# Mass spectra and higher-order couplings for AdS vacua using Exceptional Field Theory

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“Supergravity techniques and the CFT bootstrap”
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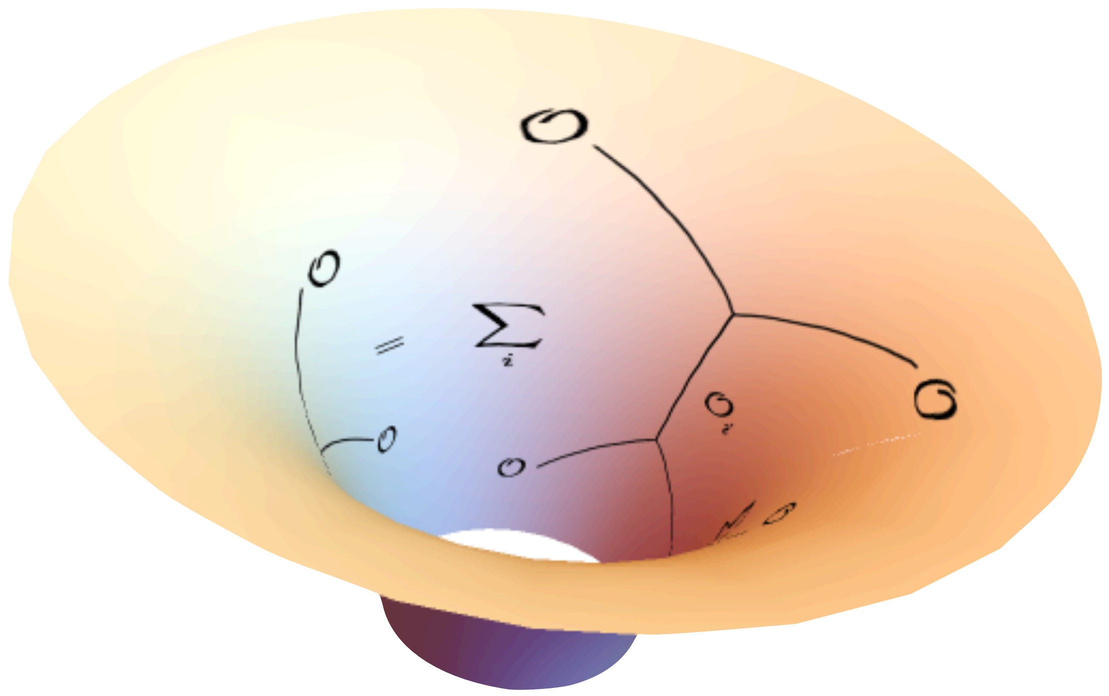


# plan: mass spectra and higher-order couplings

## motivation

- compactification, Kaluza-Klein spectra

## tools

- consistent truncations
- exceptional field theory

## examples

- $AdS_p \times S^q$ and deformations/squashings $AdS_p \times \Sigma_q$

## cubic and higher order couplings

- near extremal n-point couplings
- explicit cubic couplings

based on work with Emanuel Malek, Bastien Duboeuf,

Nikolay Bobev, Camille Eloy, Michele Galli, Adolfo Guarino, Alfredo Giambrone, Olaf Hohm, Gabriel Larios, Hermann Nicolai, Brandon Robinson, Colin Sterckx, Mario Trigiante, Jesse van Muiden

# motivation compactification & Kaluza-Klein spectra

- background $\mathscr{M}_{10} = \mathrm{AdS}_m \times \mathscr{M}_n$
- expanding fields in harmonics on the internal space

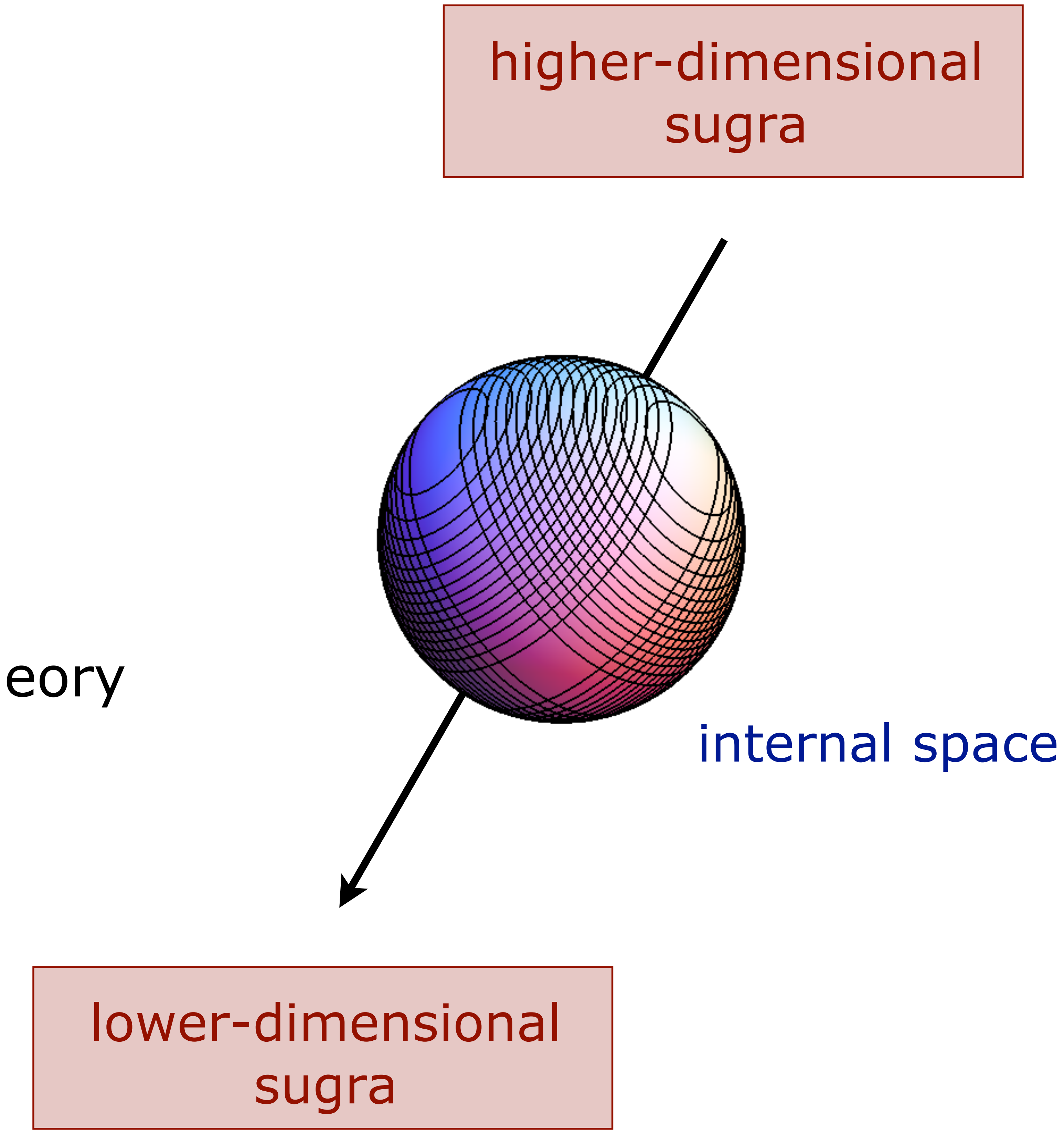
  e.g. scalar field

$$\phi(x,y) = \sum_{\Sigma} \phi_{\Sigma}(x)\, \mathcal{Y}^{\Sigma}(y)$$

fluctuations

harmonics

- dynamics of the KK fluctuations is described by a lower-dimensional theory
- infinitely many fields (KK towers of fluctuations $\{\phi_{\Sigma}, \dots\}$)
- dual to single trace CFT operators $\mathcal{O}_{\phi_{\Sigma}}$
- mass spectrum of the KK-fluctuations (—> conformal dimensions)
- higher order couplings (—> n-point correlators)



- in general: complicated problem
  - gauge fixing and field redefinitions
  - diagonalize various Laplacians on the internal manifold
  - disentangle mass eigenstates from different higher-dimensional origin
  - flux compactifications: higher-dimensional p-forms

- —> new tools !

# new tools: consistent truncations from ExFT ⟶ Dan's talk

ExFT: duality covariant reformulation of D=10/11 sugra

$$\boxed{\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).}$$

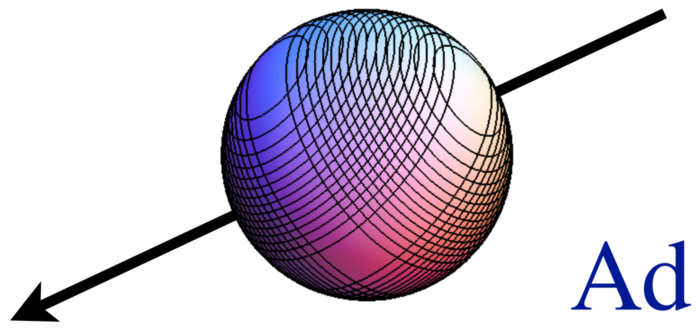
dictionary

generalized Scherk-Schwarz
reduction of ExFT

$$E_6/\text{USp}(8) \qquad \mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y)$$

$$\mathbf{27} \qquad \mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x)$$

IIB sugra



$\text{AdS}_5 \times S^5$

D=5 maximal sugra

$E_{6(6)}$ valued twist matrix $U_M{}^N(Y)$ and scale factor $\rho(Y)$

consistency equations (generalized Leibniz parallelizable)

$$\mathscr{L}_{\mathscr{U}_A} \mathscr{U}_B{}^M = X_{AB}{}^C \mathscr{U}_C{}^M$$

$$\left[(U^{-1})_M{}^P (U^{-1})_N{}^L \partial_P U_L{}^K\right]_{\mathbf{351}} \;=\; \rho\, X_{MN}{}^K$$

embedding tensor of the D=5
gauged supergravity

# new tools: consistent truncations from ExFT → Dan's talk

ExFT: duality covariant reformulation of D=10/11 sugra

$$\boxed{\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^{N}_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).}$$

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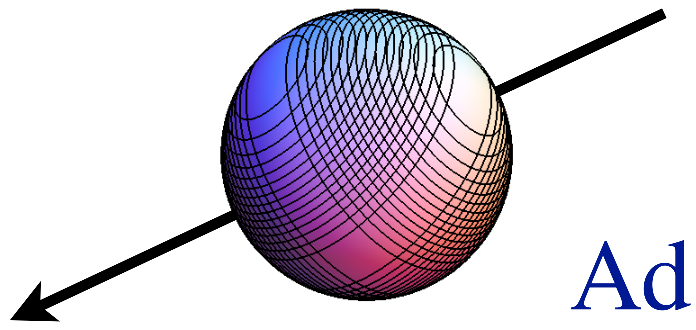
$$\begin{array}{lrcl} \mathrm{E}_6/\mathrm{USp}(8) & \mathcal{M}_{MN}(x,Y) & = & U_M{}^K(Y)\, M_{KL}(x)\, U_N{}^L(Y) \\ \mathbf{27} & \mathcal{A}_\mu{}^M(x,Y) & = & \rho^{-1}(Y)\, (U^{-1})_K{}^M(Y)\, A_\mu{}^K(x) \end{array}$$

