2d maximal supergravities from higher dimensions

Gianluca Inverso
INFN, sez. di Padova

Supergravity techniques & the CFT bootstrap
AEI Potsdam, 09/11/23

based on works with:

Guillaume Bossard, Franz Ciceri, Axel Kleinschmidt 2209.02729, 2309.07232, 2309.07233

above + Henning Samtleben: 1811.04088, 2103.12118,

2d supergravity & KK reductions

high-dimensional
(super)gravity

internal space $\neq \mathbb{T}^n$*, fluxes*
keep subset of modes *(lightest or not)*
(many consistentcy constraints!)

Kaluza–Klein on $\mathbb{T}^n$
keep massless modes

low-dimensional
(super)gravity
global symmetry $\equiv \mathfrak{g}$

gauge $\mathfrak{g}_{\text{gauge}} \subset \mathfrak{g}$

gauged (super)gravity
(gravity + charged matter)

(based on Henning Samtleben's lecture notes)

2d supergravity & KK reductions

Prototype: 'simple' AdS solutions $\Longleftrightarrow$ SO(N) gauged (maximal) supergravities

$$\begin{aligned}
\mathrm{D3} &: \quad \mathrm{AdS}_5 \times S^5 \quad \Longleftrightarrow \quad \mathrm{SO}(6)\ \mathcal{N}=8,\ D=5\ \text{gauged sugra} \\
\mathrm{M2} &: \quad \mathrm{AdS}_4 \times S^7 \quad \Longleftrightarrow \quad \mathrm{SO}(8)\ \mathcal{N}=8,\ D=4\ \text{gauged sugra} \\
\mathrm{M5} &: \quad \mathrm{AdS}_7 \times S^4 \quad \Longleftrightarrow \quad \mathrm{SO}(5)\ \mathcal{N}=2,\ D=7\ \text{gauged sugra}
\end{aligned}$$

▹ list grows much bigger with other vacua/internal spaces/less susy

▹ 'bottom-up': found interesting gauged sugra first, then *uplifted*

▹ recent maximal examples:

- mIIA on S^6 $\Longleftrightarrow$ ISO(7), $D = 4$ [Guarino Jafferis Varela] [......]
- IIB on $S^5 \times S^1$ S-folds $\Longleftrightarrow$ $\mathrm{SO}(6) \ltimes \ldots$, $D = 4$ [GI Samtleben Trigiante] [Assel Tomasiello] [.....]

Prototype: 'simple' AdS solutions $\Longleftrightarrow$ SO(N) gauged (maximal) supergravities

$$\begin{array}{lcll} \text{D3} & : & \text{AdS}_5 \times S^5 & \Longleftrightarrow \quad \text{SO}(6)\ \mathcal{N}=8,\ D=5 \ \text{gauged sugra} \\ \text{M2} & : & \text{AdS}_4 \times S^7 & \Longleftrightarrow \quad \text{SO}(8)\ \mathcal{N}=8,\ D=4 \ \text{gauged sugra} \\ \text{M5} & : & \text{AdS}_7 \times S^4 & \Longleftrightarrow \quad \text{SO}(5)\ \mathcal{N}=2,\ D=7 \ \text{gauged sugra} \end{array}$$

▹ list grows much bigger with other vacua/internal spaces/less susy (we focus here on maximal)

▹ use gauged sugra for

• SOLUTION GENERATION
- relevant/marginal deformations $\leftrightarrow$ vacua in gauged sugra
- domain wall/RG flows
- (asymptotically AdS/Mink) black hole solutions

!! bottom-up: new gauged sugras $\Rightarrow$ new fluxed backgrounds

• SUSY & STABILITY:
- *full*, perturbative KK spectra from gauged supergravity structures
- some non-perturbative decay channels

2d supergravity & KK reductions

Prototype: 'simple' AdS solutions $\Longleftrightarrow$ SO(N) gauged (maximal) supergravities

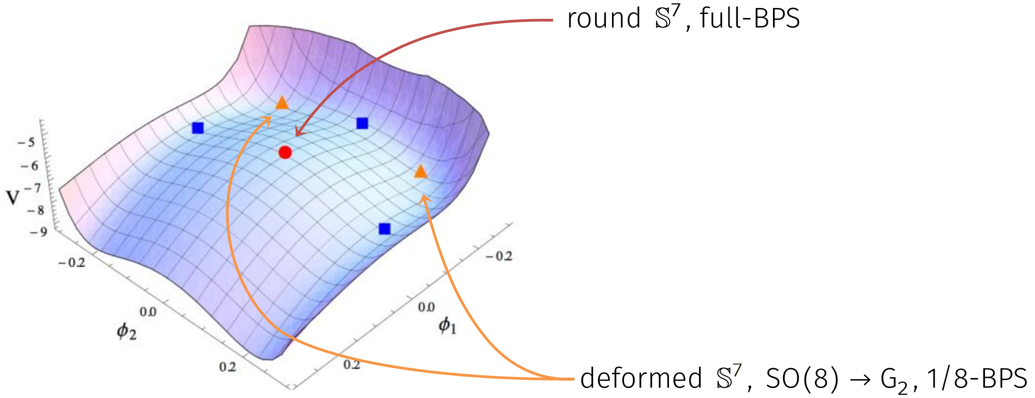
$$
\begin{array}{lcccl}
\text{D3} & : & \mathrm{AdS}_5 \times S^5 & \Longleftrightarrow & \mathrm{SO}(6)\ \mathcal{N}=8,\ D=5\ \text{gauged sugra} \\
\text{M2} & : & \mathrm{AdS}_4 \times S^7 & \Longleftrightarrow & \mathrm{SO}(8)\ \mathcal{N}=8,\ D=4\ \text{gauged sugra} \\
\text{M5} & : & \mathrm{AdS}_7 \times S^4 & \Longleftrightarrow & \mathrm{SO}(5)\ \mathcal{N}=2,\ D=7\ \text{gauged sugra}
\end{array}
$$

susy $\Rightarrow$ scalar potential $V(\phi)$

$$\frac{\delta V}{\delta \phi}(\phi_*) = 0$$

$$\Downarrow$$

$$\mathrm{AdS}_D \times \{\text{fluxed, deformed } \mathbb{S}^d\}$$



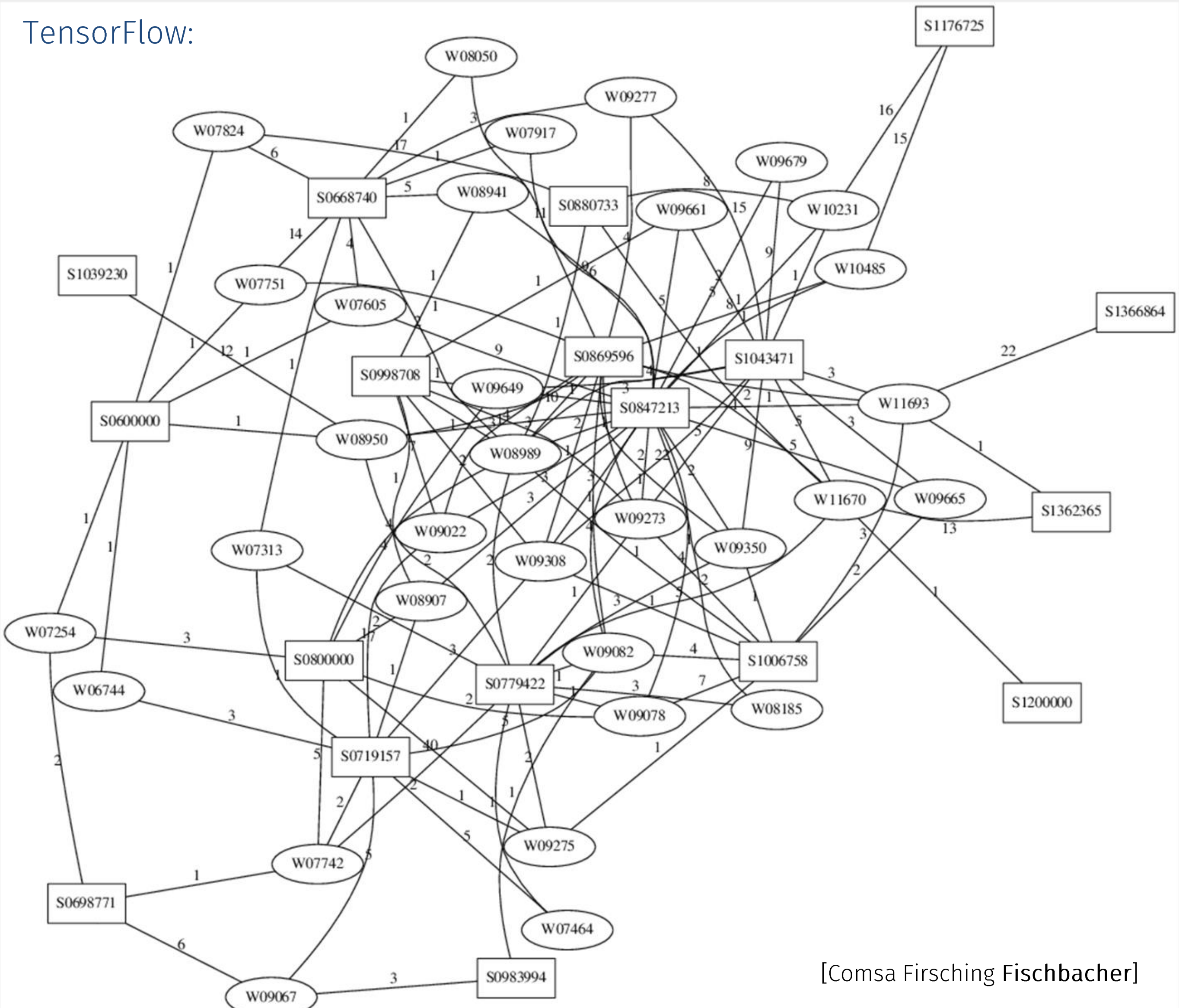
[de Wit–Nicolai, ..., ...]

