wall/RG flows
- (asymptotically AdS/Mink) black hole solutions
- !! bottom-up: new gauged sugras \Rightarrow new fluxed backgrounds

- SUSY & STABILITY:

- *full*, perturbative KK spectra from gauged supergravity structures
- some non-perturbative decay channels

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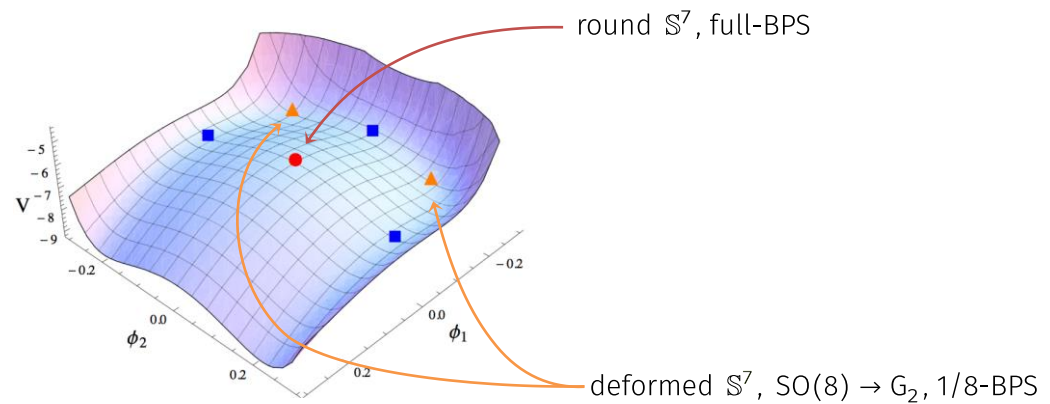
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susy \Rightarrow scalar potential $V(\phi)$

$$\frac{\delta V}{\delta \phi}(\phi_*) = 0$$

\Downarrow

$AdS_D \times \{\text{fluxed, deformed } \mathbb{S}^{d_1}\}$



[de Wit–Nicolai, ..., ...]

- we know many things on gauged sugras & KK truncations in $D \geq 3$
- $D = 2$ is more involved & less developed

AdS_2 is the realm of *decoupling limit of BHs*

D0 branes \rightarrow [conformal] $AdS_2 \times S^8$ + running dilaton + F_2 flux

- lifts to pp-wave in 11d with $SO(9)$ symmetry ($\sim S^8 \times S^1$) [Hull] [Nicolai Samtleben]
- dual: regularised supermembrane/BFSS matrix model [de Wit, Hoppe, Nicolai]
[Banks, Fischler, Schenker, Susskind]
- thermal & massive (BMN) deformations [Lin Lunin Maldacena] [...]
[Costa Greenspan Penedones Santos]

lightest states' dynamics expected \equiv 2d $\mathcal{N} = 32$ $SO(9)$ gauged sugra [Ortiz-Samtleben]
[Anabalón Ortiz Samtleben]
[Ortiz Samtleben Tsimpis]

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constructive proof
(embed all 2d field configs into 11d)

\rightarrow *Guillaume's talk*

A broader perspective:

① what is general structure of 2d gauged sugras?

(possible gaugings, dynamics, solutions, ...)

breaking 'Geroch group' / integrability

- analytic
- numeric → ML

(maximal main focus,
but applies more in general)

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② study compactifications leading to ①

- ▷ new geometries
- ▷ uplift formulæ

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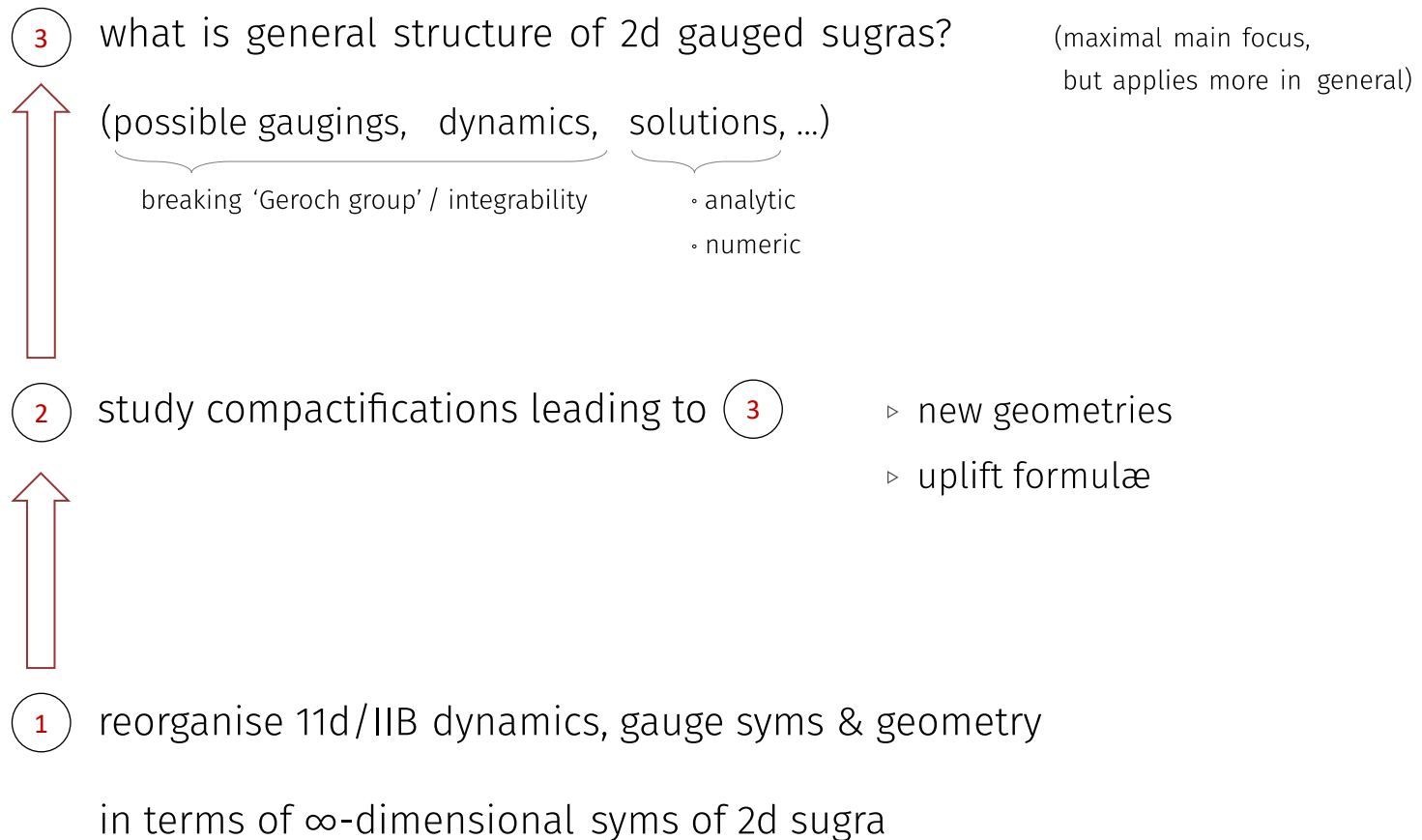
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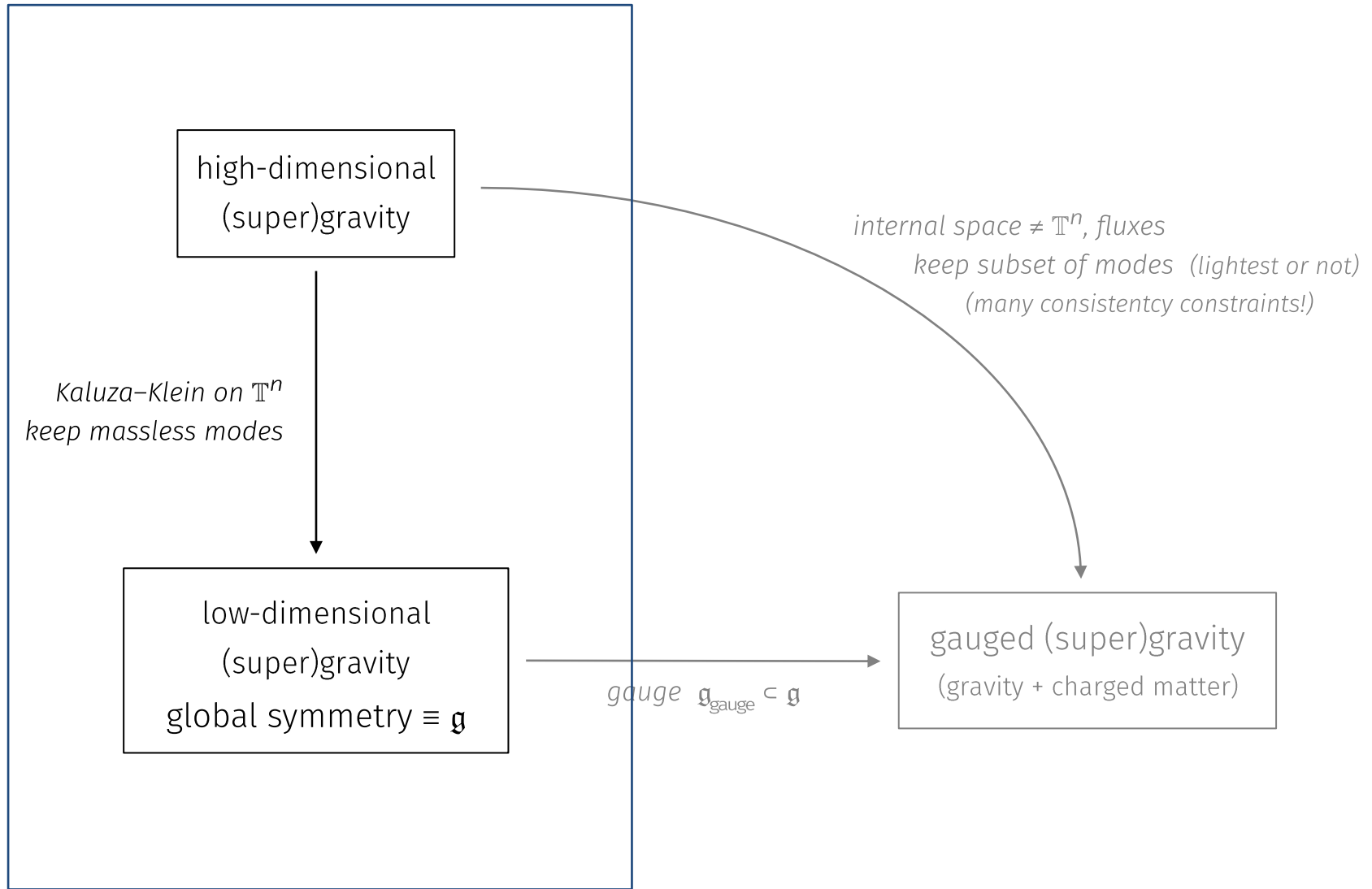
③ reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞ -dimensional syms of 2d sugra