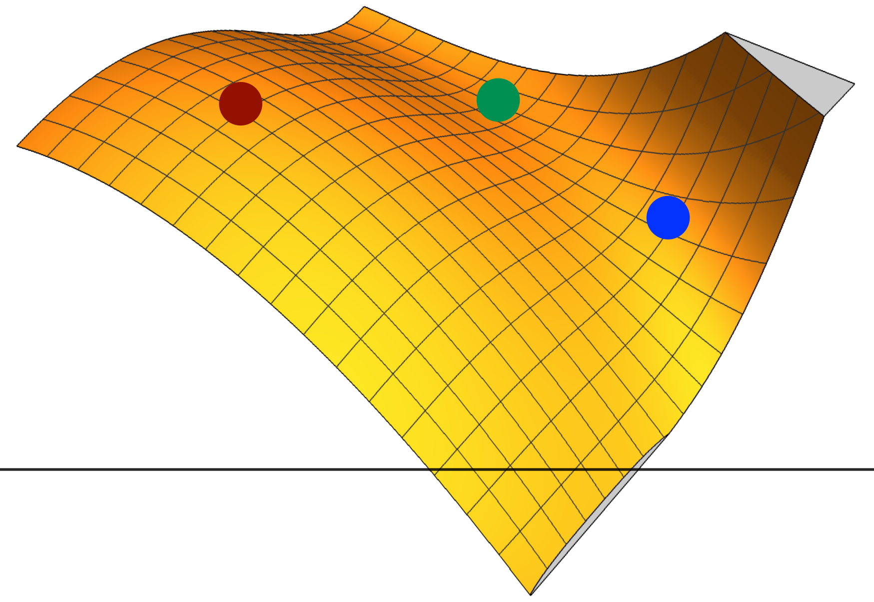
IIB sugra



$\mathrm{AdS}_5 \times S^5$

D=5 maximal sugra

- every stationary point of the D=5 scalar potential lifts to a IIB background $\mathscr{M}_{10} \;=\; \mathrm{AdS}_5 \times \Sigma_5$



- around these backgrounds: compute the masses and couplings of the 42 scalars
- instabilities in all non-supersymmetric AdS₅ vacua ! [Bobev, Fischbacher, Gautason, Pilch]

# new tools: Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

- consistent truncation to lowest KK-multiplet

$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,A_\mu{}^K(x)$$

- extend to the higher Kaluza-Klein modes (linearized)

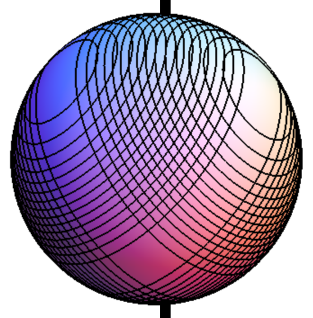
$$\mathcal{A}_\mu{}^M(x,Y) \;=\; \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma$$

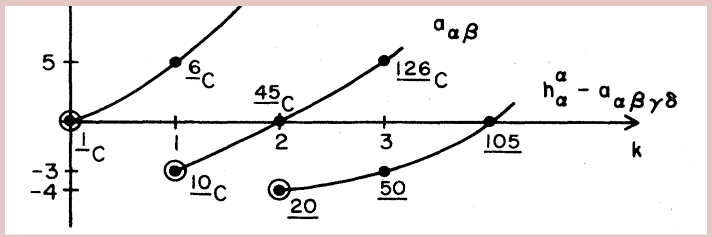
$$\mathcal{M}_{MN}(x,Y) \;=\; U_M{}^K(Y)\,U_N{}^L(Y)\Big(\delta_{KL}+\sum_\Sigma \phi^{\alpha,\Sigma}\,\mathbb{T}_{\alpha,KL}\,\mathcal{Y}^\Sigma\Big)$$

with fluctuations $\boxed{A_\mu{}^{K,\Sigma}\;,\;\phi^{\alpha,\Sigma}\;,}$

and the tower of <u>scalar</u> harmonics $\mathcal{Y}^\Sigma$

IIB sugra





- ( lowest KK multiplet ) $\otimes$ (scalar harmonics) $\xrightarrow{\text{ExFT field equations}}$ KK spectrum

trace of exceptional symmetry in the full spectrum ⟶ holography!

# new tools: Kaluza-Klein spectroscopy from ExFT

[E. Malek, HS]

$$\begin{aligned} \mathcal{A}_\mu{}^M(x,Y) &= \rho^{-1}(Y)\,(U^{-1})_K{}^M(Y)\,\sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma \\ \mathcal{M}_{MN}(x,Y) &= U_M{}^K(Y)\,U_N{}^L(Y)\Big(\delta_{KL}+\sum_\Sigma \phi^{\alpha,\Sigma}\,\mathbb{T}_{\alpha,KL}\,\mathcal{Y}^\Sigma\Big) \end{aligned}$$

- plug into the ExFT action and expand in fluctuations
- e.g. mass matrix for <u>all</u> vector fluctuations $A_\mu{}^{M,\Sigma}$

$$\begin{aligned} M_{M\Sigma,N\Omega} \;\propto\; & \frac{1}{3}\,X^{\rm s}_{ML}{}^K\,X^{\rm s}_{NK}{}^L\,\delta^{\Sigma\Omega}+2\,\big(X^{\rm s}_{MK}{}^N-X^{\rm s}_{NM}{}^K\big)\,\mathcal{T}_{K,\Omega\Sigma} \\ & -6\,\big(\mathbb{P}^K{}_M{}^L{}_N+\mathbb{P}^M{}_K{}^L{}_N\big)\,\mathcal{T}_{L,\Omega\Lambda}\mathcal{T}_{K,\Lambda\Sigma}+\frac{8}{3}\,\mathcal{T}_{N,\Omega\Lambda}\mathcal{T}_{M,\Lambda\Sigma} \end{aligned}$$

in terms of essentially five-dimensional data !

- symmetrized D=5 embedding tensor $X^{\rm s}_{MN}{}^K \equiv X_{MN}{}^K + X_{MK}{}^N$
- adjoint projector $\mathbb{P}^M{}_N{}^K{}_L = (t^\alpha)_N{}^M(t_\alpha)_L{}^K$
- representation of scalar harmonics $\mathcal{K}_M{}^m\partial_m\mathcal{Y}^\Sigma = \mathcal{T}_{M,\Sigma\Omega}\,\mathcal{Y}^\Omega$

- similar for the scalar mass matrix
- similar for fermion masses [Cesàro, Varela]
- entire KK mass spectrum!

# traditional: harmonic analysis on coset spaces

- AdS$_5$ x S$^5$: expand fluctuations in sphere harmonics (representations of SO(6))

10D scalar: $$\phi(x,y) \;=\; \sum_{\Sigma} \phi_{\Sigma}(x)\, \mathcal{Y}^{\Sigma}(y) \;=\; \sum_{n} \phi_{[n,0,0]}(x)\, \mathcal{Y}^{[n,0,0]}(y)$$

scalar harmonics

$$\mathcal{Y}^{\Sigma} = \mathcal{Y}^{[n,0,0]} = \mathcal{Y}^{((a_1}\mathcal{Y}^{a_2} \dots \mathcal{Y}^{a_n))}$$

$$\mathcal{Y}^{a}\mathcal{Y}^{a} = 1$$

10D internal metric: $$g_{kl}(x,y) \;=\; \sum_{n} g_{[n,0,0]}(x)\, \mathcal{Y}^{[n,0,0]}_{kl}(y) + \sum_{n} g_{[n,1,1]}(x)\, \mathcal{Y}^{[n,1,1]}_{kl}(y) + \sum_{n} g_{[n,2,2]}(x)\, \mathcal{Y}^{[n,2,2]}_{kl}(y)$$

tensor harmonics

- in general: several Kaluza-Klein towers for each 10D field, systematics [Salam, Strathdee]

$$S^5 \;=\; \frac{\mathrm{SO}(6)}{\mathrm{SO}(5)} \;=\; \frac{\mathrm{G}}{\mathrm{H}}$$ with embedding $\mathrm{H} \subset \mathrm{G}_{\mathrm{Lorentz,int}}$

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$$S^5 = \frac{\mathrm{SO}(6)}{\mathrm{SO}(5)} = \frac{\mathrm{G}}{\mathrm{H}}$$ with embedding $\mathrm{H} \subset \mathrm{G}_{\text{Lorentz,int}}$

10D field: $\Phi \in \mathscr{R}_{\text{Lorentz,int}} \longrightarrow \mathscr{R}_{\mathrm{H}}$

$$\mathrm{G} \longrightarrow \mathrm{H}$$

the harmonic expansion of $\Phi$ carries all representations $\mathscr{R}_{\mathrm{G}}$ of $\mathrm{G}$ such that $\mathscr{R}_{\mathrm{G}} \longrightarrow \mathscr{R}_{\mathrm{H}} \oplus \dots$

for a scalar field $\mathscr{R}_{\mathrm{H}} = \mathbf{1}$ : $\mathrm{SO}(6) \longrightarrow \mathrm{SO}(5)$

$$[n,0,0] \longrightarrow \mathbf{1} \oplus \dots$$ defines the scalar tower

$g_{kl}(x,y)$ internal metric $\mathscr{R}_{\mathrm{H}} = \mathbf{14}$ :

$$\left.\begin{array}{l} [n,0,0] \longrightarrow \mathbf{14} \oplus \dots \\ [n,1,1] \longrightarrow \mathbf{14} \oplus \dots \\ [n,2,2] \longrightarrow \mathbf{14} \oplus \dots \end{array}\right\} \longrightarrow$$ defines the towers

in closed form:

$$\underbrace{\mathbf{14}}_{\mathrm{SO}(5)} \longrightarrow \underbrace{\mathbf{20} \ominus \mathbf{6}}_{\mathrm{SO}(6)}$$

$$\left(\mathbf{20} \ominus \mathbf{6}\right) \otimes \sum_n [n,0,0] = \sum_n \overbrace{\Big([n,0,0] + [n,1,1] + [n,2,2]\Big)}$$

# structure of fluctuations

- in general: several Kaluza-Klein towers for each 10D field, systematics [Salam, Strathdee]

$$S^5 = \frac{\mathrm{SO(6)}}{\mathrm{SO(5)}} = \frac{\mathrm{G}}{\mathrm{H}}$$ with embedding $\mathrm{H} \subset \mathrm{G}_{\mathrm{Lorentz,int}}$

10D field: $\Phi \in \mathscr{R}_{\mathrm{Lorentz,int}} \longrightarrow \mathscr{R}_{\mathrm{H}}$

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the harmonic expansion of $\Phi$ carries all representations $\mathscr{R}_{\mathrm{G}}$ of $\mathrm{G}$ such that $\mathscr{R}_{\mathrm{G}} \longrightarrow \mathscr{R}_{\mathrm{H}} \oplus \dots$

- in closed form: $\mathscr{R}_{\Phi} \otimes \sum_n [n,0,0]$ $\Longleftrightarrow$ tensor product structure of fluctuations

- ( lowest KK multiplet ) $\otimes$ (scalar harmonics) $\xrightarrow{\text{ExFT field equations}}$ KK spectrum

- explicitly:

$$\mathcal{A}_\mu{}^M(x,Y) = \underbrace{\rho^{-1}(Y)\,(U^{-1})_K{}^M(Y) \sum_\Sigma A_\mu{}^{K,\Sigma}(x)\,\mathcal{Y}^\Sigma}_{\text{tensor harmonics}}$$

scalar harmonics

- ExFT basis: $\left\{ \phi^{\alpha,\Sigma}, A_\mu{}^{M,\Sigma}, \dots \right\}$

# spectrum on AdS5 x S5

[Kim, Romans, van Nieuwenhuizen, 1985]

10D scalar: $$\phi(x,y) = \sum_n \phi_{[n,0,0]}(x)\, \mathscr{Y}^{[n,0,0]}(y)$$

10D internal metric: $$g_{kl}(x,y) = \sum_n g_{[n,0,0]}(x)\, \mathscr{Y}^{[n,0,0]}_{kl}(y) + \sum_n g_{[n,1,1]}(x)\, \mathscr{Y}^{[n,1,1]}_{kl}(y) + \sum_n g_{[n,2,2]}(x)\, \mathscr{Y}^{[n,2,2]}_{kl}(y)$$

10D 4-form: $$C_{klpq}(x,y) = \sum_n c_{[n,0,0]}(x)\, \mathscr{Y}^{[n,0,0]}_{klpq}(y) + \sum_n c_{[n,1,1]}(x)\, \mathscr{Y}^{[n,1,1]}_{klpq}(y)$$