Prot

S

/8-BPS

TensorFlow:



[Comsa Firsching **Fischbacher**]

$\longrightarrow$ we know many things on gauged sugras & KK truncations in $D \geq 3$

$\longrightarrow$ $D = 2$ is more involved & less developed

2d & not 2d

AdS$_2$ is the realm of *decoupling limit of BHs*

D0 branes $\longrightarrow$ [conformal] AdS$_2$$\times S^8$ + running dilaton + F_2 flux

- lifts to pp-wave in 11d with SO(9) symmetry ($\sim S^8 \times S^1$) [Hull] [Nicolai Samtleben]
- dual: regularised supermembrane/BFSS matrix model [de Wit, Hoppe, Nicolai] [Banks, Fischler, Schenker, Susskind]
- thermal & massive (BMN) deformations [Lin Lunin Maldacena] [...] [Costa Greenspan Penedones Santos]

lightest states' dynamics expected $\equiv$ 2d $\mathcal{N}$ = 32 SO(9) gauged sugra [Ortiz–Samtleben] [Anabalon Ortiz Samtleben] [Ortiz Samtleben Tsimpis]

2d & not 2d

AdS$_2$ is the realm of *decoupling limit of BHs*

D0 branes $\longrightarrow$ [conformal] AdS$_2 \times S^8$ + running dilaton + F_2 flux

- lifts to pp-wave in 11d with SO(9) symmetry ($\sim S^8 \times S^1$) [Hull] [Nicolai Samtleben]
- dual: regularised supermembrane / BFSS matrix model [de Wit, Hoppe, Nicolai] [Banks, Fischler, Schenker, Susskind]
- thermal & massive (BMN) deformations [Lin Lunin Maldacena] [...] [Costa Greenspan Penedones Santos]

lightest states' dynamics expected $\equiv$ 2d $\mathcal{N} = 32$ SO(9) gauged sugra [Ortiz–Samtleben] [Anabalon Ortiz Samtleben] [Ortiz Samtleben Tsimpis]

$\Uparrow$

$\boxed{\text{constructive proof (embed all 2d field configs into 11d)}}$

→ *Guillaume's talk*

A broader perspective:

1 what is general structure of 2d gauged sugras? (maximal main focus, but applies more in general)

$(\underbrace{\text{possible gaugings, dynamics,}}_{\text{breaking 'Geroch group' / integrability}}\ \underbrace{\text{solutions,}}_{}\ \ldots)$

- analytic
- numeric → ML

A broader perspective:

1 what is general structure of 2d gauged sugras? (maximal main focus, but applies more in general)

$\underbrace{\text{(possible gaugings, dynamics,}}_{\text{breaking 'Geroch group' / integrability}}$ $\underbrace{\text{solutions,}}$ …)

- ◦ analytic
- ◦ numeric → ML

2 study compactifications leading to 1

- ▹ new geometries
- ▹ uplift formulæ

A broader perspective:

1 what is general structure of 2d gauged sugras? (maximal main focus, but applies more in general)

(possible gaugings, dynamics, solutions, ...)

breaking 'Geroch group' / integrability

- analytic
- numeric → ML

2 study compactifications leading to 1

- new geometries
- uplift formulæ

3 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra

A broader perspective:

3 what is general structure of 2d gauged sugras? (maximal main focus, but applies more in general)

$\underbrace{\text{(possible gaugings, dynamics,}}_{\text{breaking 'Geroch group' / integrability}}$ $\underbrace{\text{solutions, ...)}}_{}$