A broader perspective:

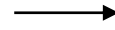


2d supergravity & KK reductions



Geroch group & E_9

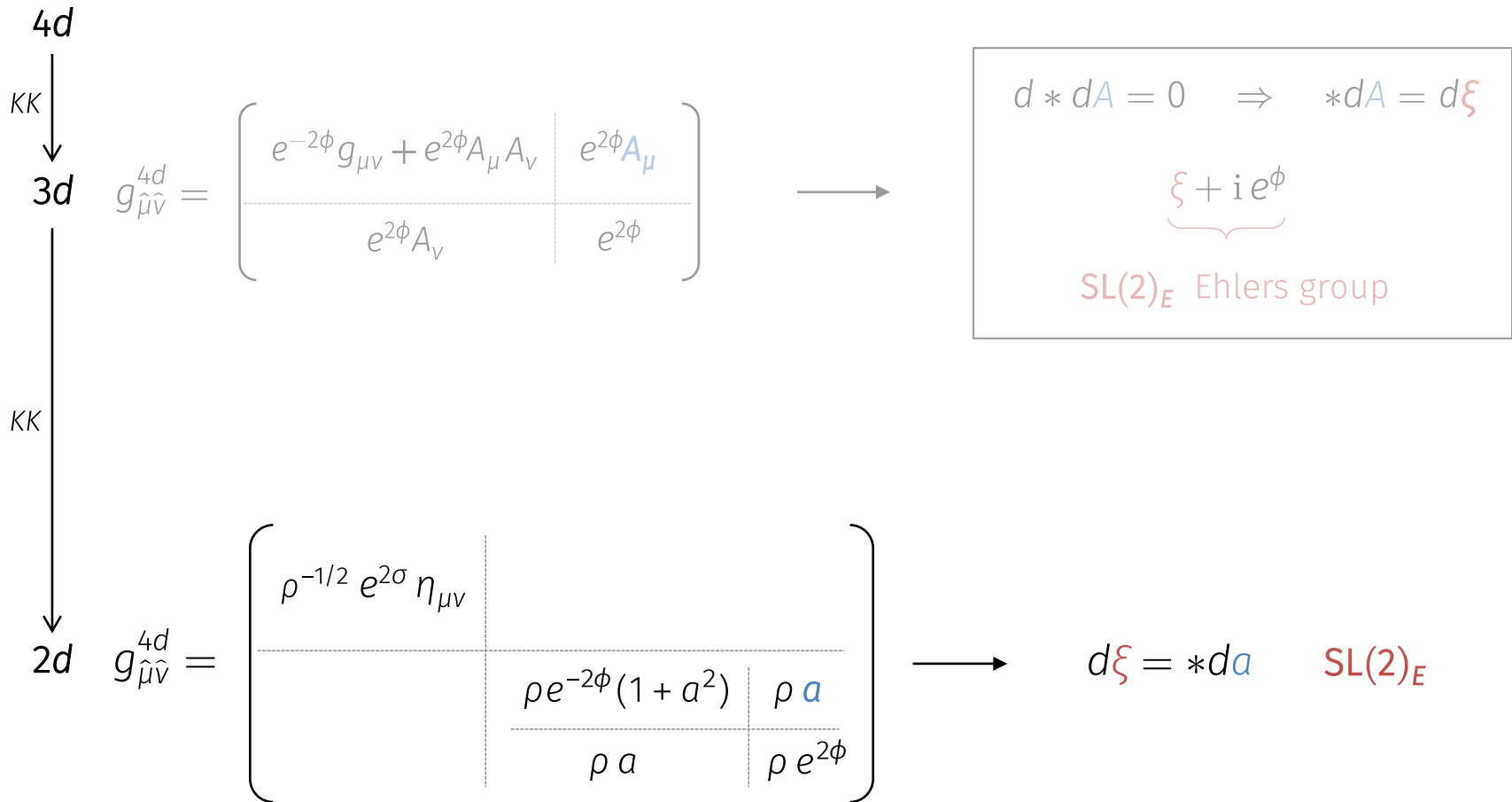
$$\begin{array}{c}
 4d \\
 \downarrow \text{KK} \\
 3d
 \end{array}
 g_{\hat{\mu}\hat{\nu}}^{4d} = \left(\begin{array}{c|c}
 e^{-2\phi} g_{\mu\nu} + e^{2\phi} A_\mu A_\nu & e^{2\phi} A_\mu \\
 \hline
 e^{2\phi} A_\nu & e^{2\phi}
 \end{array} \right)$$