- linearize & diagonalize field equations ⟶ mass spectrum

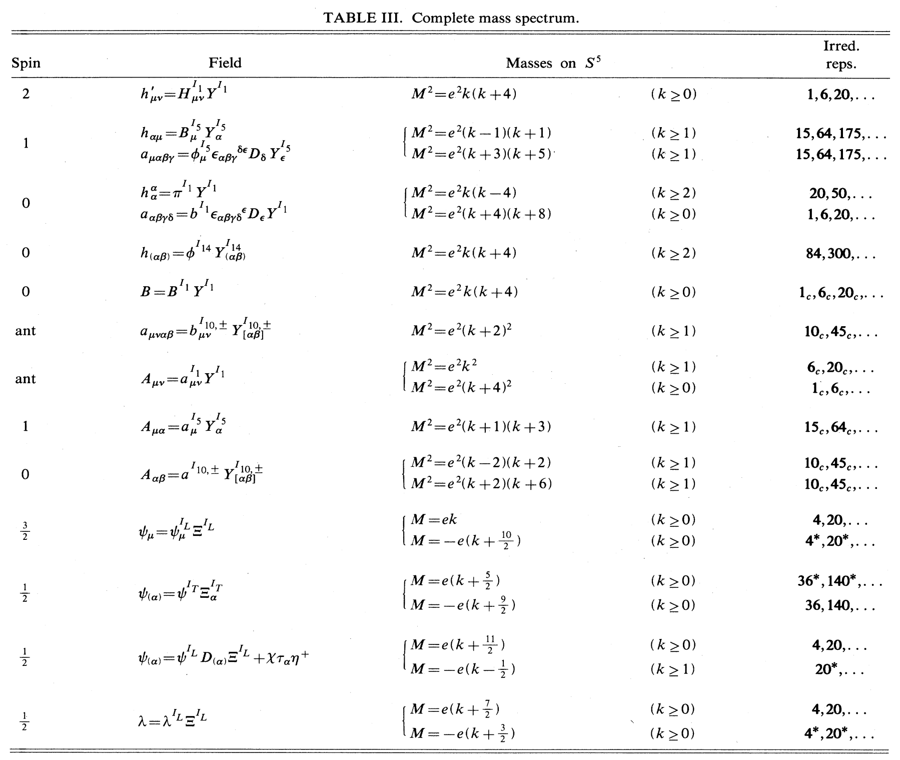

TABLE III. Complete mass spectrum.

| Spin | Field | Masses on $S^5$ | | Irred. reps. |
|---|---|---|---|---|
| 2 | $h'_{\mu\nu}=H^{I_1}_{\mu\nu}Y^{I_1}$ | $M^2=e^2k(k+4)$ | $(k\geq0)$ | 1,6,20,. . . |
| 1 | $h_{\alpha\mu}=B^{I_5}_{\mu}Y^{I_5}_{\alpha}$<br>$a_{\mu\alpha\beta\gamma}=\phi^{I_5}_{\mu}\epsilon_{\alpha\beta\gamma}{}^{\delta\epsilon}D_\delta Y^{I_5}_{\epsilon}$ | $M^2=e^2(k-1)(k+1)$<br>$M^2=e^2(k+3)(k+5)$ | $(k\geq1)$<br>$(k\geq1)$ | 15,64,175,. . .<br>15,64,175,. . . |
| 0 | $h^{\alpha}_{\alpha}=\pi^{I_1}Y^{I_1}$<br>$a_{\alpha\beta\gamma\delta}=b^{I_1}\epsilon_{\alpha\beta\gamma\delta}{}^{\epsilon}D_\epsilon Y^{I_1}$ | $M^2=e^2k(k-4)$<br>$M^2=e^2(k+4)(k+8)$ | $(k\geq2)$<br>$(k\geq0)$ | 20,50,. . .<br>1,6,20,. . . |
| 0 | $h_{(\alpha\beta)}=\phi^{I_{14}}Y^{I_{14}}_{(\alpha\beta)}$ | $M^2=e^2k(k+4)$ | $(k\geq2)$ | 84,300,. . . |
| 0 | $B=B^{I_1}Y^{I_1}$ | $M^2=e^2k(k+4)$ | $(k\geq0)$ | $1_c,6_c,20_c$,. . . |
| ant | $a_{\mu\nu\alpha\beta}=b^{I_{10,\pm}}_{\mu\nu}Y^{I_{10,\pm}}_{[\alpha\beta]}$ | $M^2=e^2(k+2)^2$ | $(k\geq1)$ | $10_c,45_c$,. . . |
| ant | $A_{\mu\nu}=a^{I_1}_{\mu\nu}Y^{I_1}$ | $M^2=e^2k^2$<br>$M^2=e^2(k+4)^2$ | $(k\geq1)$<br>$(k\geq0)$ | $6_c,20_c$,. . .<br>$1_c,6_c$,. . . |
| 1 | $A_{\mu\alpha}=a^{I_5}_{\mu}Y^{I_5}_{\alpha}$ | $M^2=e^2(k+1)(k+3)$ | $(k\geq1)$ | $15_c,64_c$,. . . |
| 0 | $A_{\alpha\beta}=a^{I_{10,\pm}}Y^{I_{10,\pm}}_{[\alpha\beta]}$ | $M^2=e^2(k-2)(k+2)$<br>$M^2=e^2(k+2)(k+6)$ | $(k\geq1)$<br>$(k\geq1)$ | $10_c,45_c$,. . .<br>$10_c,45_c$,. . . |
| $\frac{3}{2}$ | $\psi_\mu=\psi^{I_L}_{\mu}\Xi^{I_L}$ | $M=ek$<br>$M=-e(k+\frac{10}{2})$ | $(k\geq0)$<br>$(k\geq0)$ | 4,20,. . .<br>4*,20*,. . . |
| $\frac{1}{2}$ | $\psi_{(\alpha)}=\psi^{I_T}\Xi^{I_T}_{\alpha}$ | $M=e(k+\frac{5}{2})$<br>$M=-e(k+\frac{9}{2})$ | $(k\geq0)$<br>$(k\geq0)$ | 36*,140*,. . .<br>36,140,. . . |
| $\frac{1}{2}$ | $\psi_{(\alpha)}=\psi^{I_L}D_{(\alpha)}\Xi^{I_L}+\chi\tau_\alpha\eta^+$ | $M=e(k+\frac{11}{2})$<br>$M=-e(k-\frac{1}{2})$ | $(k\geq0)$<br>$(k\geq1)$ | 4,20,. . .<br>20*,. . . |
| $\frac{1}{2}$ | $\lambda=\lambda^{I_L}\Xi^{I_L}$ | $M=e(k+\frac{7}{2})$<br>$M=-e(k+\frac{3}{2})$ | $(k\geq0)$<br>$(k\geq0)$ | 4,20,. . .<br>4*,20*,. . . |

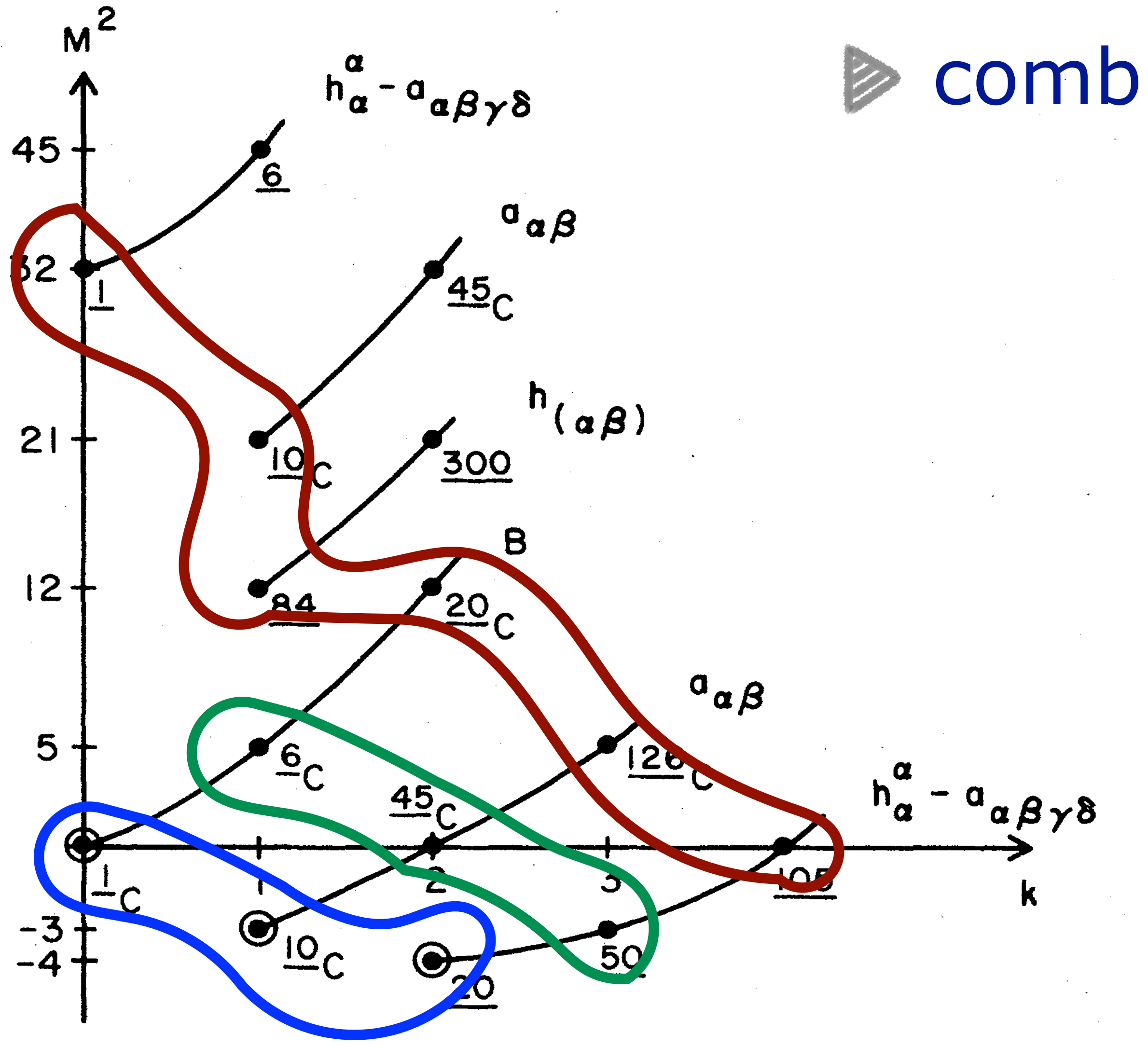

FIG. 2. Mass spectrum of scalars.

- combine into 1/2-BPS multiplets

$$\mathscr{B}_{[2,0,0](0,0)} \oplus \mathscr{B}_{[3,0,0](0,0)} \oplus \mathscr{B}_{[4,0,0](0,0)} \oplus$$

$[n,00](00)$

$[n-1,10](0\tfrac{1}{2}) + [n-1,01](\tfrac{1}{2}0)$

$[n-2,02](00) + [n-2,20](00) + [n-1,00](01) + [n-1,00](10) + [n-2,11](\tfrac{1}{2}\tfrac{1}{2})$

$[n-2,10](0\tfrac{1}{2}) + [n-3,12](0\tfrac{1}{2}) + [n-2,01](\tfrac{1}{2}0) + [n-3,21](\tfrac{1}{2}0) + [n-2,01](\tfrac{1}{2}1) + [n-2,10](1\tfrac{1}{2})$

$2[n-2,00](00) + [n-4,22](00) + [n-3,02](01) + [n-3,20](10) + 2[n-3,11](\tfrac{1}{2}\tfrac{1}{2}) + [n-2,00](11)$