- analytic
- numeric

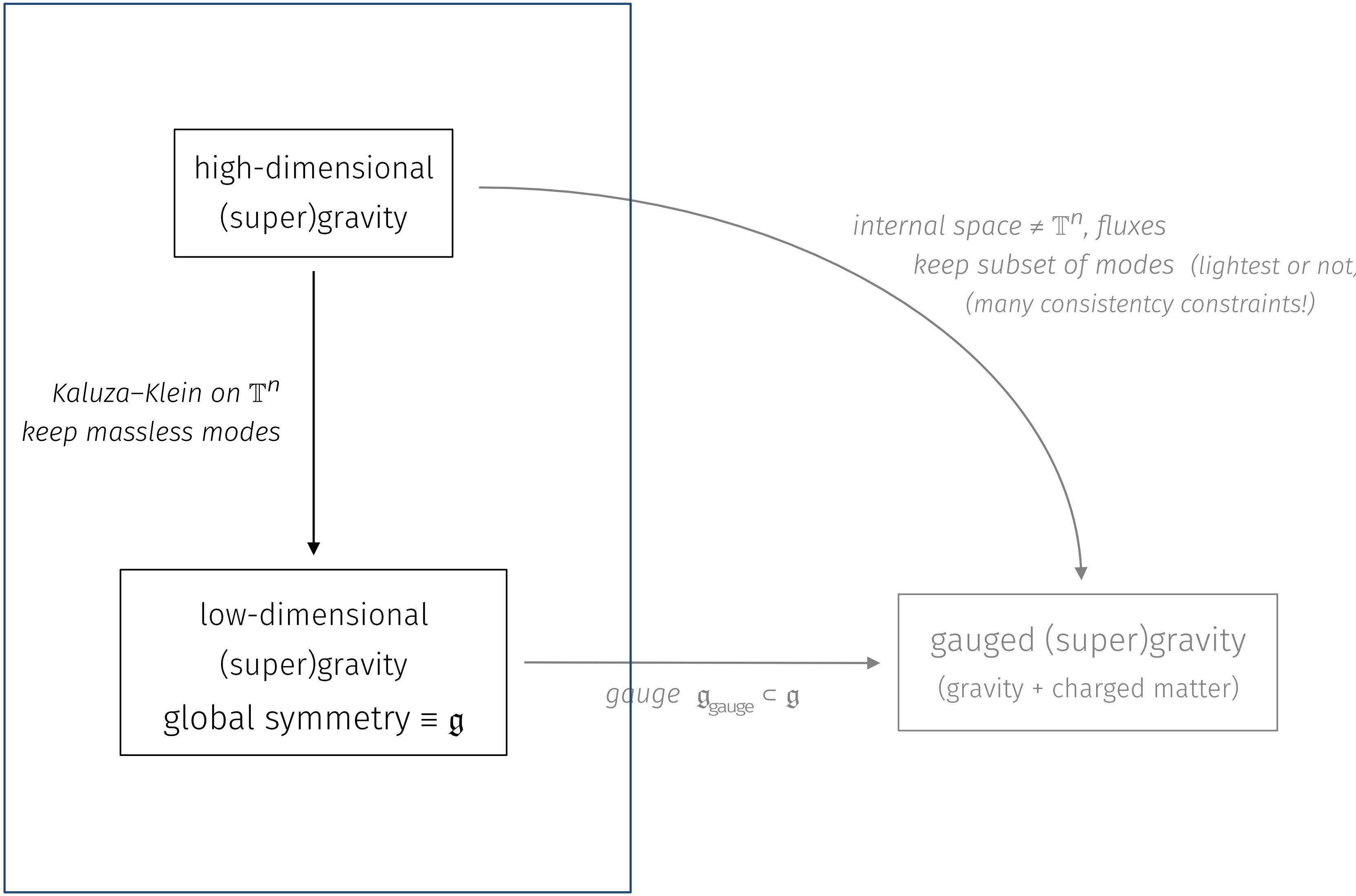
2 study compactifications leading to 3

- new geometries
- uplift formulæ

1 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra

2d supergravity & KK reductions



Geroch group & E_9

$$
\begin{array}{l}
4d \\
\big\downarrow{\scriptstyle KK} \\
3d \quad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g_{\mu\nu} + e^{2\phi} A_\mu A_\nu & e^{2\phi} \boldsymbol{A_\mu} \\ \hline e^{2\phi} A_\nu & e^{2\phi} \end{array}\right) \longrightarrow
\end{array}
$$

$$
\boxed{\begin{array}{c} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi \\ \underbrace{\xi + \mathrm{i}\, e^{\phi}}_{} \\ \mathrm{SL}(2)_E \ \text{Ehlers group} \end{array}}
$$

[Breitenlohner–Maison] [Nicolai]

Geroch group & E_9

$$4d \xrightarrow{KK} 3d \qquad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g_{\mu\nu} + e^{2\phi} A_\mu A_\nu & e^{2\phi} A_\mu \\ \hline e^{2\phi} A_\nu & e^{2\phi} \end{array}\right) \longrightarrow$$

$$\boxed{\begin{array}{c} d * dA = 0 \quad \Rightarrow \quad *dA = d\xi \\ \underbrace{\xi + \mathrm{i}\, e^{\phi}} \\ \mathrm{SL}(2)_E \ \text{Ehlers group} \end{array}}$$

$$3d \xrightarrow{KK} 2d \qquad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|cc} \rho^{-1/2} e^{2\sigma} \eta_{\mu\nu} & & \\ \hline & \rho e^{-2\phi}(1+a^2) & \rho\, a \\ & \rho\, a & \rho\, e^{2\phi} \end{array}\right) \longrightarrow \quad d\xi = *da \qquad \mathrm{SL}(2)_E$$

[Breitenlohner–Maison] [Nicolai]

$4d$

$\big\downarrow KK$

$3d$ $\quad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g_{\mu\nu} + e^{2\phi} A_\mu A_\nu & e^{2\phi} A_\mu \\ \hline e^{2\phi} A_\nu & e^{2\phi} \end{array}\right) \longrightarrow$

$$d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

$$\underbrace{\xi + \mathrm{i}\, e^{\phi}}_{}$$

$SL(2)_E$ Ehlers group

$\big\downarrow KK$

$2d$ $\quad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} \rho^{-1/2} e^{2\sigma} \eta_{\mu\nu} & \\ \hline & \boxed{\begin{array}{c|c} \rho e^{-2\phi}(1+a^2) & \rho\, a \\ \hline \rho\, a & \rho\, e^{2\phi} \end{array}} \end{array}\right) \longrightarrow \quad d\xi = *da \qquad SL(2)_E$

$SL(2)_T \times \mathbb{R}^+$

[Breitenlohner–Maison] [Nicolai]

Geroch group & E_9

$$4d \xrightarrow{KK} 3d \qquad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|c} e^{-2\phi} g_{\mu\nu} + e^{2\phi} A_\mu A_\nu & e^{2\phi} A_\mu \\ \hline e^{2\phi} A_\nu & e^{2\phi} \end{array}\right) \longrightarrow$$

$$d * dA = 0 \quad \Rightarrow \quad *dA = d\xi$$

$$\underbrace{\xi + \mathrm{i}\, e^{\phi}}$$

$SL(2)_E$ Ehlers group

$$3d \xrightarrow{KK} 2d \qquad g^{4d}_{\hat\mu\hat\nu} = \left(\begin{array}{c|cc} \rho^{-1/2}\, e^{2\sigma}\, \eta_{\mu\nu} & & \\ \hline & \rho e^{-2\phi}(1 + a^2) & \rho\, a \\ & \rho\, a & \rho\, e^{2\phi} \end{array}\right) \longrightarrow \quad d\xi = *da \qquad SL(2)_E$$

$SL(2)_T \times \mathbb{R}^+$

$$SL(2)_E \ \text{'}\times\text{'}\ SL(2)_T \times \mathbb{R}^+ \longrightarrow \text{loop group } \ \widehat{SL(2)} \rtimes \mathbb{R}^+$$

[Breitenlohner–Maison] [Nicolai]

$$
\begin{array}{c}
\textit{4d GR} \\
\downarrow \\
\textit{2d}
\end{array}
\qquad
SL(2)_E \;{}'\times'\; SL(2)_T \times \mathbb{R}^+ \longrightarrow \text{loop group} \quad \widehat{SL(2)} \rtimes \mathbb{R}^+
$$

$$
\begin{array}{c}
\textit{11d} \\
\Bigg\downarrow \\
\textit{2d}
\end{array}
\qquad
\left.
\begin{array}{ccc}
SL(2)_{Ehlers} & \longrightarrow & E_{8(8)} \\
SL(2)_T & \longrightarrow & SL(9)
\end{array}
\right\} \longrightarrow E_9
$$

[Breitenlohner–Maison] [Nicolai]

The linear system & twisted self-duality

G-invariant

$$\mathcal{L} = 2\mathrm{d}\sigma \wedge \star\mathrm{d}\rho - \rho\, \eta^{AB} P^0_A \wedge \star P^0_B$$

G/H current $\frac{1}{2}\left(\mathrm{d}v\, v^{-1} + \text{h.c.}\right)$

e.g.

- reduction of $4d$ GR: $\mathrm{SL}(2)/\mathrm{SO}(2)$
- maximal sugra: $\mathrm{E}_8/\mathrm{Spin}(16)$

The linear system & twisted self-duality

G-invariant

$$\mathcal{L} = 2\mathrm{d}\sigma \wedge \star \mathrm{d}\rho - \rho\, \eta^{AB} P_A^0 \wedge \star P_B^0$$

G/H current $\frac{1}{2}\left(\mathrm{d}v\, v^{-1} + \mathrm{h.c.}\right)$

e.g.

- reduction of $4d$ GR: $SL(2)/SO(2)$
- maximal sugra: $E_8/\mathrm{Spin}(16)$

EOM:

$$\mathrm{d} \star \mathrm{d}\rho = 0 \quad \longrightarrow \quad \mathrm{d}\tilde{\rho} = \star \mathrm{d}\rho$$

$$\mathrm{d}(\rho \star v^{-1} P^0 v) = 0 \quad \longrightarrow \quad \mathrm{d}Y_1 = 2\rho \star v^{-1} P^0 v$$

$$\longrightarrow \quad \mathrm{d}Y_2 = \left(\tfrac{1}{2}\rho^2 + \rho\tilde{\rho} \star\right) v^{-1} P^0 v - [Y_1,\ \mathrm{d}Y_1]$$

$$\longrightarrow \quad \ldots$$

The linear system & twisted self-duality

$$V(w) = v + w^{-1}V_1 + w^{-2}V_2 + \ldots$$

$$\overset{w\sim+\infty}{=} v \; e^{w^{-1}Y_1} \; e^{w^{-2}Y_2} \; \cdots \qquad\qquad w \in \mathbb{C}$$

The linear system & twisted self-duality

$$dvv^{-1} = P^0 + Q^0$$

$$V(w) = v + w^{-1}V_1 + w^{-2}V_2 + \dots$$

$$\overset{w\sim+\infty}{=} v \; e^{w^{-1}Y_1} \; e^{w^{-2}Y_2} \; \dots \qquad w \in \mathbb{C}$$

$$\hat{\mathfrak{e}}_8 \ni \quad dVV^{-1}(w) = Q^0 + \frac{1+\gamma^2}{1-\gamma^2}P^0 + \frac{2\gamma}{1-\gamma^2}\star P^0$$

$$\gamma = \frac{w-\tilde{\rho}}{\rho} \pm \sqrt{\left(\frac{w-\tilde{\rho}}{\rho}\right)^2 - 1}$$