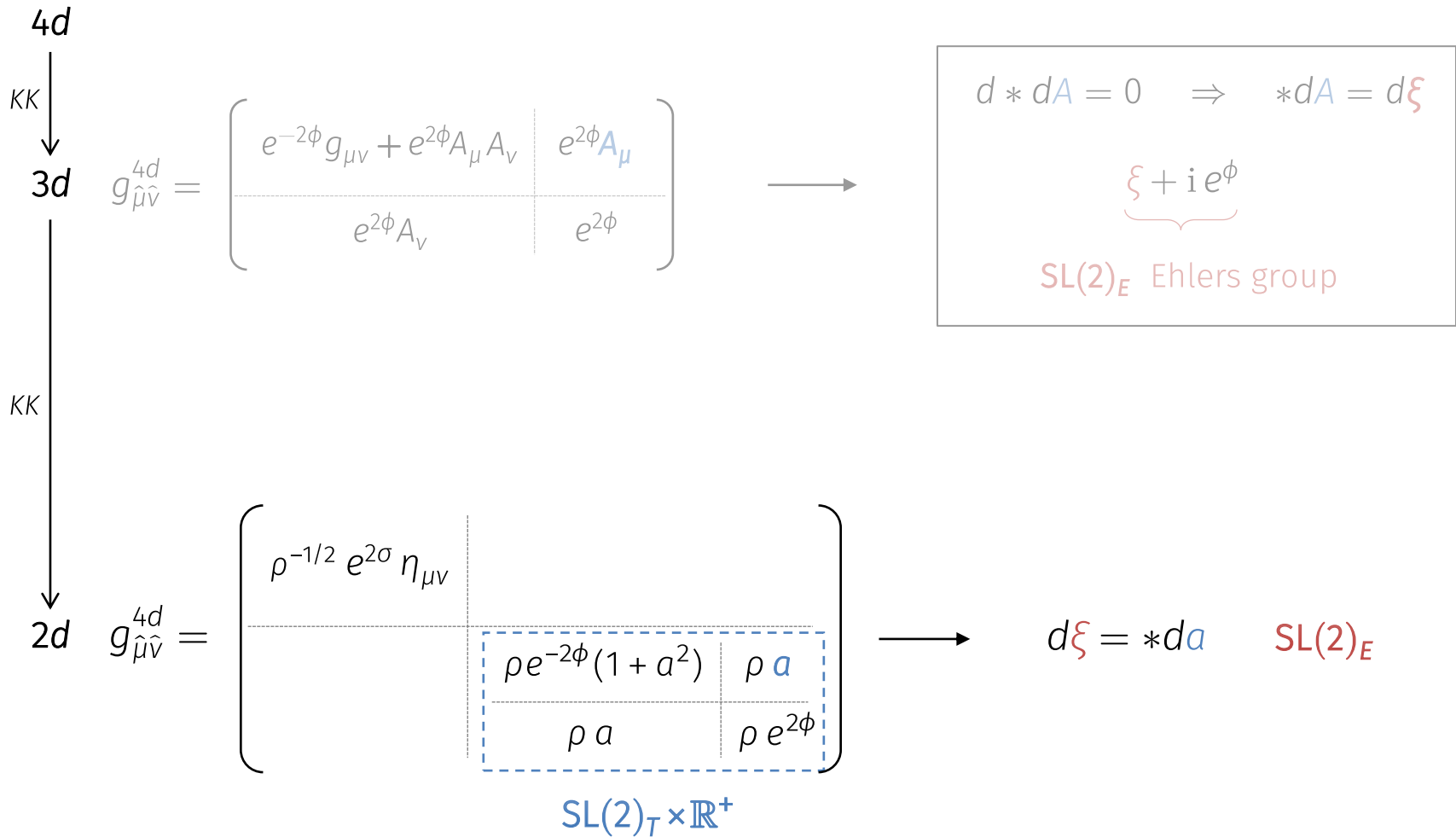
$$d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

$$\underbrace{\xi + ie^\phi}_{\text{SL}(2)_E \text{ Ehlers group}}$$

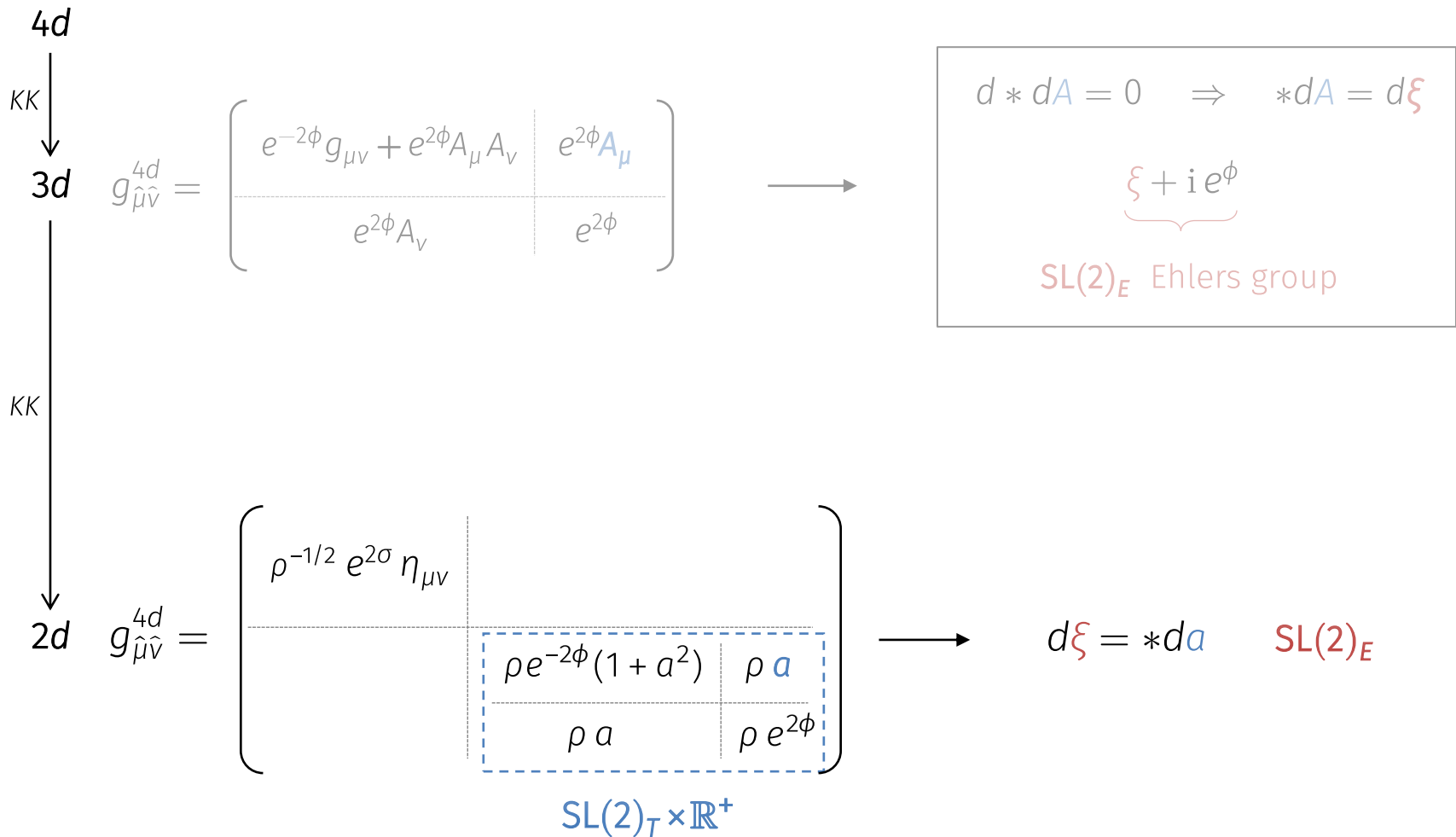
Geroch group & E₉



Geroch group & E_9



Geroch group & E₉



$$\text{SL}(2)_E \times' \text{SL}(2)_T \times \mathbb{R}^+ \longrightarrow \text{loop group } \widehat{\text{SL}(2)} \times \mathbb{R}^+$$

$$\begin{array}{c}
 4d \text{ GR} \\
 \downarrow \\
 2d
 \end{array}
 \quad
 \begin{array}{c}
 SL(2)_E \text{ '}\times\text{' } SL(2)_T \times \mathbb{R}^+ \longrightarrow \text{loop group } \widehat{SL(2)} \rtimes \mathbb{R}^+
 \end{array}$$

$$\begin{array}{c}
 11d \\
 \downarrow \\
 2d
 \end{array}
 \quad
 \begin{array}{c}
 SL(2)_{Ehlers} \longrightarrow E_{8(8)} \\
 SL(2)_T \longrightarrow SL(9)
 \end{array}
 \left. \vphantom{\begin{array}{c} SL(2)_{Ehlers} \\ SL(2)_T \end{array}} \right\} \longrightarrow E_9$$

The linear system & twisted self-duality

G-invariant

$$\mathcal{L} = 2d\sigma \wedge \star d\rho - \rho \eta^{AB} P_A^0 \wedge \star P_B^0$$

G/H current $\frac{1}{2}(dv v^{-1} + \text{h.c.})$

e.g.