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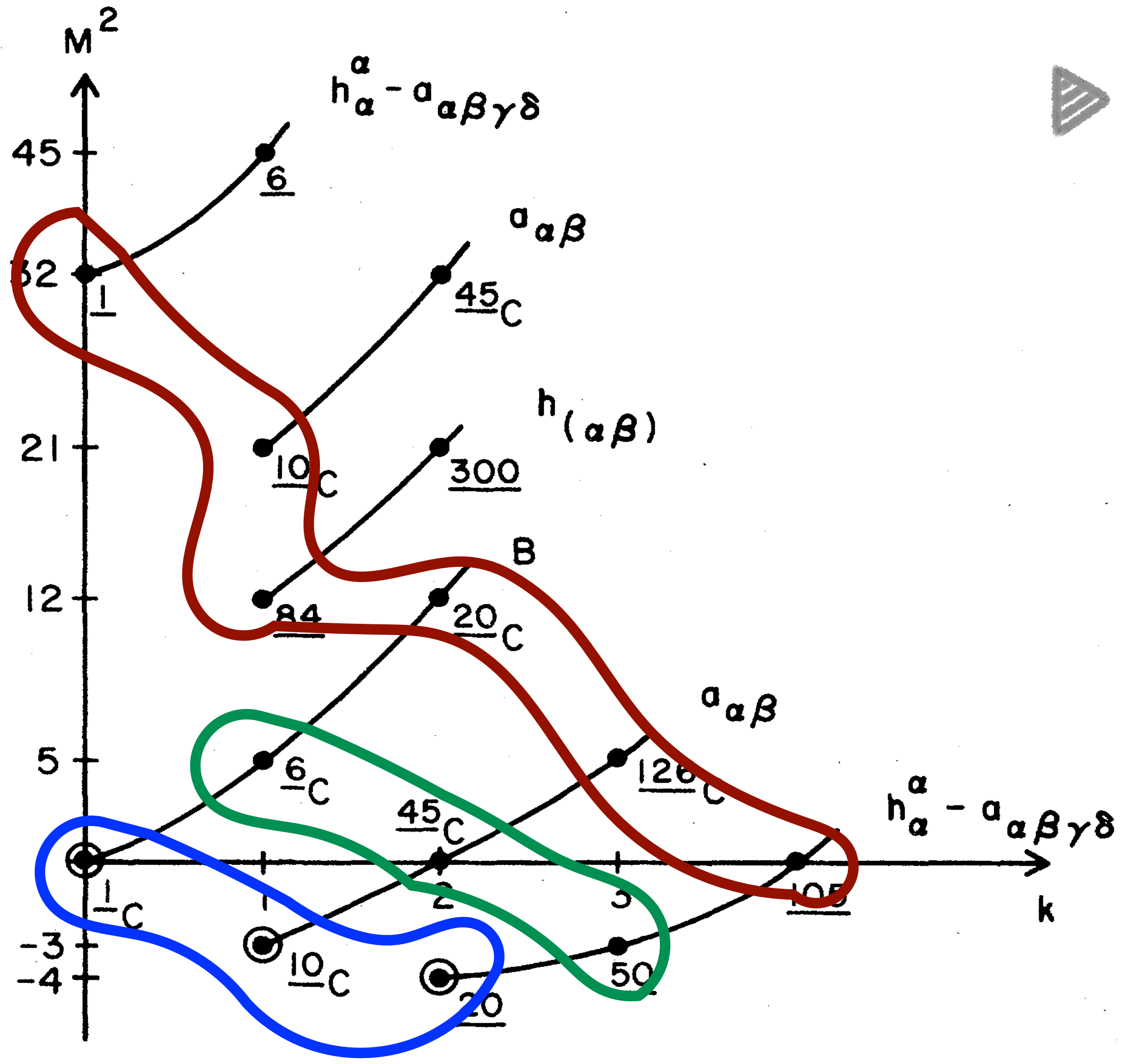
$[n-4,00](00)$

# spectrum on AdS$_5$ x S$^5$

[Kim, Romans, van Nieuwenhuizen, 1985]



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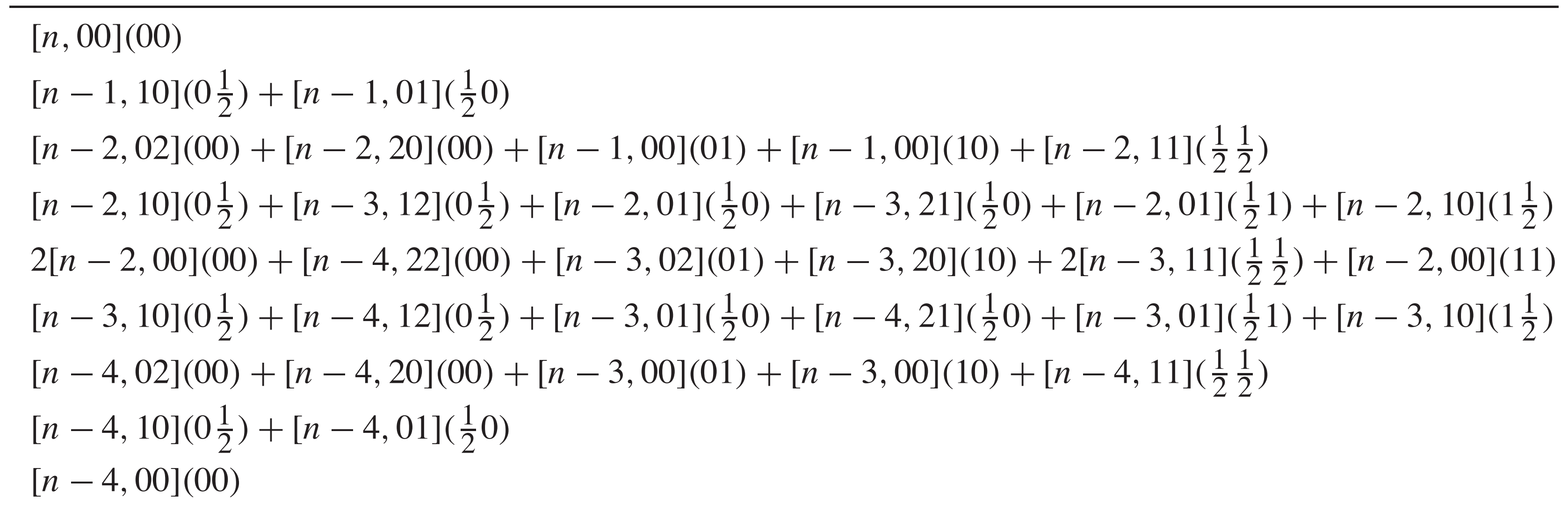

$$[n,00](00)$$
$$[n-1,10](0\tfrac{1}{2}) + [n-1,01](\tfrac{1}{2}0)$$
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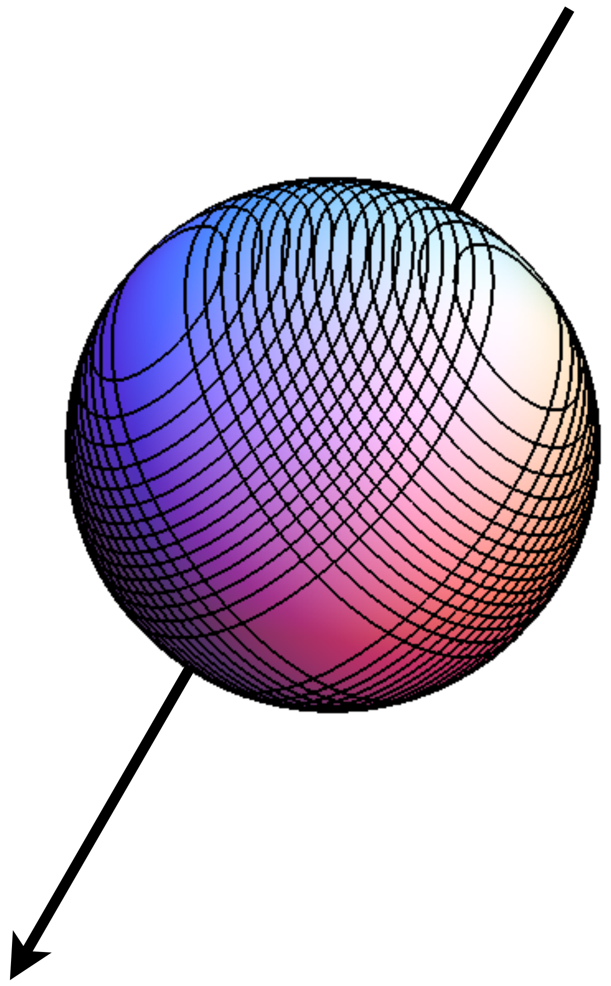
$$[n-4,10](0\tfrac{1}{2}) + [n-4,01](\tfrac{1}{2}0)$$
$$[n-4,00](00)$$

- in the ExFT basis $\left\{\phi^{\alpha,\Sigma}, A_\mu{}^{M,\Sigma}, \ldots\right\}$
    - fluctuations appear already in the diagonal basis (mass eigenstates)
    - for given $\Sigma = [n,0,0]$ the fields fill the multiplet $\mathscr{B}_{[n,0,0]}$
    - simple and compact (re-)derivation of the supergravity spectrum on S$^5$

# example: deformations of $AdS_5 \times S^5$

[with N. Bobev, E. Malek, B. Robinson, J. van Muiden]

IIB supergravity



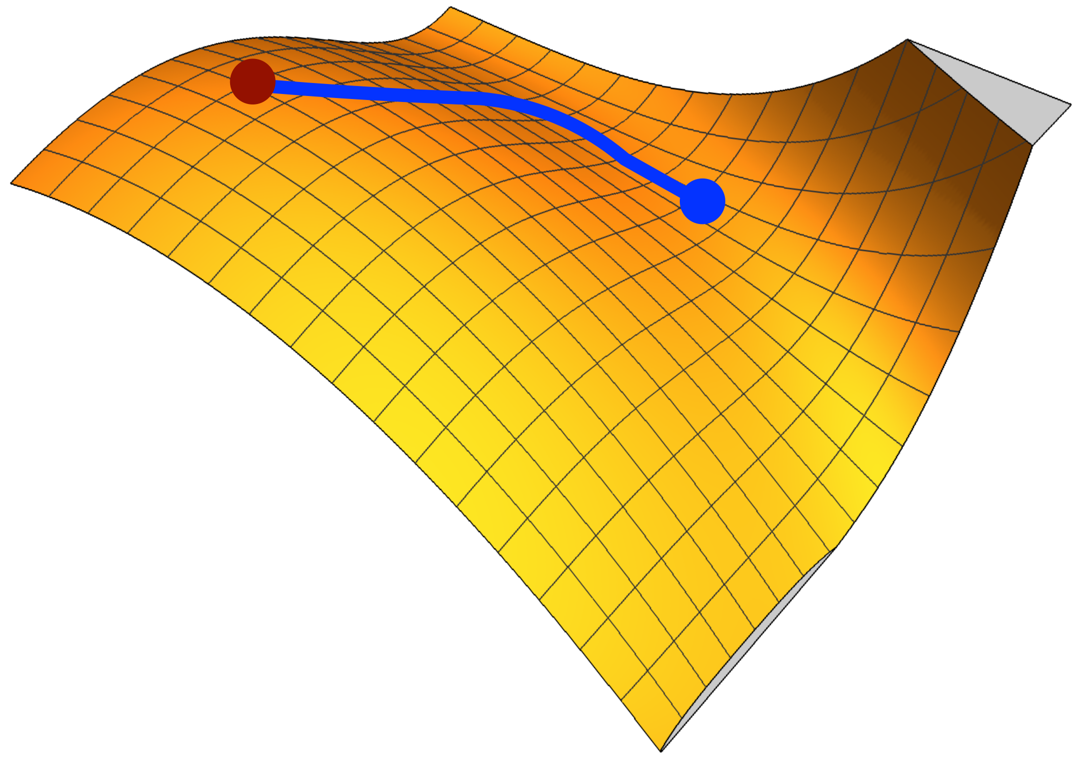
$S^5$

D=5 gauged sugra
[Gunaydin, Romans, Warner]

# example: AdS$_5$ x S$^5$ and deformations

- D=5 SO(6) gauged supergravity: 42 scalars with scalar potential

  [Gunaydin, Romans, Warner]



● $\mathcal{N} = 8$, SO(6), round $S^5$

● $\mathcal{N} = 2$, U(2), deformed $S^5$

Freedman-Gubser-Pilch-Warner flow

holographic dual of Leigh-Strassler SCFT

previously only known for the 256 dof's from the supergravity multiplet

[Freedman, Gubser, Pilch, Warner '99]