[Breitenlohner–Maison]

$\int$bility

$$\boxed{\begin{aligned} \text{EOM:} \quad & d \star d\rho = 0 \\ & d(\rho \star v^{-1}P^0 v) = 0 \end{aligned}}$$

The linear system & twisted self-duality

$$V(w) = v + w^{-1}V_1 + w^{-2}V_2 + \dots$$

$$\overset{w\sim+\infty}{=} v \; e^{w^{-1}Y_1} \; e^{w^{-2}Y_2} \; \dots \qquad\qquad w \in \mathbb{C}$$

$$\hat{\mathfrak{e}}_8 \ni \quad dVV^{-1}(w) = Q^0 + \frac{1+\gamma^2}{1-\gamma^2} P^0 + \frac{2\gamma}{1-\gamma^2} * P^0$$

$$\gamma = \frac{w-\tilde{\rho}}{\rho} \pm \sqrt{\left(\frac{w-\tilde{\rho}}{\rho}\right)^2 - 1}$$

[Breitenlohner–Maison]

...a different direction:

- if we reduce directly from 11d, E_8 is not manifest, a different GL(9) is
- $V(w)$ is in a *fixed* triangular gauge, E_8 covariant
- how to write this for *highest weight reps*? There is no “*w*” there. *Square root?*

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

$$\frac{d\gamma}{\gamma}(w) = \underbrace{\frac{1+\gamma^2}{1-\gamma^2}}_{\text{geometric series}} d\rho + \frac{2\gamma}{1-\gamma^2} *d\rho$$

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

$$\frac{d\gamma}{\gamma}(w) \overset{\gamma\sim+\infty}{=} (1+2\gamma^{-2}+2\gamma^{-4}+\ldots)\, d\rho \; + \; (2\gamma^{-1}+2\gamma^{-3}+\ldots) * d\rho$$

$$\gamma(w) \quad \xrightarrow{w\to+\infty} \quad c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \ldots$$

asymptotics

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

$$\frac{d\gamma}{\gamma}(w) \overset{\gamma \sim +\infty}{=} (1+2\gamma^{-2}+2\gamma^{-4}+\ldots)\, d\rho \; + \; (2\gamma^{-1}+2\gamma^{-3}+\ldots) * d\rho$$

$$\gamma(w) \quad \xrightarrow{w \to +\infty} \quad c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \ldots$$

$$\simeq \quad \underbrace{\ldots \; e^{\varphi_3 L_{-3}} \; e^{\varphi_2 L_{-2}} \; e^{\varphi_1 L_{-1}} \; \rho^{L_0}}_{\Gamma^{-1}} \; w \qquad\qquad L_m = -w^{m+1}\partial_w$$

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

$$d\Gamma\,\Gamma^{-1} \;=\; (L_0 + 2L_{-2} + 2L_{-4} + \ldots)\; d\rho + \; (2L_{-1} + 2L_{-3} + \ldots)\; {*d\rho}$$

$$\gamma(w) \;\xrightarrow{w\to+\infty}\; c_1 w + c_0 + c_{-1} w^{-1} + c_{-2} w^{-2} + \ldots$$

$$\simeq \;\; \underbrace{\ldots\; e^{\varphi_3 L_{-3}}\; e^{\varphi_2 L_{-2}}\; e^{\varphi_1 L_{-1}}\; \rho^{L_0}}_{\Gamma^{-1}}\; w \qquad\qquad L_m = -w^{m+1}\partial_w$$

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

$$d\Gamma\,\Gamma^{-1} \;=\; (L_0 + 2L_{-2} + 2L_{-4} + \ldots)\; d\rho + \;(2L_{-1} + 2L_{-3} + \ldots)\; {*d\rho}$$

$$\Updownarrow$$

$$*p \;=\; w\,p \;\equiv\; S_1(p) \qquad\qquad p \equiv d\Gamma\,\Gamma^{-1} + (d\Gamma\,\Gamma^{-1})^{\dagger}$$

writable in h.w.r.

$$S_p(T^A_m) = T^A_{m+p}$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(\mathrm{K}) = 0$$

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

the full system:

$$\mathcal{V} = \underbrace{\rho^{-L_0} e^{-\varphi_1 L_{-1}} e^{-\varphi_2 L_{-2}} \cdots}_{\Gamma} \underbrace{V_{e_8}\, e^{Y_{1A} T^A_{-1}} e^{Y_{2A} T^A_{-2}} \cdots\, e^{-\sigma \mathrm{K}}}_{V_{loop}}$$

$$\boxed{\star\mathcal{P} = S_1(\mathcal{P}) + \tilde{\chi}_1 \mathrm{K}}$$

auxiliary one-form, restores covariance

$$S_p(T^A_m) = T^A_{m+p}$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(\mathrm{K}) = 0$$

The linear system & twisted self-duality

[Julia–Nicolai] [Paulot]

[Bossard Ciceri GI Kleinschmidt Samtleben]

the full system:

$$\mathcal{V} = \underbrace{\rho^{-L_0}\, e^{-\varphi_1 L_{-1}}\, e^{-\varphi_2 L_{-2}} \cdots}_{\Gamma}\; V_{loop}$$

any parabolic parametrisation

$$\boxed{\star\mathcal{P} = S_1(\mathcal{P}) + \tilde{\chi}_1 \mathrm{K}}$$

auxiliary one-form,

restores covariance

$$S_p(T^A_m) = T^A_{m+p}$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(\mathrm{K}) = 0$$

The linear system & twisted self-duality

$$\mathcal{V} = \Gamma\, V_{loop} \quad \in \quad \frac{\hat{E}_8 \rtimes \mathrm{Vir}^-}{K(\mathrm{E}_9)}$$

$$\boxed{\star\mathcal{P} = S_1(\mathcal{P}) + \tilde{\chi}_1 \mathrm{K}}$$

$\hat{\mathfrak{e}}_8 \oplus \mathfrak{vir}^-$

• E_8 isometries	T^A_0
• σ shift	K
• ρ rescaling	L_0
• $\tilde{\rho}$ shift	L_{-1}
• Y_m shift $(m > 0)$	T^A_{-m}
• *hidden loop*	T^A_{+m}
• *further $\tilde{\rho}/Y$ reparam.*	L_{-m}

Pseudoaction

$$\mathrm{d}\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q} \quad = \quad 0$$

Pseudoaction

$$S_1\Big(\mathrm{d}\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q}\Big)$$

$$\uparrow$$

$$S_p(T^A_m) = T^A_{m+p}$$

$$S_p(L_m) = L_{m+p}$$

$$S_p(\mathrm{K}) = 0$$

Pseudoaction

$$S_1\Big(\mathrm{d}\mathcal{P} - \mathcal{Q}\wedge\mathcal{P} - \mathcal{P}\wedge\mathcal{Q}\Big) \quad + \quad \left(\frac{1}{2\rho}\mathcal{L}^{top}\right)\mathrm{K} \quad = \quad \text{covariant}$$

$$\uparrow$$

$S_p(T^A_m) = T^A_{m+p}$

$S_p(L_m) = L_{m+p}$

$S_p(\mathrm{K}) = 0$

Pseudoaction

$$S_1 \underbrace{\cancel{\left(\mathrm{d}\mathcal{P} - \mathcal{Q} \wedge \mathcal{P} - \mathcal{P} \wedge \mathcal{Q}\right)}}_{=\,0} \quad + \quad \left(\frac{1}{2\rho}\mathcal{L}^{top}\right) \mathrm{K} \quad = \quad invariant$$

$$\frac{1}{2\rho}\mathcal{L}^{top} \quad = \quad \mathcal{P}_{L_{-1}} \wedge \mathrm{d}\sigma \; + \sum_{n\geq 1} \left(\mathcal{P}_{L_{-n+1}} - \mathcal{P}_{L_{-1-n}}\right) \wedge \tilde{\chi}_n - \eta^{AB} \sum_{n\in\mathbb{Z}} n\, \mathcal{Q}_A^n \wedge \mathcal{P}_B^{-n-1}$$