- ▷ reduction of 4d GR: SL(2)/SO(2)
- ▷ maximal sugra: E₈/Spin(16)

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- e.g.
- ▷ reduction of 4d GR: SL(2)/SO(2)
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EOM: $d \star d\rho = 0 \quad \longrightarrow \quad d\tilde{\rho} = \star d\rho$

$d(\rho \star v^{-1} P^0 v) = 0 \quad \longrightarrow \quad dY_1 = 2\rho \star v^{-1} P^0 v$

$\longrightarrow \quad dY_2 = \left(\frac{1}{2}\rho^2 + \rho\tilde{\rho} \star\right) v^{-1} P^0 v - [Y_1, dY_1]$

$\longrightarrow \quad \dots$

The linear system & twisted self-duality

$$V(w) = v + w^{-1}V_1 + w^{-2}V_2 + \dots$$

$$\stackrel{w \sim +\infty}{=} v e^{w^{-1}Y_1} e^{w^{-2}Y_2} \dots \quad w \in \mathbb{C}$$

The linear system & twisted self-duality

$$V(w) = v + w^{-1}V_1 + w^{-2}V_2 + \dots$$

$$dVV^{-1} = P^0 + Q^0$$

$$\stackrel{w \sim +\infty}{=} v e^{w^{-1}Y_1} e^{w^{-2}Y_2} \dots \quad w \in \mathbb{C}$$

$$\hat{e}_8 \ni dVV^{-1}(w) = Q^0 + \frac{1+\gamma^2}{1-\gamma^2} P^0 + \frac{2\gamma}{1-\gamma^2} \star P^0$$

$$\gamma = \frac{w-\tilde{\rho}}{\rho} \pm \sqrt{\left(\frac{w-\tilde{\rho}}{\rho}\right)^2 - 1}$$

[Breitenlohner-Maison]

integrability

$$\text{EOM: } d \star d\rho = 0$$

$$d(\rho \star v^{-1} P^0 v) = 0$$

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[Breitenlohner–Maison]

...a different direction:

- if we reduce directly from 11d, E_8 is not manifest, a different $GL(9)$ is
- $V(w)$ is in a *fixed* triangular gauge, E_8 covariant
- how to write this for *highest weight reps*? There is no “ w ” there. *Square root*?

$$\frac{d\gamma}{\gamma}(w) = \frac{1+\gamma^2}{1-\gamma^2} d\rho + \frac{2\gamma}{1-\gamma^2} *d\rho$$



geometric series

$$\frac{d\gamma}{\gamma}(w) \stackrel{\gamma \sim +\infty}{\cong} (1 + 2\gamma^{-2} + 2\gamma^{-4} + \dots) d\rho + (2\gamma^{-1} + 2\gamma^{-3} + \dots) * d\rho$$

$$\gamma(w) \xrightarrow{w \rightarrow +\infty} c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \dots$$

↑
asymptotics

$$\frac{d\gamma}{\gamma}(w) \stackrel{\gamma \sim +\infty}{\cong} (1 + 2\gamma^{-2} + 2\gamma^{-4} + \dots) d\rho + (2\gamma^{-1} + 2\gamma^{-3} + \dots) * d\rho$$

$$\gamma(w) \xrightarrow{w \rightarrow +\infty} c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \dots$$

$$\simeq \underbrace{\dots e^{\varphi_3 L_{-3}} e^{\varphi_2 L_{-2}} e^{\varphi_1 L_{-1}} \rho^{L_0}}_{\Gamma^{-1}} w$$

$$L_m = -w^{m+1} \partial_w$$

$$d\Gamma \Gamma^{-1} = (L_0 + 2L_{-2} + 2L_{-4} + \dots) dp + (2L_{-1} + 2L_{-3} + \dots) *dp$$

$$\gamma(W) \xrightarrow{W \rightarrow +\infty} c_1 W + c_0 + c_{-1} W^{-1} + c_{-2} W^{-2} + \dots$$

$$\simeq \underbrace{\dots e^{\varphi_3 L_{-3}} e^{\varphi_2 L_{-2}} e^{\varphi_1 L_{-1}} \rho^{L_0}}_{\Gamma^{-1}} W$$

$$L_m = -W^{m+1} \partial_W$$

$$d\Gamma \Gamma^{-1} = (L_0 + 2L_{-2} + 2L_{-4} + \dots) dp + (2L_{-1} + 2L_{-3} + \dots) *dp$$

\Leftrightarrow

$$*p = w p \equiv S_1(p)$$



writable in h.w.r.

$$S_p(T_m^A) = T_{m+p}^A$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(K) = 0$$

$$p \equiv d\Gamma \Gamma^{-1} + (d\Gamma \Gamma^{-1})^\dagger$$

The linear system & twisted self-duality

[Julia-Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

the full system:

$$\mathcal{V} = \underbrace{\rho^{-L_0} e^{-\varphi_1 L_{-1}} e^{-\varphi_2 L_{-2}} \dots}_{\Gamma} V_{e_8} \underbrace{e^{Y_{1A} T_{-1}^A} e^{Y_{2A} T_{-2}^A} \dots e^{-\sigma K}}_{V_{loop}}$$

$$\star \mathcal{P} = S_1(\mathcal{P}) + \tilde{\chi}_1 K$$

$$S_p(T_m^A) = T_{m+p}^A$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(K) = 0$$

auxiliary one-form,
restores covariance

The linear system & twisted self-duality

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the full system:

$$\mathcal{V} = \underbrace{\rho^{-L_0} e^{-\varphi_1 L_{-1}} e^{-\varphi_2 L_{-2}} \dots}_{\Gamma} V_{loop} \quad \leftarrow \text{any parabolic parametrisation}$$

$$\star \mathcal{P} = S_1(\mathcal{P}) + \tilde{\chi}_1 K$$

auxiliary one-form,
restores covariance

$$S_p(T_m^A) = T_{m+p}^A$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(K) = 0$$

The linear system & twisted self-duality

$$\mathcal{V} = \Gamma V_{loop} \in \frac{\hat{E}_8 \rtimes \text{Vir}^-}{K(E_9)}$$

$$\star \mathcal{P} = S_1(\mathcal{P}) + \tilde{X}_1 K$$

$\hat{e}_8 \oplus \text{vir}^-$

- E_8 isometries T_0^A
- σ shift K
- ρ rescaling L_0
- $\tilde{\rho}$ shift L_{-1}
- Y_m shift ($m > 0$) T_{-m}^A
- *hidden loop* T_{+m}^A
- *further $\tilde{\rho}/Y$ reparam.* L_{-m}

$$d\mathcal{P} - Q \wedge \mathcal{P} - \mathcal{P} \wedge Q = 0$$

$$S_1(d\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q})$$



$$S_p(T_m^A) = T_{m+p}^A$$

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$$S_p(K) = 0$$

$$S_1(d\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q}) + \left(\frac{1}{2\rho} \mathcal{L}^{top}\right) K = \text{covariant}$$