- now: compute the full KK spectrum around the $\mathcal{N} = 2$ point

# example: AdS$_5$ x S$^5$ and deformations

- e.g. at level $n = 1$ in multiplets $D(E_0, j_1, j_2; r)$ of $\mathrm{SU}(2) \times \mathrm{SU}(2{,}2\,|\,1)$

$$\mathbf{0}\ :\ D(1+\tfrac{1}{2}\sqrt{37},0,0;1)_{\mathbb{C}} + D(1+\tfrac{1}{2}\sqrt{61},0,0;1)_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac{9}{2},\tfrac{1}{2},\tfrac{1}{2};1)_{\mathbb{C}} + 2\,D_{\mathrm{S}}(\tfrac{9}{2},\tfrac{1}{2},0;-1)_{\mathbb{C}} + D(\tfrac{9}{2},\tfrac{1}{2},0;1)_{\mathbb{C}}$$

$$\mathbf{\frac{1}{2}}\ :\ D(1+\tfrac{1}{4}\sqrt{145},\tfrac{1}{2},\tfrac{1}{2};\tfrac{1}{2})_{\mathbb{C}} + D(1+\tfrac{1}{4}\sqrt{193},0,0;\tfrac{1}{2})_{\mathbb{C}} + D(\tfrac{15}{4},\tfrac{1}{2},0;\tfrac{1}{2})_{\mathbb{C}} + D(\tfrac{17}{4},\tfrac{1}{2},0;-\tfrac{1}{2})_{\mathbb{C}}$$
$$+D_{\mathrm{S}}(\tfrac{15}{4},0,0;\tfrac{5}{2})_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac{17}{4},0,0;\tfrac{3}{2})_{\mathbb{C}}$$

$$\mathbf{1}\ :\ 2\,D(1+\sqrt{7},0,0;0) + D(1+\sqrt{7},\tfrac{1}{2},0;0)_{\mathbb{C}} + D_{\mathrm{S}}(\tfrac{7}{2},\tfrac{1}{2},0;1)_{\mathbb{C}} + D_{\mathrm{S}}(3,\tfrac{1}{2},0;2)_{\mathbb{C}}$$

$$\mathbf{\frac{3}{2}}\ :\ D_{\mathrm{S}}(\tfrac{9}{4},0,0;\tfrac{3}{2})_{\mathbb{C}}$$

in terms of semi-short and long multiplets

# example: AdS$_5$ x S$^5$ and deformations

- e.g. full tower of semi-short (protected) supermultiplets

$$\begin{array}{llll}
L\bar{B}_1(\frac{9+3n}{4};\frac{1}{2},0;\frac{n+3}{2})\otimes[\frac{n+1}{2}]\,, & B_1\bar{L}(\frac{9+3n}{4},0,\frac{1}{2};-\frac{n+3}{2})\otimes[\frac{n+1}{2}]\,, & L\bar{B}_1(\frac{6+3n}{4};0,0;\frac{n+2}{2}]\otimes[\frac{n+2}{2}]\,, & B_1\bar{L}(\frac{6+3n}{4};0,0;-\frac{n+2}{2}]\otimes[\frac{n+2}{2})\,, \\
A_1\bar{L}(\frac{6+3n}{2};\frac{1}{2},0;-n)\otimes[0]\,, & L\bar{A}_1(\frac{6+3n}{2};0,\frac{1}{2};n)\otimes[0]\,, & L\bar{B}_1(\frac{12+3n}{4};0,0;\frac{n+4}{2}]\otimes[\frac{n}{2}]\,, & B_1\bar{L}(\frac{12+3n}{4};0,0;-\frac{n+4}{2}]\otimes[\frac{n}{2})\,, \\
A_1\bar{L}(\frac{6+3n}{2};\frac{1}{2},0;-n)\otimes[0]\,, & L\bar{A}_1(\frac{6+3n}{2};0,\frac{1}{2};n)\otimes[0]\,, & L\bar{A}_2(\frac{8+3n}{4};0,0;\frac{n}{2}]\otimes[\frac{n+2}{2}]\,, & A_2\bar{L}(\frac{8+3n}{4};0,0;-\frac{n}{2}]\otimes[\frac{n+2}{2})\,, \\
L\bar{A}_2(\frac{11+3n}{4};\frac{1}{2},0;\frac{n+1}{2})\otimes[\frac{n+1}{2}]\,, & A_2\bar{L}(\frac{11+3n}{4};0,\frac{1}{2};-\frac{n+1}{2})\otimes[\frac{n+1}{2}]\,, & L\bar{A}_2(\frac{14+3n}{4};0,0;\frac{n+2}{2}]\otimes[\frac{n}{2}]\,, & A_2\bar{L}[\frac{14+3n}{4};0,0;-\frac{n+2}{2}]\otimes(\frac{n}{2})\,.
\end{array}$$

- agreement with index computation in the dual Leigh-Strassler SCFT

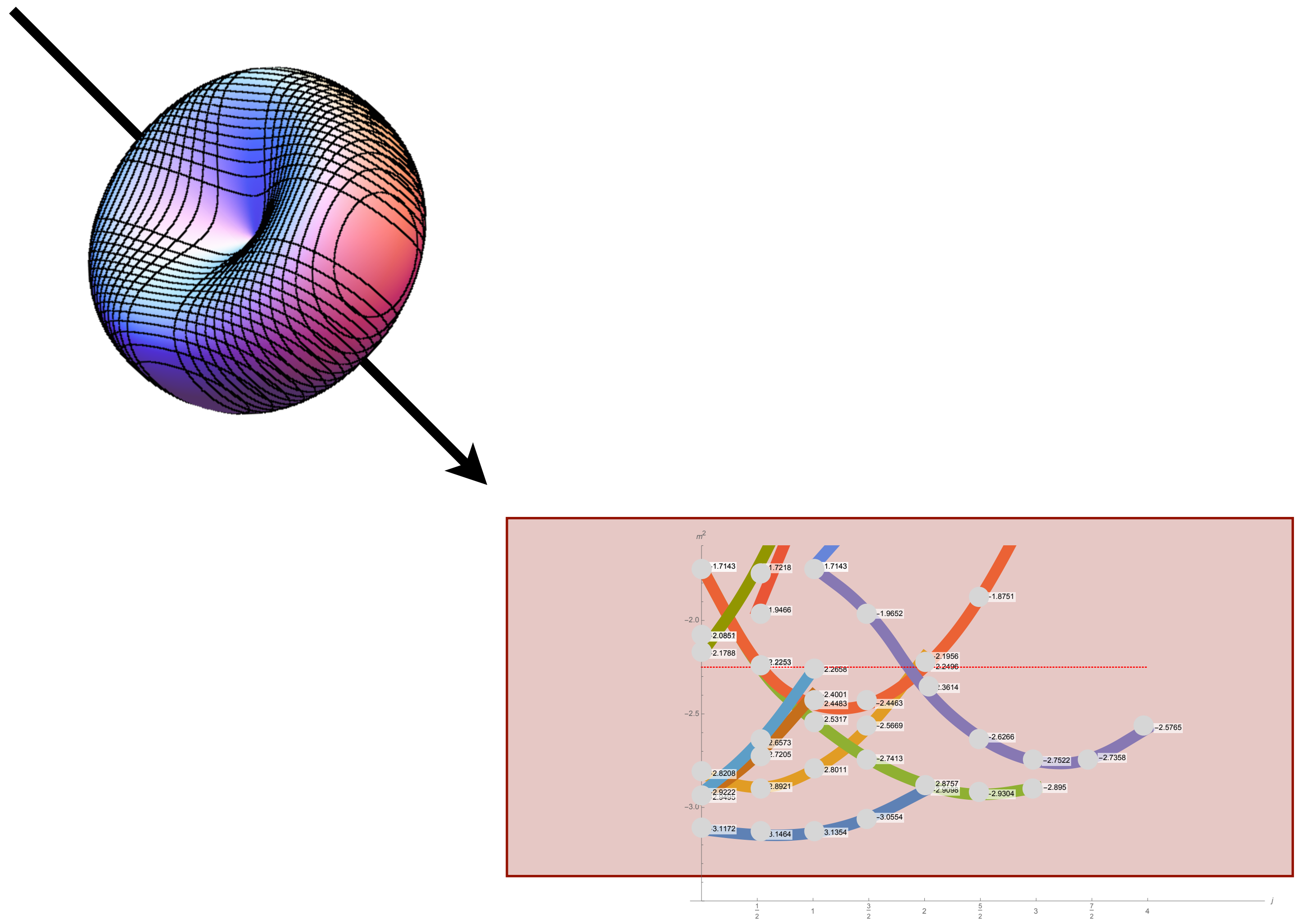
- closed formula for all (including unprotected long) multiplets

$$\Delta = 1 + \sqrt{7 - 3\,|\underbrace{j_1 + j_2}_{\text{SO(4)}}| + \tfrac{3}{4}\left(\underbrace{r^2 - 2(p+2y)^2}_{\text{U(1)}_R \text{ x U(1)}} + 2\,\underset{\text{KK-level}}{n}(n+4) - 4\underset{\text{SU(2)}}{k}(k+1)\right)}\,.$$

# example: AdS$_4$ x S$^7$ and deformations

[with A. Guarino, E. Malek, H. Nicolai]

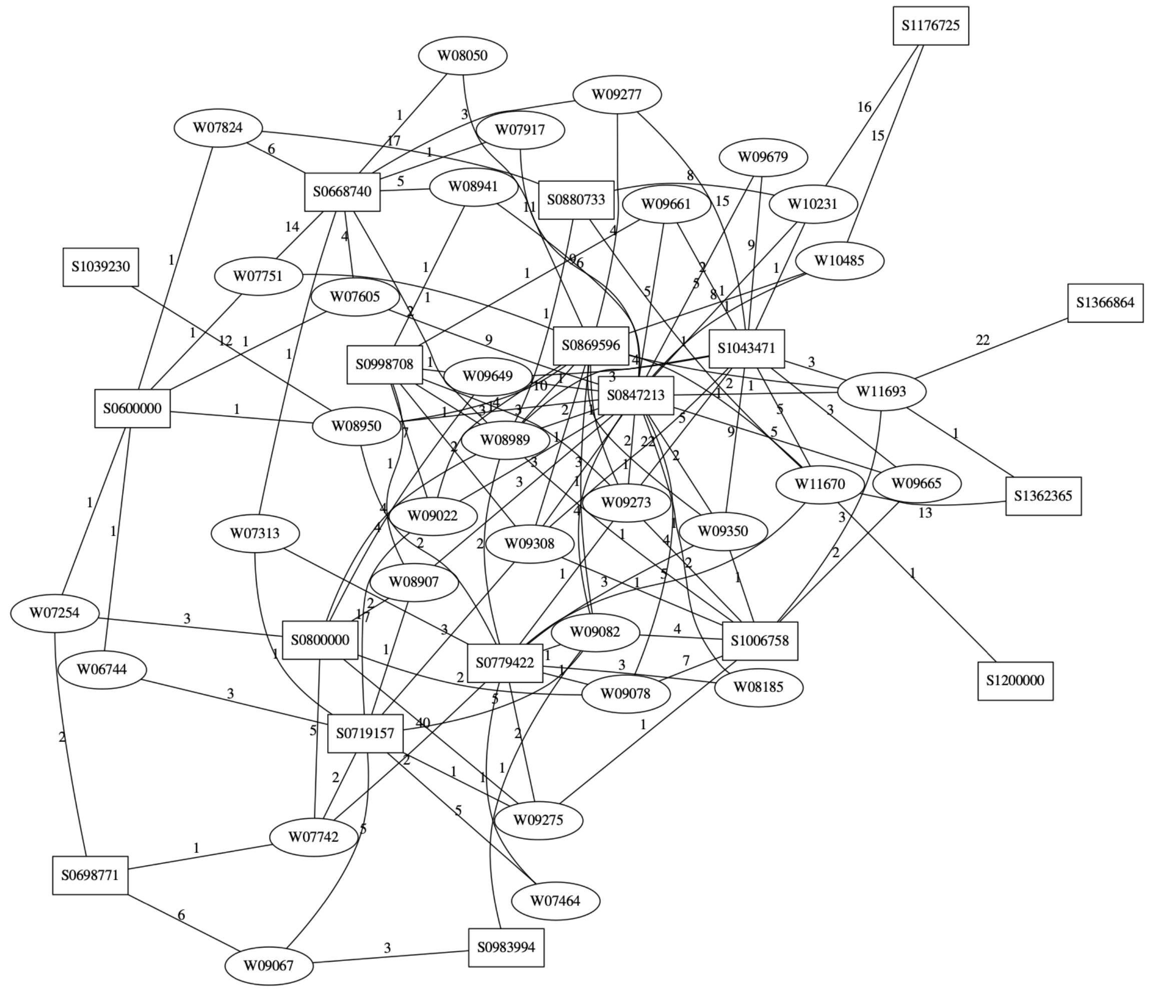
D=11 sugra



tools for <u>non-supersymmetric</u> vacua (where masses are not controlled by symmetry)