Pseudoaction

$$\underbrace{\cancel{S_1\left(\mathrm{d}\mathcal{P} - \mathcal{Q}\wedge\mathcal{P} - \mathcal{P}\wedge\mathcal{Q}\right)}}_{=\,0} \quad + \quad \left(\frac{1}{2\rho}\mathcal{L}^{top}\right)\mathrm{K} \quad = \quad invariant$$

$$\boxed{\frac{1}{2\rho}\mathcal{L}^{top} \quad = \quad \mathbf{D}\tilde{\chi}_1}$$

Duality frames & physical Lagrangians

$$\begin{tikzcd}
\text{11d} \arrow[d] & & \\
\text{3d} \arrow[r] & \text{dualise KK vectors \& } C_{\mu m n} \arrow[r] & \mathrm{E}_8 \arrow[d] \\
 & & \text{2d}
\end{tikzcd}$$

$$\mathcal{L} = \sqrt{-g}\,\left(\rho R - \rho\,\eta^{AB} P_{\mu\,A}{}^{0} P^{\mu}{}_{B}{}^{0}\right)$$

$$\begin{tikzcd}
11\text{d} \arrow[d] \arrow[dd, bend right] & & \\
3\text{d} \arrow[r] & \text{dualise KK vectors \& } C_{\mu mn} \arrow[r] & E_8 \arrow[d] \\
2\text{d} & & 2\text{d}
\end{tikzcd}$$

$$\mathcal{L} = \sqrt{-g}\,\left(\rho R - \rho\,\eta^{AB} P_{\mu A}{}^{0} P^{\mu}{}_{B}{}^{0}\right)$$

$$\mathcal{L} = \sqrt{-g}\,\left(\rho R + \frac{1}{4}\rho\, g^{\mu\nu}\partial_\mu \mathrm{m}^{IJ}\partial_\nu \mathrm{m}_{IJ} - \frac{1}{12}\rho^{1/3} g^{\mu\nu}\partial_\mu a_{I_1I_2I_3}\partial_\nu a_{J_1J_2J_3}\mathrm{m}^{I_1J_1}\mathrm{m}^{I_2J_2}\mathrm{m}^{I_3J_3}\right)$$

$$+\frac{1}{6^4}\varepsilon^{\mu\nu}\varepsilon^{I_1\ldots I_9} a_{I_1I_2I_3}\partial_\mu a_{I_4I_5I_6}\partial_\nu a_{I_7I_8I_9} \qquad \mathrm{m} \in \frac{SL(9)}{SO(9)}$$

covariant w.r.t. $SL(9) \notin E_8$ *(spectrally flowed)*

Duality frames & physical Lagrangians

pseudoaction → physical action by *completing the self-duality squares*

$$\mathcal{L}^{pseudo} = \mathcal{L}^{phys} + \sum^{\infty} \Big(\text{twisted self-duality}\Big)^2$$

gravity/dilaton sector:

$$2\rho\, P_0\, \tilde{\chi}_1 + 2\rho \sum_{n\geq 1} P_n\, (\tilde{\chi}_{n+1} - \tilde{\chi}_{n-1}) \quad = \quad \underbrace{2\mathrm{d}\rho \wedge \star\mathrm{d}\sigma}_{\sqrt{-g}\ \rho\ R} + 2\rho \sum_{n=0}^{+\infty} (\star P_n - P_{n+1})(\tilde{\chi}_n - \star\tilde{\chi}_{n+1})$$

Duality frames & physical Lagrangians

pseudoaction → physical action by *completing the self-duality squares*

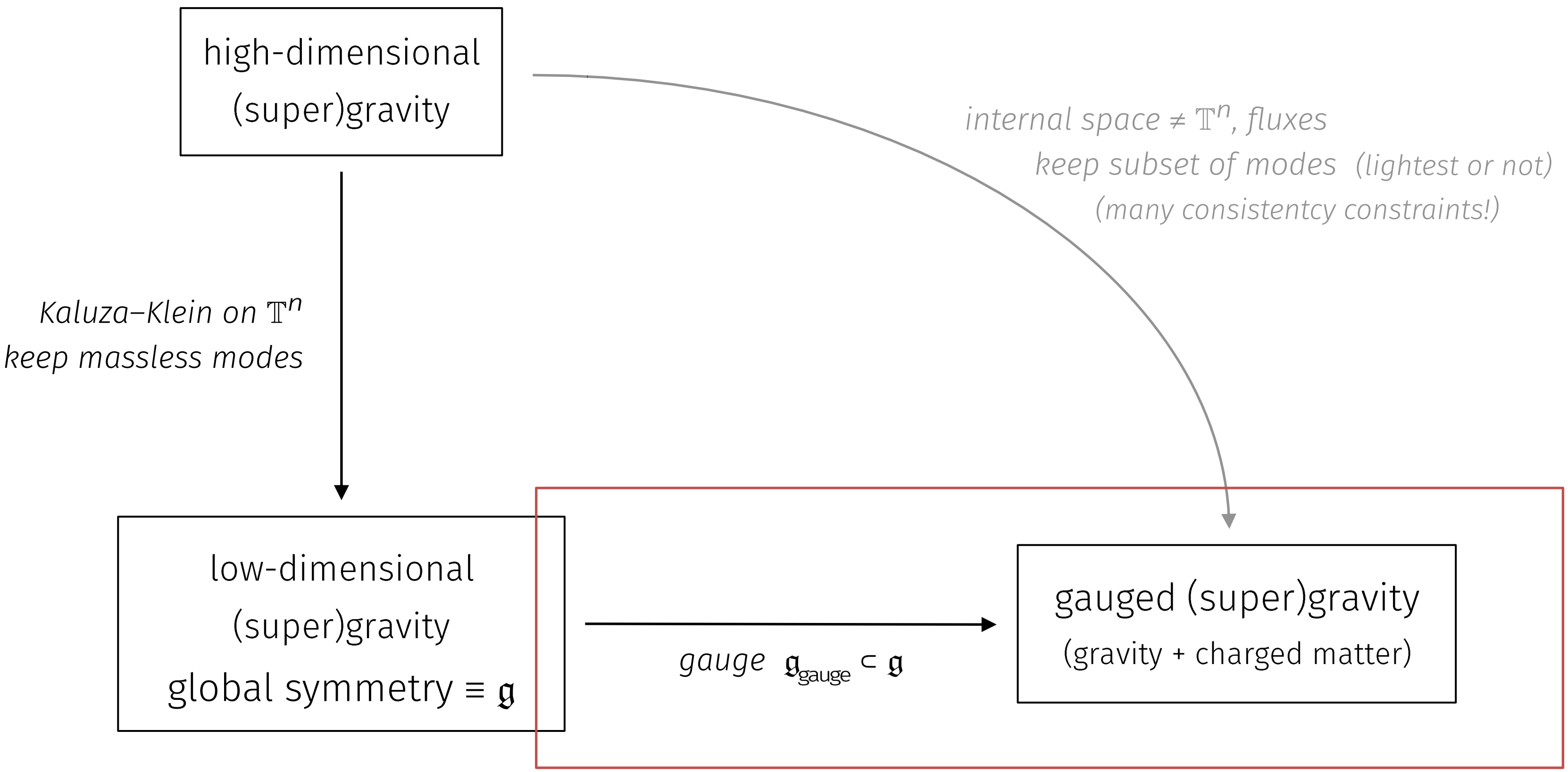
$$\mathcal{L}^{pseudo} = \mathcal{L}^{phys} + \sum^{\infty} \left(\text{twisted self-duality}\right)^2$$

full theory: depends on parabolic gauge

- $V_{loop} = V_{e_8}\, e^{Y_{1A} T^A_{-1}}\, e^{Y_{2A} T^A_{-2}} \cdots \longrightarrow \mathcal{L} = \sqrt{-g}\left(\rho R - \rho\, \eta^{AB} P_{\mu\, A}{}^{0} P^{\mu}{}_{B}{}^{0}\right)$
- $V_{loop} = V_{sl_9}\;\; e^{a^{IJK} T_{-1/3\, IJK}}\;\; e^{b_{IJK} T^{IJK}_{-2/3}} \cdots$

$$\longrightarrow \mathcal{L} = \sqrt{-g}\left(\rho R + \frac{1}{4}\rho\, g^{\mu\nu}\partial_\mu \mathrm{m}^{IJ}\partial_\nu \mathrm{m}_{IJ} - \frac{1}{12}\rho^{1/3} g^{\mu\nu}\partial_\mu a^{I_1I_2I_3}\partial_\nu a^{J_1J_2J_3}\mathrm{m}_{I_1J_1}\mathrm{m}_{I_2J_2}\mathrm{m}_{I_3J_3}\right)$$
$$+ \frac{1}{6^4}\varepsilon^{\mu\nu}\varepsilon_{I_1\dots I_9} a^{I_1I_2I_3}\partial_\mu a^{I_4I_5I_6}\partial_\nu a^{I_7I_8I_9}$$