↑

$$S_p(T_m^A) = T_{m+p}^A$$

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$$\cancel{S_1(d\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q})} + \left(\frac{1}{2\rho} \mathcal{L}^{top} \right) K = \text{invariant}$$

$= 0$

$$\frac{1}{2\rho} \mathcal{L}^{top} = \mathcal{P}_{L-1} \wedge d\sigma + \sum_{n \geq 1} (\mathcal{P}_{L-n+1} - \mathcal{P}_{L-1-n}) \wedge \tilde{\chi}_n - \eta^{AB} \sum_{n \in \mathbb{Z}} n \mathcal{Q}_A^n \wedge \mathcal{P}_B^{-n-1}$$

$$\cancel{S_1(d\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q})} + \left(\frac{1}{2\rho} \mathcal{L}^{top} \right) \mathbb{K} = \text{invariant}$$

$= 0$

$$\frac{1}{2\rho} \mathcal{L}^{top} = \mathbf{D}\tilde{\chi}_1$$

Duality frames & physical Lagrangians

11d



3d



dualise KK vectors & $C_{\mu mn}$



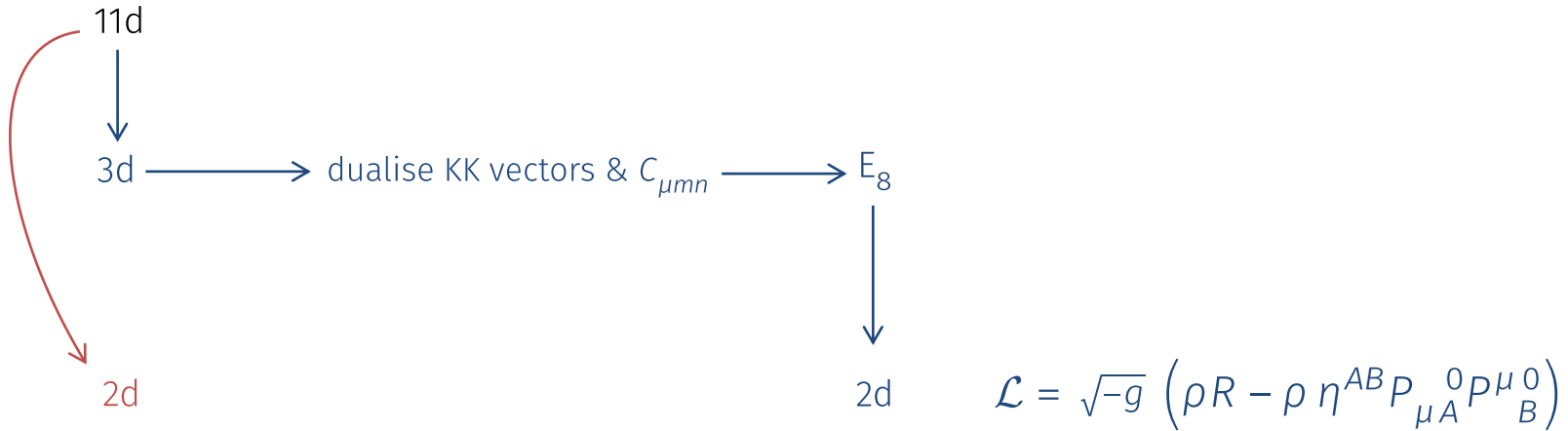
E_8



2d

$$\mathcal{L} = \sqrt{-g} \left(\rho R - \rho \eta^{AB} P_{\mu A}{}^0 P^{\mu 0}{}_B \right)$$

Duality frames & physical Lagrangians



$$\mathcal{L} = \sqrt{-g} \left(\rho R + \frac{1}{4} \rho g^{\mu\nu} \partial_\mu m^{IJ} \partial_\nu m_{IJ} - \frac{1}{12} \rho^{1/3} g^{\mu\nu} \partial_\mu a_{I_1 I_2 I_3} \partial_\nu a_{J_1 J_2 J_3} m^{I_1 J_1} m^{I_2 J_2} m^{I_3 J_3} \right) \\ + \frac{1}{64} \varepsilon^{\mu\nu} \varepsilon^{I_1 \dots I_9} a_{I_1 I_2 I_3} \partial_\mu a_{I_4 I_5 I_6} \partial_\nu a_{I_7 I_8 I_9} \quad m \in \frac{SL(9)}{SO(9)}$$

covariant w.r.t. $SL(9) \not\subseteq E_8$ (spectrally flowed)

Duality frames & physical Lagrangians

pseudoaction \rightarrow physical action by *completing the self-duality squares*

$$\mathcal{L}^{pseudo} = \mathcal{L}^{phys} + \sum_{\infty} (\text{twisted self-duality})^2$$

gravity/dilaton sector:

$$2\rho P_0 \tilde{X}_1 + 2\rho \sum_{n \geq 1} P_n (\tilde{X}_{n+1} - \tilde{X}_{n-1}) = \underbrace{2d\rho \wedge \star d\sigma}_{\sqrt{-g} \rho R} + 2\rho \sum_{n=0}^{+\infty} (\star P_n - P_{n+1}) (\tilde{X}_n - \star \tilde{X}_{n+1})$$

Duality frames & physical Lagrangians

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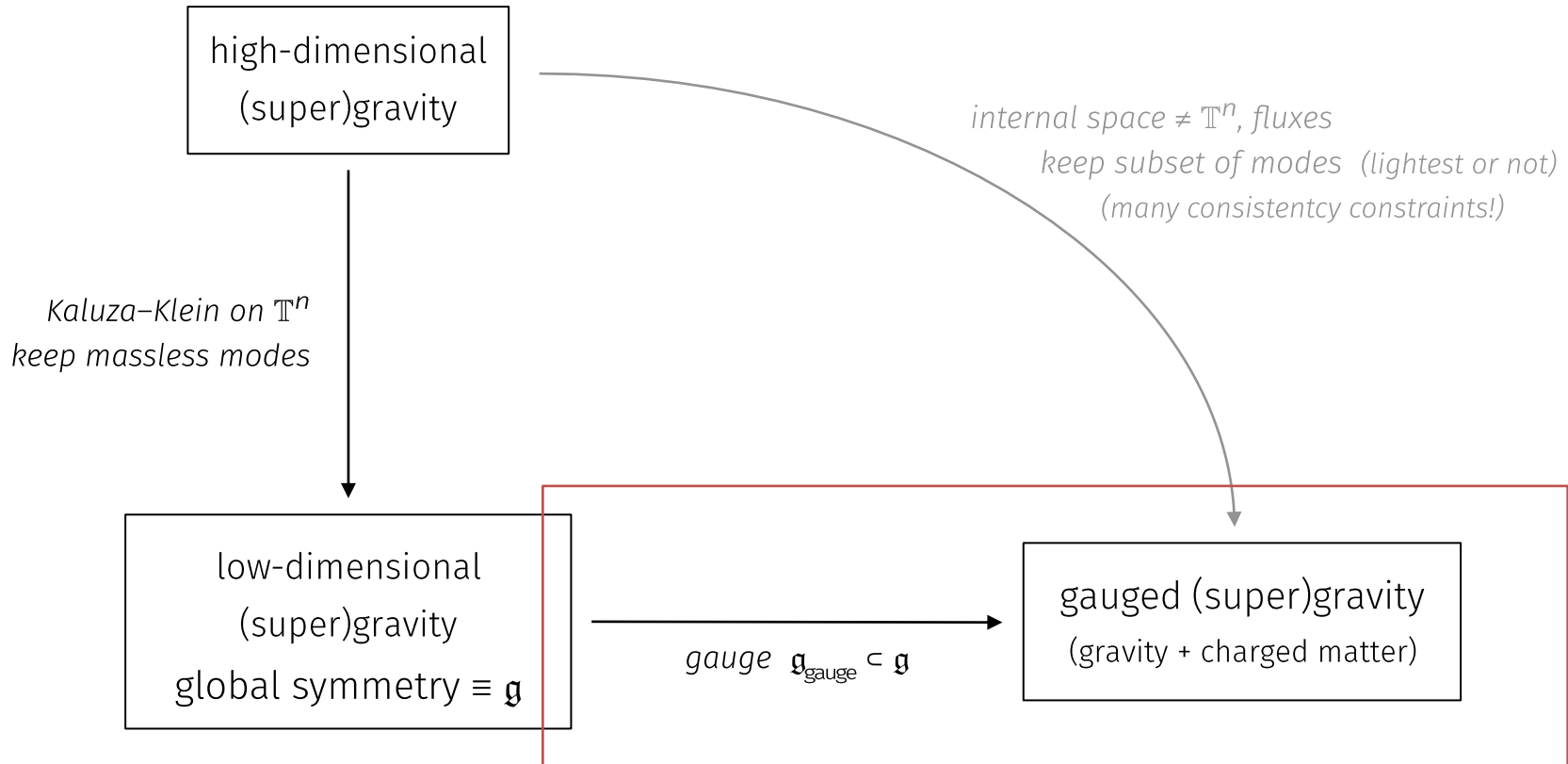
full theory: depends on parabolic gauge