# example: non-supersymmetric AdS$_4$ vacua SO(3) x SO(3)

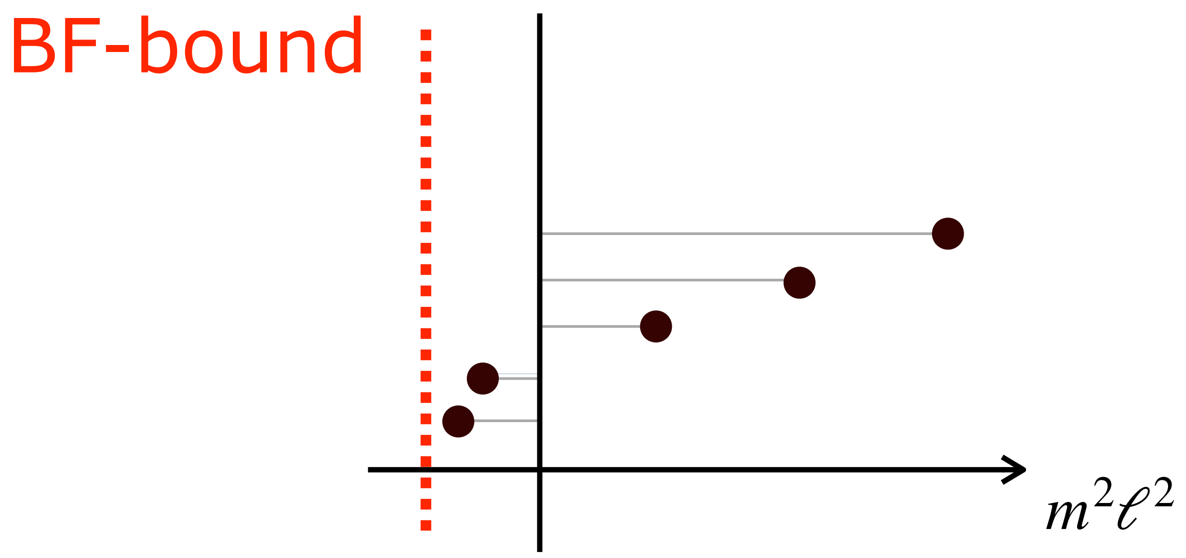
- in D=4 SO(8) supergravity [de Wit, Nicolai]
  the supergravity potential has been carefully scanned for AdS$_4$ vacua
  [Comsa, Firsching, Fischbacher]



all non-supersymmetric vacua are unstable already within D=4 supergravity, i.e. have instabilities within the lowest Kaluza-Klein multiplet

- except for a distinguished SO(3) x SO(3) invariant extremal point [Warner]
  - stable within D=4 supergravity [Fischbacher, Pilch, Warner]
  - uplift to D=11 supergravity [Godazgar, Godazgar, Krüger, Nicolai, Pilch]
  - brane-jet instabilities [Bena, Pilch, Warner]

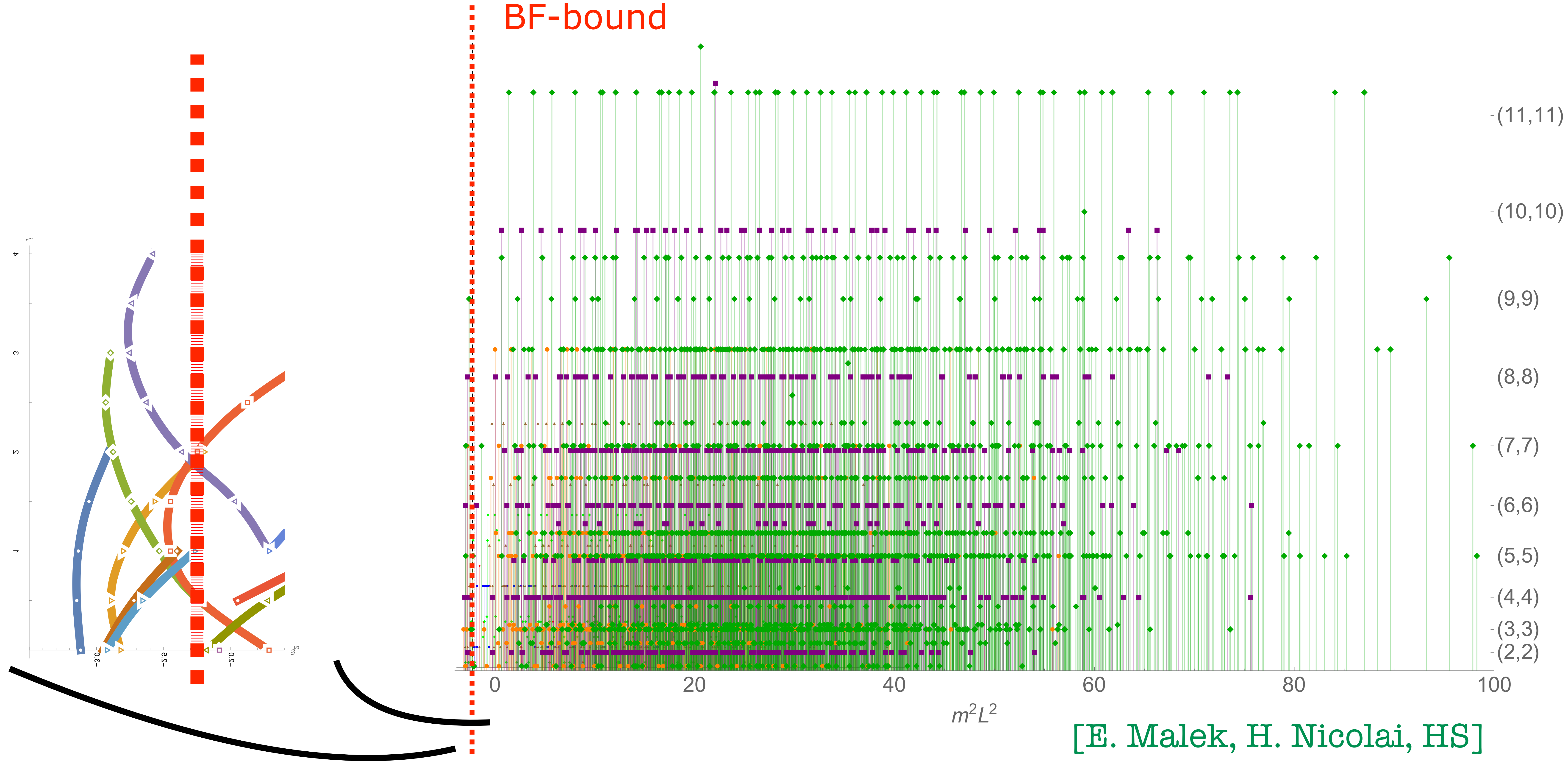
70 scalars:



beyond ?

# example: non-supersymmetric AdS4 vacua SO(3) x SO(3)

- ExFT formulas: full scalar Kaluza-Klein spectrum up to level 6 ( ~ 100.000 scalar fields), from D=4 data



[E. Malek, H. Nicolai, HS]

instabilities starting from KK level 2

$\Longrightarrow$ In D=4, SO(8) supergravity, all known non-supersymmetric vacua are perturbatively unstable!

# example: non-supersymmetric AdS4 vacua in ISO(7) supergravity

- massive IIA admits a consistent truncation on S6 [Guarino, Jafferis, Varela]

  ⟶ to (dyonic) ISO(7) gauged supergravity [Dall'Agata, Inverso]

  with $\mathcal{N} = 3$ AdS4 vacuum

- the D=4 scalar potential carries a wealth of AdS vacua:

  ⟶ non-supersymmetric vacua, stable within D=4 supergravity

- most symmetric: $\mathcal{N} = 0$ G2 vacuum, deformed S6

  ⟶ no brane-jet instabilities [Guarino, Tarrio, Varela]

- ExFT analysis yields the full KK spectrum!

  analytic mass formula for all scalars:

$$\boxed{m^2\,\ell^2 = (n+2)\,(n+3) - \frac{3}{2}\,\mathscr{C}_{[n_1,n_2]}}$$

[A. Guarino, E. Malek, HS]

$$\begin{aligned}\langle \mathscr{J}\mathscr{M}\mathscr{J}\rangle \;\propto\; & \frac{1}{5}\left(X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BE}}{}^{\underline{F}} + X_{\underline{EA}}{}^{\underline{F}}X_{\underline{EB}}{}^{\underline{F}} + X_{\underline{EF}}{}^{\underline{A}}X_{\underline{EF}}{}^{\underline{B}} + 5\,X_{\underline{AE}}{}^{\underline{F}}X_{\underline{BF}}{}^{\underline{E}}\right)\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Sigma}\\ & + \frac{2}{5}\left(X_{\underline{AC}}{}^{\underline{E}}X_{\underline{BD}}{}^{\underline{E}} - X_{\underline{AE}}{}^{\underline{C}}X_{\underline{BE}}{}^{\underline{D}} - X_{\underline{EA}}{}^{\underline{C}}X_{\underline{EB}}{}^{\underline{D}}\right)\mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{CD},\Sigma}\\ & - \frac{4}{5}\left(X_{\underline{AC}}{}^{\underline{D}}\mathscr{T}_{\underline{B},\Omega\Sigma} + 6\,X_{\underline{AC}}{}^{\underline{B}}\mathscr{T}_{\underline{D},\Omega\Sigma}\right)\mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{CD},\Omega}\\ & - \frac{4}{5}\left(X_{\underline{CA}}{}^{\underline{B}}\mathscr{T}_{\underline{C},\Omega\Sigma} + 6\,X_{\underline{BC}}{}^{\underline{A}}\mathscr{T}_{\underline{C},\Omega\Sigma}\right)\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Omega}\\ & + 12\,\mathscr{J}_{\underline{AD},\Sigma}\,\mathscr{J}_{\underline{BD},\Omega}\mathscr{T}_{\underline{A},\Omega\Lambda}\mathscr{T}_{\underline{B},\Lambda\Sigma} - \mathscr{J}_{\underline{AB},\Sigma}\,\mathscr{J}_{\underline{AB},\Omega}\mathscr{T}_{\underline{C},\Omega\Lambda}\mathscr{T}_{\underline{C},\Lambda\Sigma}\end{aligned}$$