$D = 2$ gauged supergravity



$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

$D = 2$ gauged supergravity

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

- $D > 2$: SUSY + $\Theta \implies$ full Lagrangian

lectures: [Samtleben '08][Trigiante '16]

$$\boxed{\partial \to \mathcal{D}}$$

$$\Downarrow$$

$$\boxed{\delta_{\text{SUSY}} \mathcal{L} \neq 0 \quad \Rightarrow \quad \text{correct} \quad \delta_{\text{SUSY}}(\text{fermions}) \propto \Theta}$$

$$\Downarrow$$

$$\boxed{\text{add } V(\phi) \propto \Theta^2 \implies \delta_{\text{SUSY}} \mathcal{L} = 0}$$

$D = 2$ gauged supergravity

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^M \Theta_{M\alpha} T^\alpha$$

- $D = 2$ SUSY too difficult: $K(\mathfrak{e}_9)$ representation theory

$\boxed{\partial \to \mathcal{D}}$

$\cancel{\Downarrow}$

$\boxed{\delta_{\text{SUSY}} \mathcal{L} \neq 0 \quad \Rightarrow \quad \text{correct} \quad \delta_{\text{SUSY}}(\text{fermions}) \propto \Theta}$ not available for generic Θ in $D = 2$

$\cancel{\Downarrow}$

? $V(\phi)$?

[Samtleben–Weidner]: construction up to $V(\phi)$; fixed $K(\mathrm{e}_9)$ gauge / duality frame

2d & not 2d

3 what is general structure of 2d gauged sugras?

2 study compactifications leading to 3

1 reorganise 11d / IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra

start from 11d sugra

$$\left.\begin{array}{l} ds^2_{11d} \\ C_{(3)} \\ C_{(6)} \\ \vdots \end{array}\right\} \Longrightarrow \left\{\begin{array}{ll} g_{\mu\nu}(x,y) & D\text{-dimensional metric} \\ \mathcal{M}_{MN}(x,y) & \text{scalars parametrising } E_d/K(E_d) \\ A_\mu^M(x,y) & \\ B_{\mu\nu}^{MN}(x,y) & p\text{-forms in } E_d \text{ representations} \\ \vdots & \end{array}\right.$$

$$\left.\begin{array}{l} \xi^m(x,y) \\ \lambda_{mn}(x,y) \\ \lambda_{mnpqr}(x,y) \\ \vdots \end{array}\right\} \Lambda^M(x,y)$$

$$\boxed{\begin{array}{c} \mathcal{L}_\Lambda \phi = \Lambda^M \partial_M \phi + \mathbb{P}_\alpha(\partial\Lambda)\,\delta^\alpha_{\mathfrak{e}_d \oplus \mathbb{R}}\,\phi \\ \textit{generalised diffeomorphisms} \end{array}}$$

[...] [Berman Perry] [...] [Berman Cederwall Kleinschmidt Thompson]
[Coimbra Strickland-Constable Waldram] [...] [Hohm Samtleben]

$$\mathcal{L}_\Lambda \phi = \Lambda^M \partial_M \phi + \mathbb{P}_\alpha(\partial\Lambda)\, \delta^\alpha_{e_d \oplus \mathbb{R}}\, \phi$$

generalised diffeomorphisms

$$\partial_M = \begin{pmatrix} \frac{\partial}{\partial y^m} \\ 0 \\ \vdots \\ 0 \end{pmatrix}$$

determined by *algebraic* condition

$$\mathbb{P}(\partial_M \otimes \partial_N) = 0$$ *section constraint*

- two solutions: 11d or IIB supergravity
- we can keep *formal E_d covariance* and treat both *at same time*

[Berman Perry] [...] [Berman Cederwall Kleinschmidt Thompson]
[Coimbra Strickland-Constable Waldram] [...] [Hohm Samtleben]

E_9 exceptional field theory *(very briefly)*

$$\Lambda^M \in \textit{basic } E_9 \text{ irrep}: \quad T^A_m|0\rangle = (K-1)|0\rangle = 0 \ , \quad m \geq 0 \qquad (\ \Lambda^M \leftrightarrow |\Lambda\rangle\ )$$

$$\mathfrak{e}_8 \oplus L_0 \longrightarrow \mathbf{1}_0 + \mathbf{248}_{-1} + (\mathbf{1} + \mathbf{248} + \mathbf{3875})_{-2} + \ldots$$

[Bossard Cederwall Kleinschmidt Palmkvist Samtleben]

$$\boxed{\mathcal{L}_{\Lambda,\Sigma} = \Lambda^M \partial_M + \Big(\mathbb{P}_\alpha(\partial\Lambda) + \underbrace{\Sigma_\alpha}_{\text{`ancillaries'}}\Big)\,\delta^\alpha_{\hat{\mathfrak{e}}_8 + \mathfrak{vir}^-}}$$

Dynamics

1. covariantise maximal sugra pseudo-action w.r.t. generalised diffeos
2. add invariant *internal curvature*, $\mathcal{O}(\partial_M^2)$

[Bossard Ciceri GI Kleinschmidt Samtleben]

1. covariantise w.r.t. generalised diffeos $\qquad \partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$

$$\mathcal{S}_1\Big(\underbrace{\cancel{\mathcal{D}\mathcal{P} - \mathcal{Q}\wedge\mathcal{P} - \mathcal{P}\wedge\mathcal{Q} + \text{field strenghts}}}_{=0}\Big) + \frac{1}{2\rho}\mathcal{L}_{\text{pseudo}}\, K = \text{gendiffeo covariant}$$

E_9 exceptional field theory *(very briefly)*

[Bossard Ciceri GI Kleinschmidt Samtleben]

1. covariantise w.r.t. generalised diffeos $\qquad \partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$

$$\frac{1}{2\rho}\mathcal{L}_{\text{pseudo}} = \mathcal{P}_{L_{-1}} \wedge \mathcal{D}\sigma + \sum_{n\geq 1}\left(\mathcal{P}_{L_{-n+1}} - \mathcal{P}_{L_{-1-n}}\right) \wedge \tilde{\chi}_n - \eta^{AB} \sum_{n\in\mathbb{Z}} n\,\mathcal{Q}_A^n \wedge \mathcal{P}_B^{-n-1}$$

+ field strenghts

[Bossard Ciceri GI Kleinschmidt Samtleben]

1. covariantise w.r.t. generalised diffeos $\quad \partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$

$$\frac{1}{2\rho}\mathcal{L}_{\text{pseudo}} = \mathbf{D}\tilde{\chi}_1 \quad + \text{field strenghts}$$

E_9 exceptional field theory *(very briefly)*

[Bossard Ciceri GI Kleinschmidt Samtleben]

1. covariantise w.r.t. generalised diffeos $\qquad \partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - \mathcal{L}_{\mathbb{A}_\mu}$

$$\frac{1}{2\rho}\mathcal{L}_{\text{pseudo}} = \mathbf{D}\tilde{\chi}_1 \quad + \text{field strenghts}$$

2. add invariant *internal curvature*, $\mathcal{O}(\partial_M^2)$

   built from scratch!