$$\circ V_{loop} = V_{e_8} e^{Y_{1A} T_{-1}^A} e^{Y_{2A} T_{-2}^A} \dots \longrightarrow \mathcal{L} = \sqrt{-g} \left(\rho R - \rho \eta^{AB} P_{\mu A}^0 P_{\mu B}^0 \right)$$

$$\circ V_{loop} = V_{sl_9} e^{a^{IJK} T_{-1/3 IJK}} e^{b_{IJK} T_{-2/3}^{IJK}} \dots$$

$$\begin{aligned} \mathcal{L} = \sqrt{-g} & \left(\rho R + \frac{1}{4} \rho g^{\mu\nu} \partial_{\mu} m^{IJ} \partial_{\nu} m_{IJ} - \frac{1}{12} \rho^{1/3} g^{\mu\nu} \partial_{\mu} a^{1'2'3} \partial_{\nu} a^{1'2'3} m_{1'1'} m_{2'2'} m_{3'3'} \right) \\ & + \frac{1}{6^4} \varepsilon^{\mu\nu} \varepsilon_{1\dots 9} a^{1'2'3} \partial_{\mu} a^{4'5'6} \partial_{\nu} a^{7'8'9} \end{aligned}$$

$D = 2$ gauged supergravity



$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

$D = 2$ gauged supergravity

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

▷ $D > 2$: SUSY + $\Theta \implies$ full Lagrangian

lectures: [Samtleben '08][Trigiante '16]

$$\partial \rightarrow \mathcal{D}$$



$$\delta_{\text{SUSY}} \mathcal{L} \neq 0 \implies \text{correct } \delta_{\text{SUSY}}(\text{fermions}) \propto \Theta$$

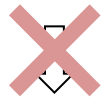


$$\text{add } V(\phi) \propto \Theta^2 \implies \delta_{\text{SUSY}} \mathcal{L} = 0$$

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

- ▷ $D = 2$ SUSY too difficult: $K(\mathfrak{e}_9)$ representation theory

$$\partial \rightarrow \mathcal{D}$$



$$\delta_{\text{SUSY}} \mathcal{L} \neq 0 \Rightarrow \text{correct } \delta_{\text{SUSY}}(\text{fermions}) \propto \Theta$$

not available for generic Θ in $D = 2$



? $V(\phi)$?

[Samtleben–Weidner]: construction up to $V(\phi)$; fixed $K(\mathfrak{e}_9)$ gauge / duality frame

③ what is general structure of 2d gauged sugras?



② study compactifications leading to ③

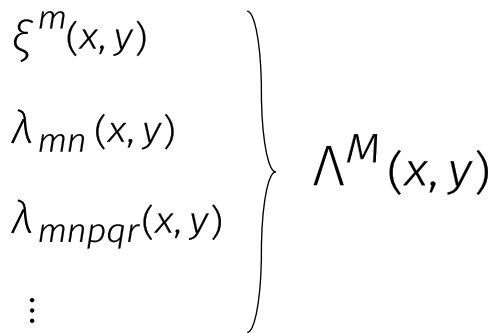
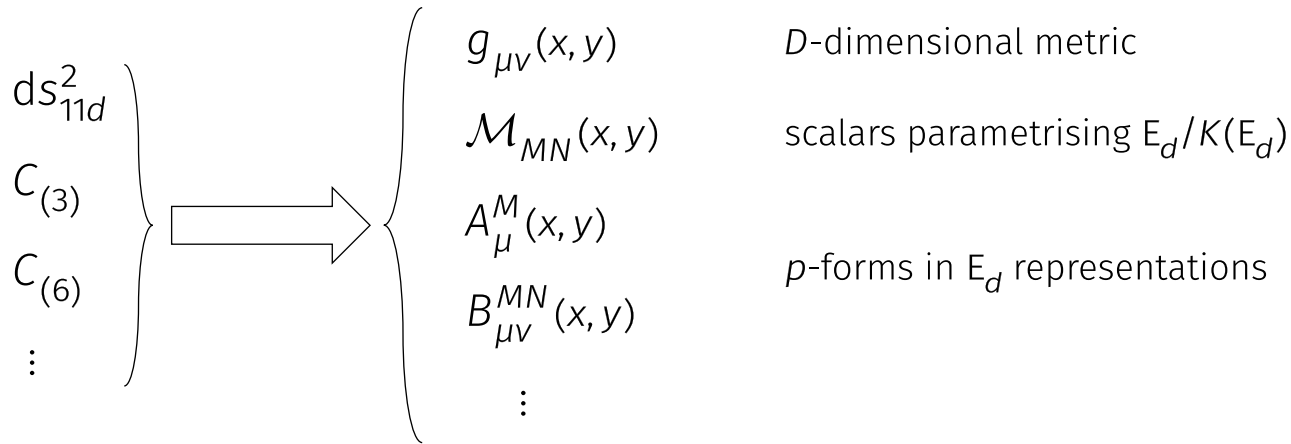


① reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞ -dimensional syms of 2d sugra

E_9 exceptional field theory (very briefly)

start from 11d sugra



$$\mathcal{L}_\Lambda \phi = \Lambda^M \partial_M \phi + \mathbb{P}_\alpha(\partial \Lambda) \delta_{e_d \oplus \mathbb{R}}^\alpha \phi$$

generalised diffeomorphisms

[...] [Berman Perry] [...] [Berman Cederwall Kleinschmidt Thompson]
 [Coimbra Strickland-Constable Waldram] [...] [Hohm Samtleben]

$$\mathcal{L}_\Lambda \phi = \Lambda^M \partial_M \phi + \mathbb{P}_\alpha (\partial \Lambda) \delta_{e_d \oplus \mathbb{R}}^\alpha \phi$$

generalised diffeomorphisms

$$\partial_M = \begin{pmatrix} \frac{\partial}{\partial y^m} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

determined by *algebraic* condition

$$\mathbb{P}(\partial_M \otimes \partial_N) = 0 \quad \text{section constraint}$$

- ▷ two solutions: 11d or IIB supergravity
- ▷ we can keep *formal* E_d covariance and treat both *at same time*

E_9 exceptional field theory (very briefly)

$$\Lambda^M \in \text{basic } E_9 \text{ irrep: } T_m^A |0\rangle = (K-1)|0\rangle = 0, \quad m \geq 0 \quad (\Lambda^M \leftrightarrow |\Lambda\rangle)$$

$$\mathfrak{e}_8 \oplus L_0 \begin{array}{l} \downarrow \\ \rightarrow \mathbf{1}_0 + 248_{-1} + (\mathbf{1} + 248 + 3875)_{-2} + \dots \end{array}$$