KK level $n$, G2 Casimir $\mathscr{C}_{[n_1,n_2]}$

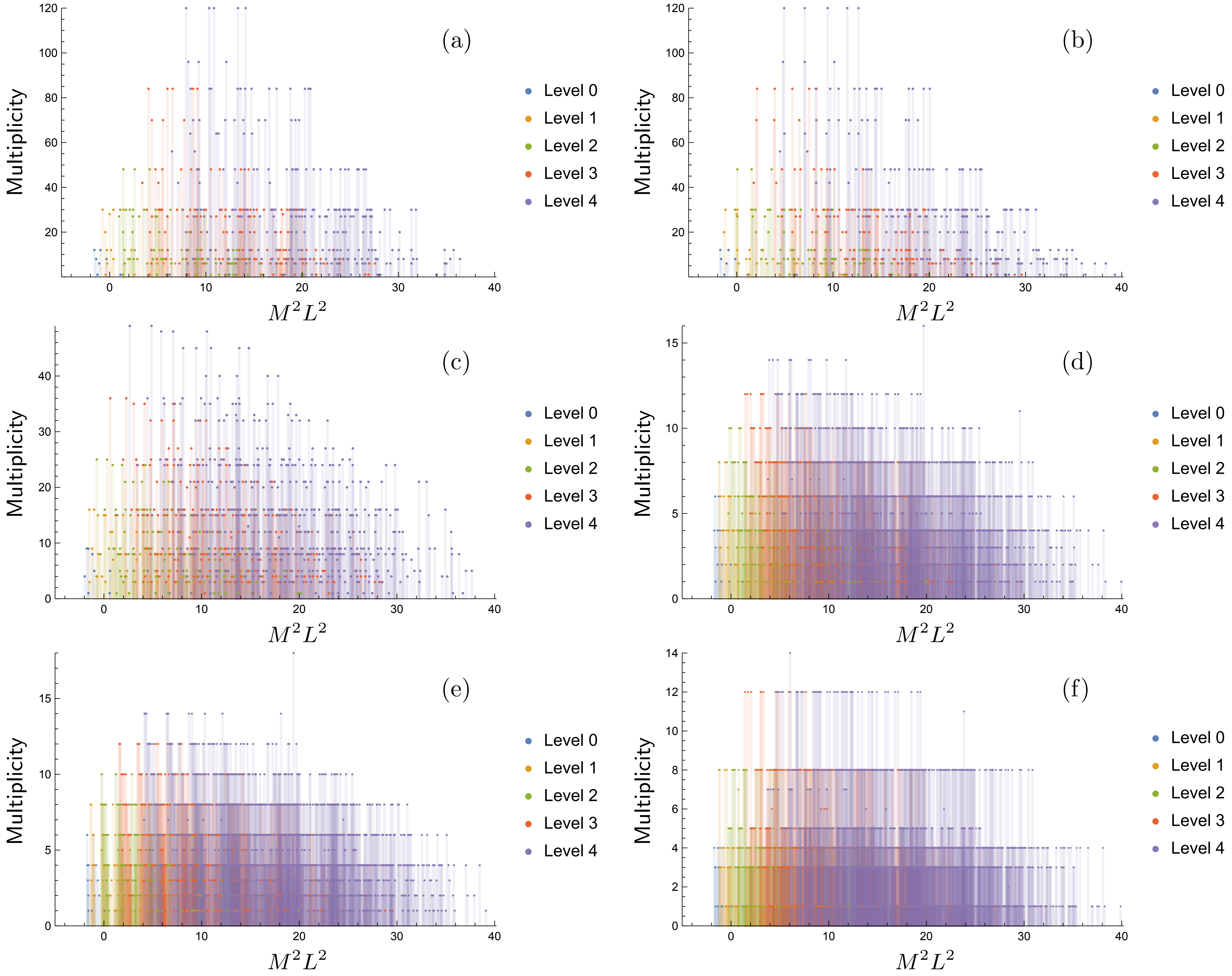
- proves stability of the KK spectrum: $m^2\,\ell^2 \geq m^2_{\rm BF}\,\ell^2$

- (perturbatively) stable non-supersymmetric AdS4 vacuum

- bubble instabilities… [Bomans, Cassani, Dibitetto, Petri]

# example: non-supersymmetric AdS₄ vacua in ISO(7) supergravity

- likewise: KK-spectra for more non-supersymmetric vacua (numerical) with remaining SU(3), SO(4), U(2), SO(3): all (perturbatively) stable!



## cubic and higher order couplings

[with B. Duboeuf, E. Malek]

# n-point couplings

- higher couplings around AdS5 x S5 all fields in representations of SO(6)

$$\mathscr{L}_{\phi^n} = g_{I_1 I_2 \ldots I_n}\, \phi^{I_1} \phi^{I_2} \ldots \phi^{I_n}$$

  - information on the holographic n-pt functions $\langle \mathcal{O}_{I_1} \mathcal{O}_{I_2} \cdots \mathcal{O}_{I_n} \rangle$

- previously
  - expand fields into S5 harmonics and integrate IIB Lagrangian over S5
  - expand/diagonalize/disentangle IIB field equations
  - gauge fixing, non-linear field redefinitions
  - achieved for cubic and (some) quartic couplings, "heroic efforts" [Arutyunov, Frolov] [Lee, Minwalla, Rangamani, Seiberg]

- in ExFT framework
  - 'standard' two-derivative action $\boxed{\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).}$
  - & basis of fluctuations $\mathcal{M}_{MN} = U_M{}^K(Y)\, U_N{}^L(Y) \Big( \delta_{KL} + \sum_\Sigma \bigcirc\!\!\!\!\!\!\!\phantom{x}\phi^{\alpha,\Sigma}\, \mathbb{T}_{\alpha,KL}\, \mathcal{Y}^\Sigma \Big)$
  - tensor product structure (lowest KK multiplet) ⊗ (scalar harmonics)

# near-extremal n-point couplings

- in ExFT framework
  - ‘standard’ two-derivative action $\boxed{\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).}$
  - & basis of fluctuations $\mathcal{M}_{MN} = U_M{}^K(Y)\, U_N{}^L(Y) \Big(\delta_{KL} + \sum_\Sigma \boxed{\phi^{\alpha,\Sigma}}\, \mathbb{T}_{\alpha,KL}\, \mathcal{Y}^\Sigma\Big)$
  - tensor product structure (lowest KK multiplet) $\otimes$ (scalar harmonics)
  - n-point couplings $\mathscr{L}_{\phi^n} = \underbrace{g_{\alpha_1\Sigma_1,\alpha_2\Sigma_2,\cdots,\alpha_n\Sigma_n}}\, \phi^{\alpha_1\Sigma_1}\, \phi^{\alpha_2\Sigma_2} \,\ldots\, \phi^{\alpha_n\Sigma_n}$

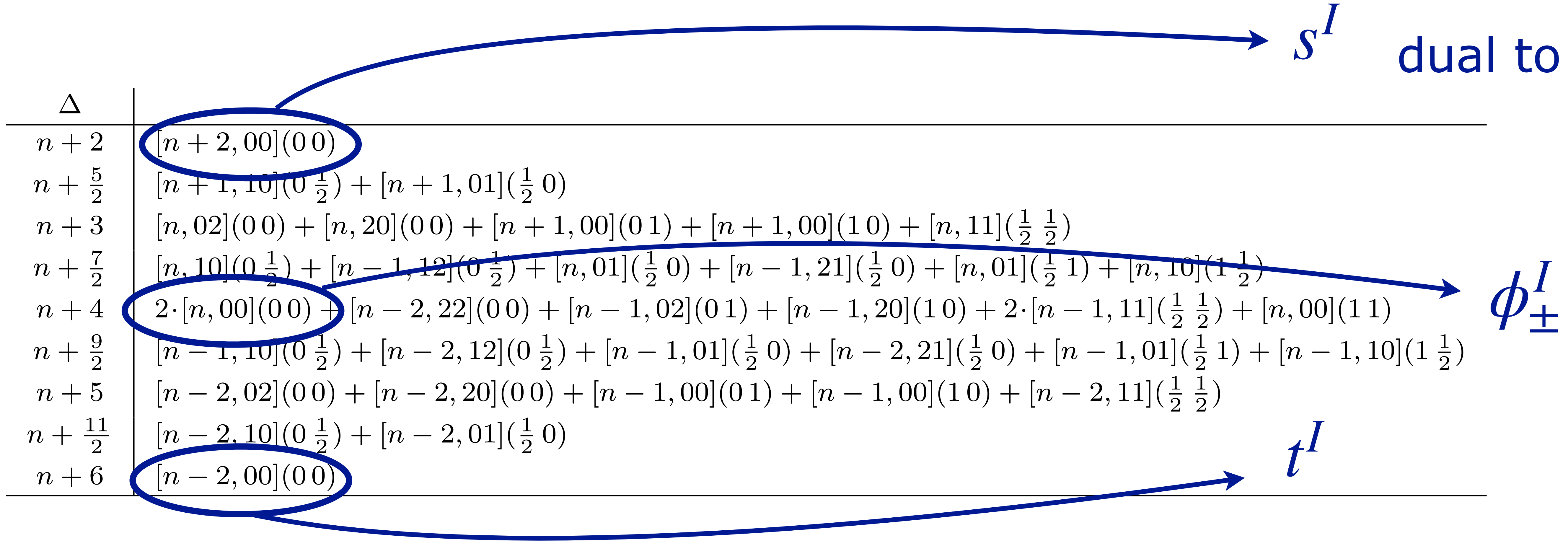
$\longrightarrow$ carries $\displaystyle\int_{S^5} \mathcal{Y}^{\Sigma_1} \mathcal{Y}^{\Sigma_2} \ldots \mathcal{Y}^{\Sigma_n} \equiv c_{\Sigma_1\Sigma_2\cdots\Sigma_n}$

SO(6) invariant tensor

  - non-vanishing n-point coupling requires $c_{\Sigma_1\Sigma_2\ldots\Sigma_n}$ to exist!

# near-extremal n-point couplings

- example: scalars in [m,0,0] of SO(6) $\mathcal{R}^I\ :\ \ [m{,}0{,}0]\ :\ \ |I| \equiv m \qquad \mathcal{Y}^I = \mathcal{Y}^{(\!(a_1}\mathcal{Y}^{a_2}\ldots\,\mathcal{Y}^{a_m)\!)} \qquad \mathcal{Y}^a\mathcal{Y}^a = 1$



| $\Delta$ | |
|---|---|
| $n+2$ | $[n+2,00](0\,0)$ |
| $n+\frac{5}{2}$ | $[n+1,10](0\,\frac{1}{2}) + [n+1,01](\frac{1}{2}\,0)$ |
| $n+3$ | $[n,02](0\,0) + [n,20](0\,0) + [n+1,00](0\,1) + [n+1,00](1\,0) + [n,11](\frac{1}{2}\,\frac{1}{2})$ |
| $n+\frac{7}{2}$ | $[n,10](0\,\frac{1}{2}) + [n-1,12](0\,\frac{1}{2}) + [n,01](\frac{1}{2}\,0) + [n-1,21](\frac{1}{2}\,0) + [n,01](\frac{1}{2}\,1) + [n,10](1\,\frac{1}{2})$ |
| $n+4$ | $2\cdot[n,00](0\,0) + [n-2,22](0\,0) + [n-1,02](0\,1) + [n-1,20](1\,0) + 2\cdot[n-1,11](\frac{1}{2}\,\frac{1}{2}) + [n,00](1\,1)$ |
| $n+\frac{9}{2}$ | $[n-1,10](0\,\frac{1}{2}) + [n-2,12](0\,\frac{1}{2}) + [n-1,01](\frac{1}{2}\,0) + [n-2,21](\frac{1}{2}\,0) + [n-1,01](\frac{1}{2}\,1) + [n-1,10](1\,\frac{1}{2})$ |
| $n+5$ | $[n-2,02](0\,0) + [n-2,20](0\,0) + [n-1,00](0\,1) + [n-1,00](1\,0) + [n-2,11](\frac{1}{2}\,\frac{1}{2})$ |
| $n+\frac{11}{2}$ | $[n-2,10](0\,\frac{1}{2}) + [n-2,01](\frac{1}{2}\,0)$ |
| $n+6$ | $[n-2,00](0\,0)$ |

$s^I$ dual to chiral primary $\mathcal{O}_{s^I}$

$\phi^I_\pm$

$t^I$

within the ExFT basis $\phi^{\alpha,\Sigma}$ : $\quad \alpha\,:\ \mathbf{42} \longrightarrow [2{,}0{,}0]_0 \oplus [0{,}0{,}2]_{+1} \oplus [0{,}2{,}0]_{-1} \oplus [0{,}0{,}0]_{\pm 2} \qquad \Sigma\,:\ [n{,}0{,}0]_0$

$$s^I :\ [n+2{,}0{,}0]\ \in\ [2{,}0{,}0]\otimes[n{,}0{,}0]\ :\quad \phi^{(\!(ab\,,\,\Sigma)\!)} \ = \ \phi^{(\!(ab\,,\,a_1\ldots a_n)\!)} \qquad |I| = |\Sigma| + 2$$

$$\phi^I_\pm :\ [n{,}0{,}0]\ \in\ [0{,}0{,}0]\otimes[n{,}0{,}0]\ :\quad \phi^{\pm,\Sigma} \ = \ \phi^{\pm,a_1\ldots a_n} \qquad |I| = |\Sigma|$$

$$t^I :\ [n-2{,}0{,}0]\ \in\ [2{,}0{,}0]\otimes[n{,}0{,}0]\ :\quad \phi^{\underbrace{ab\,,\,\Sigma}} \ = \ \phi^{ab\,,\,aba_1\ldots a_{n-2}} \qquad |I| = |\Sigma| - 2$$

- consider a coupling among the $s^I$ : $\mathcal{G}(s^{I_1}, s^{I_2}, \ldots, s^{I_n})$