- $\mathcal{P}_{\underline{M}} =$ internal current
- $\mathcal{P}^{(+1)}_{\underline{M}} = \mathcal{S}_1(\mathcal{P}_{\underline{M}}) + \tilde{\chi}_{\underline{M}} K$

$$\rho V_{\text{ExFT}} = \underbrace{\eta^{\alpha\beta}\mathcal{P}_{\underline{M}\,\alpha}\mathcal{P}_{\underline{M}\,\beta} - 2\,\mathcal{P}_{\underline{M}\,\alpha}\mathcal{P}_{\underline{N}\,\beta}\left(T^{\beta\dagger}T^{\alpha}\right)_{\underline{MN}} - 2\,\mathcal{P}_{\underline{M}\,\alpha}\,T^{\alpha\underline{M}}{}_{\underline{N}}\,\partial_{\underline{N}}\rho}_{\textit{standard terms}} \; \underbrace{+2\,\mathcal{P}^{(+1)}_{\underline{M}\,\alpha}\mathcal{P}^{(+1)}_{\underline{N}\,\beta}\left(T^{\beta\dagger}T^{\alpha}\right)_{\underline{MN}}}_{\Sigma\ \textit{gauge invariance}}$$

2d & not 2d

3 what is general structure of 2d gauged sugras?

2 study compactifications leading to 3

1 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra

$$\text{EOM}\begin{pmatrix}\text{10d/11d sugra}\\ \text{on fluxed bckgd}\end{pmatrix} \propto \text{EOM}\begin{pmatrix}\text{low-dim. sugra}\\ \text{with gauge group } G\end{pmatrix} \times \left\langle\begin{matrix}\text{geometric data}\\ \text{of fluxed bckgd}\end{matrix}\right\rangle$$

$$= 0 \quad \Longleftarrow \quad = 0$$

example: *Scherk–Schwarz*

$$ds^2 = \Delta(x,y) g(x,y)_{\mu\nu} dx^\mu dx^\nu + \mathbf{g(x,y)_{mn}} (dy^m + \mathbf{K(x,y)_\mu{}^m} dx^\mu)(dy^n + K(x,y)_\nu{}^n dx^\nu)$$

- take internal space $=$ Lie group G
- left-invariant forms: $e^{\underline{m}}{}_m(y)\ T_{\underline{m}}\, dy^m$
- factorise: $g_{mn}(x,y) = g_{\underline{mn}}(x)\ e^{\underline{m}}{}_m(y)\ e^{\underline{n}}{}_n(y)$

$$K^m_\mu(x,y) = a^{\underline{m}}_\mu(x)\ e_{\underline{m}}{}^m(y)$$

$$\boxed{[e_{\underline{m}}, e_{\underline{n}}] = f_{\underline{mn}}{}^{\underline{p}} e_{\underline{p}}}$$

gauge algebra

2d supergravity & KK reductions

[...]

[Lee Strickland-Constable Waldram '14]

[Hohm Samtleben '14]

generalised Scherk–Schwarz $\mathcal{U} \in \mathrm{E}_d$

$$\left.\begin{array}{l} ds^2_{11d} \\ C_{(3)} \\ C_{(6)} \\ \vdots \end{array}\right\} \Longrightarrow \left\{\begin{array}{lcl} A_\mu^M(x,y) & = & a_\mu^A(x)\, \overbrace{r^{-1}(y)\,\mathcal{U}^{-1}{}_A{}^M(y)}^{E_A{}^M(y)} \\ g_{\mu\nu}(x,y) & = & g_{\mu\nu}(x)\; r^{-2}(y) \\ \mathcal{M}_{MN}(x,y) & = & M_{AB}(x)\; \mathcal{U}^A{}_M(y)\,\mathcal{U}^B{}_N(y) \\ \vdots & & \vdots \end{array}\right.$$

$$\boxed{\mathcal{L}_{E_A} E_B \;=\; -\Theta_{A\alpha}\left(T^\alpha\right)_B{}^C E_C}$$

$$\updownarrow$$

$$\partial_\mu \longrightarrow \mathcal{D}_\mu = \partial_\mu - A_\mu^A\, \Theta_{A\,\alpha}\, T^\alpha$$

2d supergravity & KK reductions

- $\mathcal{L}_{E_A} E_B \;=\; -\underbrace{\Theta_{A\alpha}}_{\mathbb{P}\left(W_{A\alpha}\right)}\left(T^{\alpha}\right)_B{}^C E_C \qquad W_{A\alpha} \sim \partial E$

- $D = 2: \quad \Theta_{A\alpha} \;\to\; \langle\Theta_{\alpha}| = \underbrace{\eta_{-1\alpha\beta}}_{\uparrow\ \hat{\mathfrak{e}}_8 \oplus \langle L_{-1}\rangle \text{ invariant}} \boldsymbol{\langle\theta|} T^{\beta}$ [Samtleben–Weidner]

new

$$\langle\theta| \;=\; \langle W_{\alpha}|S_1(T^{\alpha}) + \langle W^{+}|$$

$\langle\vartheta| = \langle W_{\alpha}|T^{\alpha}$

trombone $\to 0$

[Bossard Cederwall Kleinschmidt Palmkvist Samtleben]

[Bossard Ciceri GI Kleinschmidt]

$$\boxed{\partial \;\to\; \mathcal{D}}$$

$$\boxed{\delta_{\text{SUSY}}\,\mathcal{L} \neq 0 \quad\Rightarrow\quad \text{correct} \quad \delta_{\text{SUSY}}(\text{fermions}) \;\propto\; \Theta}$$

? $V(\phi)$?

The scalar potential of $D = 2$ gauged maximal supergravities

$$\rho V_{\text{ExFT}} = \eta^{\alpha\beta}\mathcal{P}_{\underline{M}\alpha}\mathcal{P}_{\underline{M}\beta} - 2\,\mathcal{P}_{\underline{M}\alpha}\mathcal{P}_{\underline{N}\beta}\left(T^{\beta\dagger}T^{\alpha}\right)_{\underline{MN}} - 2\,\mathcal{P}_{\underline{M}\alpha}\,T^{\alpha\underline{M}}{}_{\underline{N}}\partial_{\underline{N}}\rho + 2\,\mathcal{P}^{(+1)}_{\underline{M}\alpha}\mathcal{P}^{(+1)}_{\underline{N}\beta}\left(T^{\beta\dagger}T^{\alpha}\right)_{\underline{MN}} \quad (2)$$

$$\Downarrow$$

$$\boxed{2\rho V_{\text{g.sugra}} = \rho^{-2}\langle\theta|M^{-1}|\theta\rangle + \eta_{-2\,\alpha\beta}\langle\theta|T^{\alpha}M^{-1}T^{\beta\dagger}|\theta\rangle}$$

$\forall\ D = 2$ gauged maximal supergravity with higher dimensional origin

$$\mathcal{L}_{pseudo} = \mathcal{L}_{top} - V_{g.sugra}$$

$$\frac{1}{2\rho}\mathcal{L}_{top} \quad = \quad \mathbf{D}\tilde{\chi}_1 + \langle\theta|\mathbf{O}(M)|F\rangle \longrightarrow \textit{physical Lagrangians}$$

$$2\rho V_{\text{g.sugra}} \quad = \quad \rho^{-2}\langle\theta|M^{-1}|\theta\rangle \; + \; \eta_{-2\,\alpha\beta}\langle\theta|T^{\alpha}M^{-1}T^{\beta^{\dagger}}|\theta\rangle$$

$\forall$ $D = 2$ gauged maximal supergravity with higher dimensional origin

$D = 2$ maximal supergravities from inner space

$$\mathcal{L}_{pseudo} = \mathcal{L}_{top} - V_{g.sugra}$$

$$\frac{1}{2\rho}\mathcal{L}_{top} \quad = \quad \mathbf{D}\tilde{\chi}_1 + \langle\theta|\mathbf{O}(M)|F\rangle \longrightarrow \textit{physical Lagrangians}$$

$$2\rho V_{\text{g.sugra}} \quad = \quad \rho^{-2}\langle\theta|M^{-1}|\theta\rangle \; + \; \eta_{-2\,\alpha\beta}\langle\theta|T^{\alpha}M^{-1}T^{\beta\dagger}|\theta\rangle$$

$\forall$ $D = 2$ gauged maximal supergravity with higher dimensional origin

Even in higher D, no one has been able to classify them

(no such thing as "All gauged maximal supergravities with uplift")

...but we do know several things!