[Bossard Cederwall Kleinschmidt Palmkvist Samtleben]

$$\mathcal{L}_{\Lambda, \Sigma} = \Lambda^M \partial_M + \left(\mathbb{P}_\alpha(\partial\Lambda) + \underset{\substack{\uparrow \\ \text{'ancillaries'}}}{\Sigma_\alpha} \right) \delta_{\mathfrak{e}_8 + \mathfrak{vir}}^\alpha$$

DYNAMICS

- 1 covariantise maximal ^{pseudo-}sugra-action w.r.t. generalised diffeos
- 2 add invariant *internal curvature*, $\mathcal{O}(\partial_M^2)$

① **covariantise** w.r.t. generalised diffeos

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$$

$$\mathcal{S}_1 \left(\underbrace{\mathcal{D}\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q}}_{=0} + \text{field strenghts} \right) + \frac{1}{2\rho} \mathcal{L}_{\text{pseudo}} K = \text{gendiffeo covariant}$$

① **covariantise** w.r.t. generalised diffeos

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$$

$$\frac{1}{2\rho} \mathcal{L}_{\text{pseudo}} = \mathcal{P}_{L_{-1}} \wedge \mathcal{D}\sigma + \sum_{n \geq 1} (\mathcal{P}_{L_{-n+1}} - \mathcal{P}_{L_{-1-n}}) \wedge \tilde{\chi}_n - \eta^{AB} \sum_{n \in \mathbb{Z}} n \mathcal{Q}_A^n \wedge \mathcal{P}_B^{-n-1}$$

+ field strengths

① **covariantise** w.r.t. generalised diffeos

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$$

$$\frac{1}{2\rho} \mathcal{L}_{\text{pseudo}} = \mathbf{D}\tilde{\chi}_1 + \text{field strenghts}$$

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2 add invariant **internal curvature**, $\mathcal{O}(\partial_M^2)$

built from scratch!

• $\mathcal{P}_{\underline{M}}$ = internal current

• $\mathcal{P}_{\underline{M}}^{(+1)} = \mathcal{S}_1(\mathcal{P}_{\underline{M}}) + \tilde{\chi}_{\underline{M}} K$

$$\rho V_{\text{EXFT}} = \underbrace{\eta^{\alpha\beta} \mathcal{P}_{\underline{M}\alpha} \mathcal{P}_{\underline{M}\beta} - 2 \mathcal{P}_{\underline{M}\alpha} \mathcal{P}_{\underline{N}\beta} (T^{\beta\dagger} T^\alpha)_{\underline{MN}} - 2 \mathcal{P}_{\underline{M}\alpha} T^{\alpha M}{}_{\underline{N}} \partial_{\underline{N}} \rho}_{\text{standard terms}} + \underbrace{+ 2 \mathcal{P}_{\underline{M}\alpha} \mathcal{P}_{\underline{N}\beta}^{(+1)} (T^{\beta\dagger} T^\alpha)_{\underline{MN}}}_{\Sigma \text{ gauge invariance}}$$

③ what is general structure of 2d gauged sugras?



② study compactifications leading to ③



① reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞ -dimensional syms of 2d sugra

2d supergravity & KK reductions

$$\text{EOM} \left(\begin{array}{l} 10\text{d}/11\text{d sugra} \\ \text{on fluxed bckgd} \end{array} \right) = 0 \quad \longleftarrow \quad \text{EOM} \left(\begin{array}{l} \text{low-dim. sugra} \\ \text{with gauge group } G \end{array} \right) = 0 \quad \times \quad \left\langle \begin{array}{l} \text{geometric data} \\ \text{of fluxed bckgd} \end{array} \right\rangle$$

example: *Scherk-Schwarz*

$$ds^2 = \Delta(x, y) g(x, y)_{\mu\nu} dx^\mu dx^\nu + \mathbf{g}(x, y)_{mn} (dy^m + K(x, y)_\mu{}^m dx^\mu) (dy^n + K(x, y)_\nu{}^n dx^\nu)$$

▷ take internal space = Lie group G

▷ left-invariant forms: $e^{\underline{m}}_m(y) T_{\underline{m}} dy^m$

▷ factorise: $g_{mn}(x, y) = \mathbf{g}_{\underline{mn}}(x) e^{\underline{m}}_m(y) e^{\underline{n}}_n(y)$

$$K_\mu{}^m(x, y) = \mathbf{a}_\mu^{\underline{m}}(x) e_{\underline{m}}{}^m(y)$$

$$[e_{\underline{m}}, e_{\underline{n}}] = f_{\underline{mn}}{}^{\underline{p}} e_{\underline{p}}$$

gauge algebra

generalised Scherk–Schwarz $\mathcal{U} \in E_d$

$$\left. \begin{array}{l} ds_{11d}^2 \\ C_{(3)} \\ C_{(6)} \\ \vdots \end{array} \right\} \longrightarrow \left\{ \begin{array}{l} A_{\mu}^M(x, y) = a_{\mu}^A(x) \overbrace{r^{-1}(y) \mathcal{U}^{-1}{}^M_A(y)}^{E_A{}^M(y)} \\ g_{\mu\nu}(x, y) = g_{\mu\nu}(x) r^{-2}(y) \\ \mathcal{M}_{MN}(x, y) = M_{AB}(x) \mathcal{U}^A{}_M(y) \mathcal{U}^B{}_N(y) \\ \vdots \qquad \qquad \qquad \vdots \end{array} \right.$$

$$\mathcal{L}_{E_A} E_B = -\Theta_{A\alpha} (T^{\alpha})_B{}^C E_C$$



$$\partial_{\mu} \longrightarrow \mathcal{D}_{\mu} = \partial_{\mu} - A_{\mu}^A \Theta_{A\alpha} T^{\alpha}$$

2d supergravity & KK reductions

- $$\mathcal{L}_{E_A E_B} = - \underbrace{\Theta_{A\alpha} (T^\alpha)_B^C}_{\mathbb{P}(W_{A\alpha})} E_C$$

$$W_{A\alpha} \sim \partial E$$

- $$D = 2: \quad \Theta_{A\alpha} \rightarrow \langle \Theta_\alpha | = \eta_{-1\alpha\beta} \langle \theta | T^\beta$$

$$\uparrow$$

$$\hat{\mathfrak{e}}_8 \oplus \langle L_{-1} \rangle \text{ invariant}$$

[Samtleben–Weidner]

$$\langle \theta | = \langle W_\alpha | S_1(T^\alpha) + \overset{\text{new}}{\langle W^+ |}$$

$$\langle \theta | = \langle W_\alpha | T^\alpha$$

↖ trombone → 0

[Bossard Cederwall Kleinschmidt Palmkvist Samtleben]

[Bossard Ciceri GI Kleinschmidt]

The scalar potential of $D = 2$ gauged maximal supergravities

$$\partial \rightarrow \mathcal{D}$$



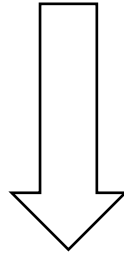
$$\delta_{\text{SUSY}} \mathcal{L} \neq 0 \Rightarrow \text{correct } \delta_{\text{SUSY}}(\text{fermions}) \propto \Theta$$



$$? V(\phi) ?$$

The scalar potential of $D = 2$ gauged maximal supergravities

$$\textcircled{2} \quad \rho V_{\text{EXFT}} = \eta^{\alpha\beta} \mathcal{P}_{\underline{M}\alpha} \mathcal{P}_{\underline{M}\beta} - 2 \mathcal{P}_{\underline{M}\alpha} \mathcal{P}_{\underline{N}\beta} (T^{\beta\dagger} T^\alpha)_{\underline{MN}} - 2 \mathcal{P}_{\underline{M}\alpha} T^{\alpha M} \underline{N} \partial_{\underline{N}} \rho + 2 \mathcal{P}_{\underline{M}\alpha}^{(+1)} \mathcal{P}_{\underline{N}\beta}^{(+1)} (T^{\beta\dagger} T^\alpha)_{\underline{MN}}$$



$$2\rho V_{\text{g.sugra}} = \rho^{-2} \langle \theta | M^{-1} | \theta \rangle + \eta_{-2\alpha\beta} \langle \theta | T^\alpha M^{-1} T^{\beta\dagger} | \theta \rangle$$