# near-extremal n-point couplings

- example: scalars in [m,0,0] of SO(6) $\mathcal{R}^I$ : $[m,0,0]$ : $|I| \equiv m$ $\mathcal{Y}^I = \mathcal{Y}^{(\!(a_1}\mathcal{Y}^{a_2}\dots\,\mathcal{Y}^{a_m)\!)}$ $\mathcal{Y}^a\mathcal{Y}^a = 1$

  within the ExFT basis $\phi^{\alpha,\Sigma}$ : $s^I$ : $[n+2,0,0] \in [2,0,0]\otimes[n,0,0]$ : $\phi^{(\!(ab\,,\,\Sigma)\!)} = \phi^{(\!(ab\,,\,a_1\dots a_n)\!)}$ $|I| = |\Sigma| + 2$

- consider a coupling among the $s^I$ : $\mathcal{G}(s^{I_1}, s^{I_2}, \dots, s^{I_n})$
  - SO(6) group theory: a non-vanishing coupling requires $\mathbf{1} \in I_1 \otimes I_2 \otimes \cdots \otimes I_n$

    thus
    $$\boxed{\Big(\sum_{j\neq i} |I_j|\Big) - |I_i| < 0 \quad\Longrightarrow\quad \mathcal{G}(s^{I_1}, s^{I_2}, \dots, s^{I_n}) = 0}$$
  - in the ExFT basis $\mathcal{G}(\phi^{\alpha_1,\Sigma_1}, \phi^{\alpha_2,\Sigma_2}, \dots, \phi^{\alpha_n,\Sigma_n})$

    a non-vanishing coupling requires moreover $\mathbf{1} \in \Sigma_1 \otimes \Sigma_2 \otimes \cdots \otimes \Sigma_n$

$$\boxed{\Big(\sum_{j\neq i} |I_j|\Big) - |I_i| \;\leq\; 2\,(n-3) \quad\Longrightarrow\quad \mathcal{G}(s^{I_1}, s^{I_2}, \dots, s^{I_n}) = 0}$$

  - cubic extremal couplings vanish!
  - n-point near-extremal couplings vanish!

conjectured in
[D'Hoker, Erdmenger, Freedman, Perez-Victoria, 2000]
[D'Hoker, Pioline, 2000]

# near-extremal n-point couplings

$$\boxed{\Big(\sum_{j\neq i} |I_j|\Big) - |I_i| \;\leq\; 2\,(n-3) \qquad \Longrightarrow \qquad \mathscr{G}(s^{I_1}, s^{I_2}, \dots, s^{I_n}) = 0}$$

- cubic extremal couplings vanish!
- n-point near-extremal couplings vanish!

conjectured in
[D'Hoker, Erdmenger, Freedman, Perez-Victoria, 2000]
[D'Hoker, Pioline, 2000]

- moreover:
  - proof of analogous conjectures for near-extremal couplings on AdS$_7$ x S$^4$ and AdS$_4$ x S$^7$ [D'Hoker, Pioline, 2000]
  - similar results for couplings $\mathscr{G}(s^{I_1}, \dots, s^{I_m}, t^{J_1}, \dots, t^{J_n})$ and scalars in other representations
  - similar results for couplings involving spin-1 and spin-2 fields
  - apply to all vacua of the theory, e.g. $\mathscr{N}=2$ holographic dual of Leigh-Strassler SCFT

$$\begin{array}{llll}
L\bar{B}_1(\frac{9+3n}{4}; \frac{1}{2}, 0; \frac{n+3}{2}) \otimes [\frac{n+1}{2}]\,, & B_1\bar{L}(\frac{9+3n}{4}, 0, \frac{1}{2}; -\frac{n+3}{2}) \otimes [\frac{n+1}{2}]\,, & L\bar{B}_1(\frac{6+3n}{4}; 0, 0; \frac{n+2}{2}] \otimes [\frac{n+2}{2}]\,, & B_1\bar{L}(\frac{6+3n}{4}; 0, 0; -\frac{n+2}{2}] \otimes [\frac{n+2}{2})\,, \\
A_1\bar{L}(\frac{6+3n}{2}; \frac{1}{2}, 0; -n) \otimes [0]\,, & L\bar{A}_1(\frac{6+3n}{2}; 0, \frac{1}{2}; n) \otimes [0]\,, & L\bar{B}_1(\frac{12+3n}{4}; 0, 0; \frac{n+4}{2}] \otimes [\frac{n}{2}]\,, & B_1\bar{L}(\frac{12+3n}{4}; 0, 0; -\frac{n+4}{2}] \otimes [\frac{n}{2})\,, \\
A_1\bar{L}(\frac{6+3n}{2}; \frac{1}{2}, 0; -n) \otimes [0]\,, & L\bar{A}_1(\frac{6+3n}{2}; 0, \frac{1}{2}; n) \otimes [0]\,, & L\bar{A}_2(\frac{8+3n}{4}; 0, 0; \frac{n}{2}] \otimes [\frac{n+2}{2}]\,, & A_2\bar{L}(\frac{8+3n}{4}; 0, 0; -\frac{n}{2}] \otimes [\frac{n+2}{2})\,, \\
L\bar{A}_2(\frac{11+3n}{4}; \frac{1}{2}, 0; \frac{n+1}{2}) \otimes [\frac{n+1}{2}]\,, & A_2\bar{L}(\frac{11+3n}{4}; 0, \frac{1}{2}; -\frac{n+1}{2}) \otimes [\frac{n+1}{2}]\,, & L\bar{A}_2(\frac{14+3n}{4}; 0, 0; \frac{n+2}{2}] \otimes [\frac{n}{2}]\,, & A_2\bar{L}[\frac{14+3n}{4}; 0, 0; -\frac{n+2}{2}] \otimes (\frac{n}{2})\,.
\end{array}$$

semi-short supermultiplets at a given level $|\Sigma| = n$

# explicit cubic couplings

- universal formulas for cubic couplings $\mathscr{L}_{\phi^3} \;=\; g_{\alpha\Sigma,\beta\Delta,\gamma\Omega}\, \phi^{\alpha,\Sigma}\, \phi^{\beta,\Delta}\, \phi^{\gamma,\Omega}$

$$\mathcal{L}_{\phi^3} \;=\; \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Omega}\left\{c^{\Sigma\Delta\Omega}\,\mathbb{X}_{\alpha\beta\gamma}+c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Sigma}\,\mathbb{X}_{A\,\alpha\beta\gamma}+c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Theta}\mathcal{T}_B{}^{\Theta\Sigma}\,\mathbb{X}_{AB\,\alpha\beta\gamma}\right\}$$

representation matrix on harmonics

symmetric SO(6) tensors

- with tensors

$$\begin{aligned}
\mathbb{X}_{AB\,\alpha\beta\gamma} &= 6\,\mathbb{T}_{\gamma\alpha\beta\,A}{}^{B}-\frac{3}{2}\,\mathbb{T}_{\gamma\,A}{}^{B}\,\eta_{\alpha\beta}\\
\mathbb{X}_{A\,\alpha\beta\gamma} &= -X_{BC}{}^{D}\,\mathbb{T}_{[\alpha\gamma]\,D}{}^{C}\,\mathbb{T}_{\beta A}{}^{B}-X_{BC}{}^{D}\,\mathbb{T}_{\alpha\,D}{}^{B}\mathbb{T}_{\beta\gamma\,A}{}^{C}+X_{AB}{}^{C}\,\mathbb{T}_{\beta\alpha\gamma\,C}{}^{B}\;+\;\dots\\
\mathbb{X}_{\alpha\beta\gamma} &= \frac{3}{5}\,X_{AB}{}^{C}X_{DE}{}^{F}\;\times\;\left\{\delta^{BE}\mathbb{T}_{\alpha\,A}{}^{B}\mathbb{T}_{\beta\gamma\,F}{}^{C}+\delta^{AD}\mathbb{T}_{\alpha\,B}{}^{E}\mathbb{T}_{\beta\gamma\,F}{}^{C}+\delta_{CF}\mathbb{T}_{\beta\,A}{}^{D}\mathbb{T}_{\alpha\gamma\,B}{}^{E}\;+\;\dots\;\right\}
\end{aligned}$$

products of E₆ generators

embedding tensor

- reproduce and extend the known results on AdS₅ x S⁵
- valid for all vacua in the theory

# explicit cubic couplings

- universal formulas for cubic couplings $\mathscr{L}_{\phi^3} \;=\; g_{\alpha\Sigma,\beta\Delta,\gamma\Omega}\,\phi^{\alpha,\Sigma}\,\phi^{\beta,\Delta}\,\phi^{\gamma,\Omega}$

$$\mathcal{L}_{\phi^3} \;=\; \phi^{\alpha\Sigma}\phi^{\beta\Delta}\phi^{\gamma\Omega}\left\{c^{\Sigma\Delta\Omega}\,\mathbb{X}_{\alpha\beta\gamma} + c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Sigma}\,\mathbb{X}_{A\,\alpha\beta\gamma} + c^{\Delta\Omega\Lambda}\,\mathcal{T}_A{}^{\Lambda\Theta}\mathcal{T}_B{}^{\Theta\Sigma}\,\mathbb{X}_{AB\,\alpha\beta\gamma}\right\}$$

representation matrix on harmonics

symmetric SO(6) tensors

- example on AdS5 x S5:

$$\mathcal{G}(s^{I_1},s^{I_2},s^{I_3}) = \left(\frac{\sigma}{2}+2\right)\left(\frac{\sigma}{2}+1\right)\,a(n_1,n_2,n_3)\;\mathcal{C}^{I_1I_2I_3}\;s^{I_1}s^{I_2}s^{I_3}$$

$n_i = |\Sigma_i| = |I_i| - 2$ $\qquad$ $\sigma = n_1 + n_2 + n_3$ $\qquad$ $\int_{S^5} \mathcal{Y}^{I_1}\mathcal{Y}^{I_2}\mathcal{Y}^{I_3} \;=\; a(n_1,n_2,n_3)\,\mathcal{C}^{I_1I_2I_3}$

translate to compare to previous results: [Lee, Minwalla, Rangamani, Seiberg] [Arutyunov, Frolov]

$$\mathcal{G}(s^{I_1},s^{I_2},s^{I_3}) = \frac{\tilde{\sigma}\,\alpha_1\,\alpha_2\,\alpha_3}{16\,(k_1+1)(k_2+1)(k_3+1)}\left(\frac{\tilde{\sigma}^2}{4}-1\right)\left(\frac{\tilde{\sigma}^2}{4}-4\right)a(k_1,k_2,k_3)\;\mathcal{C}^{I_1I_2I_3}\;\tilde{s}^{I_1}\tilde{s}^{I_2}\tilde{s}^{I_3}$$

$k_i = |I_i|$ $\qquad$ $\alpha_i = \frac{1}{2}\tilde{\sigma} - k_i$ $\qquad$ $\tilde{\sigma} = k_1 + k_2 + k_3$

with ‘unexpected’ zeros for the extremal case $\alpha_i = 0$

# conclusions

- new tools from ExFT for the analysis of Kaluza-Klein spectra and couplings
  - 'standard' two-derivative action $\boxed{\mathcal{L} \equiv \hat{R} + \frac{1}{24} g^{\mu\nu} \mathcal{D}_\mu \mathcal{M}^{MN} \mathcal{D}_\nu \mathcal{M}_{MN} - \frac{1}{4} \mathcal{M}_{MN} \mathcal{F}^{\mu\nu M} \mathcal{F}^N_{\mu\nu} + e^{-1} \mathcal{L}_{\text{top}} - V(\mathcal{M}, e).}$
  - & basis of fluctuations $\mathcal{M}_{MN} = U_M{}^K(Y)\, U_N{}^L(Y) \Big( \delta_{KL} + \sum_\Sigma \boxed{\phi^{\alpha,\Sigma}}\, \mathbb{T}_{\alpha,KL}\, \mathcal{Y}^\Sigma \Big)$

- Kaluza-Klein spectra entirely encoded in 5-dim data:
  - embedding tensor $X_{MN}{}^K$ of the lower-dimensional supergravity
  - representation $(\mathcal{T}_M)_\Sigma{}^\Lambda$ of the scalar harmonics

- access to vacua
  - with few or no (super-)symmetries
  - within and beyond consistent truncations

- extension to cubic and higher couplings

—> universal patterns in mass spectra & n-point couplings: holography!