$D = 2$ maximal supergravities from inner space

[GI]
[Bugden Hulik Valach Waldram]
[Hulik Malek Valach Waldram]

$$\mathcal{L}_{E_A} E_B \quad = \quad -\Theta_{A\alpha}\left(T^{\alpha}\right)_B{}^C E_C$$

Necessary & sufficient conditions for $D \geq 4$

- iternal space $= \dfrac{\mathrm{G}_{gauge}}{H}$ [Grana Minasian Petrini Waldram]
  (difficulty: find H)

- $\mathbb{P}^{MN}_{section}\Theta_{M\underline{a}}\,\Theta_{N\underline{b}} = 0$ + other algebraic constraints
  projected on coset generators

- $E_A{}^M(y)$ can be constructed explicitly

vielbein $\in$ $\mathrm{GL}(d) \subset \mathrm{E}_d \times \mathbb{R}^+$

$$E_A{}^M \;=\; L^{-1}{}_A{}^{\underline{N}} \;\; e^{-1}{}_{\underline{N}}{}^N \;\; C_N{}^M$$

G_{gauge}/H representative

$C_N{}^M\partial_M = \partial_N \leftarrow$ found via '*untwisting*'

$D = 2$ maximal supergravities from inner space

[GI]
[Bugden Hulik Valach Waldram]
[Hulik Malek Valach Waldram]

$$\mathcal{L}_{E_A} E_B = -\Theta_{A\alpha}(T^\alpha)_B{}^C E_C$$

Necessary & sufficient conditions for $D \geq 4$

- iternal space $= \dfrac{G_{gauge}}{H}$ [Grana Minasian Petrini Waldram]
  (difficulty: find H)

- $\mathbb{P}^{MN}_{section}\Theta_{M\underline{a}}\,\Theta_{N\underline{b}} = 0$ + other algebraic constraints
  projected on coset generators

- $E_A{}^M(y)$ can be constructed explicitly

$D < 4$

still OK

to do

vielbein $\in GL(d) \subset E_d \times \mathbb{R}^+$

$$E_A{}^M = L^{-1}{}_A{}^{\underline{N}}\;\; e^{-1}{}_{\underline{N}}{}^N\;\; C_N{}^M$$

G_{gauge}/H representative

$C_N{}^M\partial_M = \partial_N \leftarrow$ found via '*untwisting*'

$D = 2$ maximal supergravities from inner space

vielbein $\in \mathrm{GL}(d) \subset \mathrm{E}_d \times \mathbb{R}^+$

$$E_A{}^M = L^{-1}{}_A{}^{\underline{N}} \; e^{-1}{}_{\underline{N}}{}^N \; C_N{}^M$$

G_{gauge}/H representative

$C_N{}^M \partial_M = \partial_N \leftarrow$ found via *'untwisting'*

to do

- important technical tool: twisting of genLie derivative

$$\mathcal{L}^{(F)}_\Lambda V^M = \mathcal{L}_\Lambda V^M - \Lambda^P V^Q F_{PQ}{}^M$$

  express internal fluxes as deformations of $\mathcal{L}$

- $F_{MN}{}^P$ subject to algebraic & diff constraints (*Bianchi identities* $\Rightarrow F$ = torsion of C)

- $D = 3, 2$: genLie $\rightarrow$ 'generalised Dorfman product' [Hohm Musaev Samtleben]
  - $\rightarrow$ classify most general twistings
  - $\rightarrow$ relate twistings to 'torsion of C'
  - $\rightarrow$ deduce algebraic uplift conditions on $\Theta_{M\alpha}$

(w.i.p. with Davide Rovere)

$D = 2$ maximal supergravities from inner space

Necessary conditions for $D = 2$

$$\mathcal{L}_{E_A} E_B = -\underbrace{\Theta_{A\alpha}}_{\mathbb{P}(W_{A\alpha})} (T^\alpha)_B{}^C E_C$$

$$W_{A\alpha} \sim \partial E$$

on section!

→ covariant conditions $0 = \eta_{-3\,\alpha\beta} \langle\theta| T^\alpha \otimes \langle\theta| T^\beta = \eta_{-4\,\alpha\beta} \langle\theta| T^\alpha \otimes \langle\theta| T^\beta \ldots$

[D=3: Eloy, Galli, Malek]

→ brute-force projection *(11d or IIB section, 11d displayed)* (same for D =3, 4, … : [Hassler, Yakatani])

$$\begin{aligned}\langle\theta| = \langle 0|^I \Theta_I^{(\frac{4}{9})} + \frac{1}{2}\langle 1/3|_{IJ} \Theta^{(\frac{7}{9})IJ} + \frac{1}{24}\langle 2/3|^{IJKL} \Theta_{IJKL}^{(\frac{10}{9})} + \frac{1}{2}\langle 1|_K^{IJ} \Theta_{IJ}^{(\frac{13}{9})K} \\ + \frac{1}{6}\langle 4/3|_{IJK}^L \Theta_L^{(\frac{16}{9})IJK} + \frac{1}{6}\langle 5/3|^{I,JKL} \Theta_{I,JKL}^{(\frac{19}{9})} + \langle 2|_K^{IJ} \Theta_{IJ}^{(\frac{22}{9})K}\end{aligned}$$

finitely many components!

$D = 2$ maximal supergravities from inner space

$$\langle\theta| = \langle 0|^{I}\Theta_I^{(\frac{4}{9})} + \frac{1}{2}\langle 1/3|_{IJ}\Theta^{(\frac{7}{9})IJ} + \frac{1}{24}\langle 2/3|^{IJKL}\Theta_{IJKL}^{(\frac{10}{9})} + \frac{1}{2}\langle 1|_K^{IJ}\Theta_{IJ}^{(\frac{13}{9})K}$$
$$+ \frac{1}{6}\langle 4/3|_{IJK}^{L}\Theta_L^{(\frac{16}{9})IJK} + \frac{1}{6}\langle 5/3|^{I,JKL}\Theta_{I,JKL}^{(\frac{19}{9})} + \langle 2|_K^{IJ}\Theta_{IJ}^{(\frac{22}{9})K}$$

- finitely many allowed couplings, subject to *quadratic constraints*

$$\eta_{-1\,\alpha\beta}\langle\theta|T^\alpha \otimes \langle\theta|T^\beta = 0 = \eta_{-3\,\alpha\beta}\langle\theta|T^\alpha \otimes \langle\theta|T^\beta = \eta_{-4\,\alpha\beta}\langle\theta|T^\alpha \otimes \langle\theta|T^\beta \ \dots$$

- looking for AdS_2: $\langle\theta|T^\delta|\theta\rangle + \eta_{-2\,\alpha\beta}\langle\theta|T^\delta T^\alpha T^{\beta^\dagger}|\theta\rangle = 0 \qquad T^\delta \in \hat{\mathfrak{e}}_8$ `going to the origin'

- solve analytically: small subsectors
  or *numerically*...

Concluding...

3 what is general structure of 2d gauged sugras?

(maximal main focus, but applies more in general)

(possible gaugings, dynamics, solutions, ...)

breaking 'Geroch group' / integrability

- analytic
- numeric → ML

2 study compactifications leading to 3

1 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra $\longrightarrow$ E_9 exceptional field theory

Concluding...

3 what is general structure of 2d gauged sugras?

(possible gaugings, dynamics, solutions, ...)

breaking 'Geroch group' / integrability

- analytic
- numeric → ML

2 study compactifications leading to 3

1 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra $\longrightarrow$ E_9 exceptional field theory

- plethora of 2d models to study
  (even just for D0 brane/SO(9) gauged sugra...)
- uplift conditions
  & twist matrix construction
- perturbative expansion using ExFT
  → KK spectra & higher order couplings
- generalised G structures
  & less-susy truncations

Henning's talk

Dan's talk

Concluding...

3 what is general structure of 2d gauged sugras?

$\underbrace{\text{(possible gaugings,} \quad \text{dynamics,}}_{\text{breaking 'Geroch group' / integrability}} \quad \underbrace{\text{solutions,}}_{\substack{\circ\ \text{analytic} \\ \circ\ \text{numeric} \to \text{ML}}} \text{...)}$

2 study compactifications leading to 3

1 reorganise 11d/IIB dynamics, gauge syms & geometry

in terms of ∞-dimensional syms of 2d sugra $\longrightarrow$ E_9 exceptional field theory

- plethora of 2d models to study
  (even just for D0 brane/SO(9) gauged sugra...)
- uplift conditions
  & twist matrix construction
- perturbative expansion using ExFT
  → KK spectra & higher order couplings
- generalised G structures
  & less-susy truncations

Henning's talk

Dan's talk

Thanks!