$\forall D = 2$ gauged maximal supergravity with higher dimensional origin

$D = 2$ maximal supergravities from inner space

$$\mathcal{L}_{pseudo} = \mathcal{L}_{top} - V_{g.sugra}$$

$$\frac{1}{2\rho} \mathcal{L}_{top} = \mathbf{D}\tilde{\chi}_1 + \langle \theta | \mathbf{O}(M) | F \rangle \longrightarrow \text{physical Lagrangians}$$

$$2\rho V_{g.sugra} = \rho^{-2} \langle \theta | M^{-1} | \theta \rangle + \eta_{-2\alpha\beta} \langle \theta | T^\alpha M^{-1} T^{\beta\dagger} | \theta \rangle$$

$\forall D = 2$ gauged maximal supergravity with higher dimensional origin

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$\forall D = 2$ gauged maximal supergravity with higher dimensional origin

Even in higher D , no one has been able to classify them
(no such thing as “All gauged maximal supergravities with uplift”)

...but we do know several things!

$D = 2$ maximal supergravities from inner space

[GI]

[Bugden Hulik Valach Waldram]

[Hulik Malek Valach Waldram]

$$\mathcal{L}_{E_A} E_B = -\Theta_{A\alpha} (T^\alpha)_B{}^C E_C$$

Necessary & sufficient conditions for $D \geq 4$

▷ internal space = $\frac{G_{gauge}}{H}$ [Grana Minasian Petrini Waldram]
 (difficulty: find H)

▷ $\mathbb{P}_{section}^{MN} \Theta_{M\underline{a}} \Theta_{N\underline{b}} = 0$ + other algebraic constraints
 ↗ ↗ projected on coset generators

▷ $E_A{}^M(y)$ can be constructed explicitly

$$E_A{}^M = L^{-1}{}_A{}^{\underline{N}} e^{-1}{}_{\underline{N}}{}^M C_N{}^M$$

G_{gauge}/H representative ↗ $L^{-1}{}_A{}^{\underline{N}}$

vielbein $\in GL(d) \subset E_d \times \mathbb{R}^+$ ↘ $e^{-1}{}_{\underline{N}}{}^M$

$C_N{}^M \partial_M = \partial_N$ ← found via 'untwisting' ↗

D = 2 maximal supergravities from inner space

[GI]

[Bugden Hulik Valach Waldram]

[Hulik Malek Valach Waldram]

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 ↗ ↖ projected on coset generators
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$D < 4$

still OK

} *to do*

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D = 2 maximal supergravities from inner space

$$E_A^M = L^{-1}{}^N_A e^{-1}{}^N_C C_N^M$$

G_{gauge}/H representative \nearrow
 vielbein $\in GL(d) \subset E_d \times \mathbb{R}^+$
 $C_N^M \partial_M = \partial_N \leftarrow$ found via 'untwisting'
 to do

- important technical tool: *twisting* of genLie derivative

$$\mathcal{L}_\Lambda^{(F)} V^M = \mathcal{L}_\Lambda V^M - \Lambda^P V^Q F_{PQ}{}^M$$

express internal fluxes as deformations of \mathcal{L}

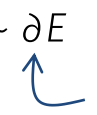
- $F_{MN}{}^P$ subject to algebraic & diff constraints (*Bianchi identities* $\Rightarrow F =$ torsion of C)

- $D = 3, 2$: genLie \rightarrow 'generalised Dorfman product' \rightarrow classify most general twistings
 [Hohm Musaev Samtleben] \rightarrow relate twistings to 'torsion of C '
 \rightarrow deduce algebraic uplift conditions on $\Theta_{M\alpha}$

D = 2 maximal supergravities from inner space

Necessary conditions for D = 2

$$\mathcal{L}_{E_A} E_B = - \underbrace{\Theta_{A\alpha} (T^\alpha)_B^C}_{\mathbb{P}(W_{A\alpha})} E_C$$

$W_{A\alpha} \sim \partial E$
 on section!

→ covariant conditions $0 = \eta_{-3\alpha\beta} \langle \theta | T^\alpha \otimes \langle \theta | T^\beta = \eta_{-4\alpha\beta} \langle \theta | T^\alpha \otimes \langle \theta | T^\beta \dots$

[D=3: Eloy, Galli, Malek]

→ brute-force projection (11d or IIB section, 11d displayed)

(same for D = 3, 4, ... : [Hassler, Yakatani])

$$\begin{aligned} \langle \theta | = & \langle 0 |^I \Theta_I^{(\frac{4}{9})} + \frac{1}{2} \langle 1/3 |_{IJ} \Theta^{(\frac{7}{9})IJ} + \frac{1}{24} \langle 2/3 |^{IJKL} \Theta_{IJKL}^{(\frac{10}{9})} + \frac{1}{2} \langle 1 |^I_{JK} \Theta_{IJ}^{(\frac{13}{9})K} \\ & + \frac{1}{6} \langle 4/3 |^L_{IJK} \Theta_L^{(\frac{16}{9})IJK} + \frac{1}{6} \langle 5/3 |^{I,J,KL} \Theta_{I,J,KL}^{(\frac{19}{9})} + \langle 2 |^I_{JK} \Theta_{IJ}^{(\frac{22}{9})K} \end{aligned}$$

finitely many components!

$D = 2$ maximal supergravities from inner space

$$\begin{aligned} \langle \theta | = \langle 0 |^I \Theta_I^{(\frac{4}{9})} + \frac{1}{2} \langle 1/3 |_{IJ} \Theta^{(\frac{7}{9})IJ} + \frac{1}{24} \langle 2/3 |^{IJKL} \Theta_{IJKL}^{(\frac{10}{9})} + \frac{1}{2} \langle 1 |^I \Theta_{IJ}^{(\frac{13}{9})K} \\ + \frac{1}{6} \langle 4/3 |_{IJK}^L \Theta_L^{(\frac{16}{9})IJK} + \frac{1}{6} \langle 5/3 |^{I,J,KL} \Theta_{I,J,KL}^{(\frac{19}{9})} + \langle 2 |^I \Theta_{IJ}^{(\frac{22}{9})K} \end{aligned}$$

- finitely many allowed couplings, subject to *quadratic constraints*

$$\eta_{-1\alpha\beta} \langle \theta | T^\alpha \otimes \langle \theta | T^\beta = 0 = \eta_{-3\alpha\beta} \langle \theta | T^\alpha \otimes \langle \theta | T^\beta = \eta_{-4\alpha\beta} \langle \theta | T^\alpha \otimes \langle \theta | T^\beta \dots$$

- looking for AdS_2 : $\langle \theta | T^\delta | \theta \rangle + \eta_{-2\alpha\beta} \langle \theta | T^\delta T^\alpha T^{\beta\dagger} | \theta \rangle = 0 \quad T^\delta \in \hat{\mathfrak{e}}_8 \quad \text{'going to the origin'}$

- solve analytically: small subsectors
or *numerically*...

③ what is general structure of 2d gauged sugras?

(maximal main focus,
but applies more in general)

↑
(possible gaugings, dynamics, solutions, ...)
└─── breaking 'Geroch group' / integrability ───┬───
└─── analytic ───┬───
└─── numeric → ML ───┬───

② study compactifications leading to ③



① reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞ -dimensional syms of 2d sugra $\rightarrow E_9$ exceptional field theory

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- plethora of 2d models to study
(even just for $D0$ brane/ $SO(9)$ gauged sugra...)
- uplift conditions
& twist matrix construction
- perturbative expansion using ExFT
→ KK spectra & higher order couplings
- generalised G structures *Henning's talk*
& less-susy truncations *Dan's talk*

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Thanks!

