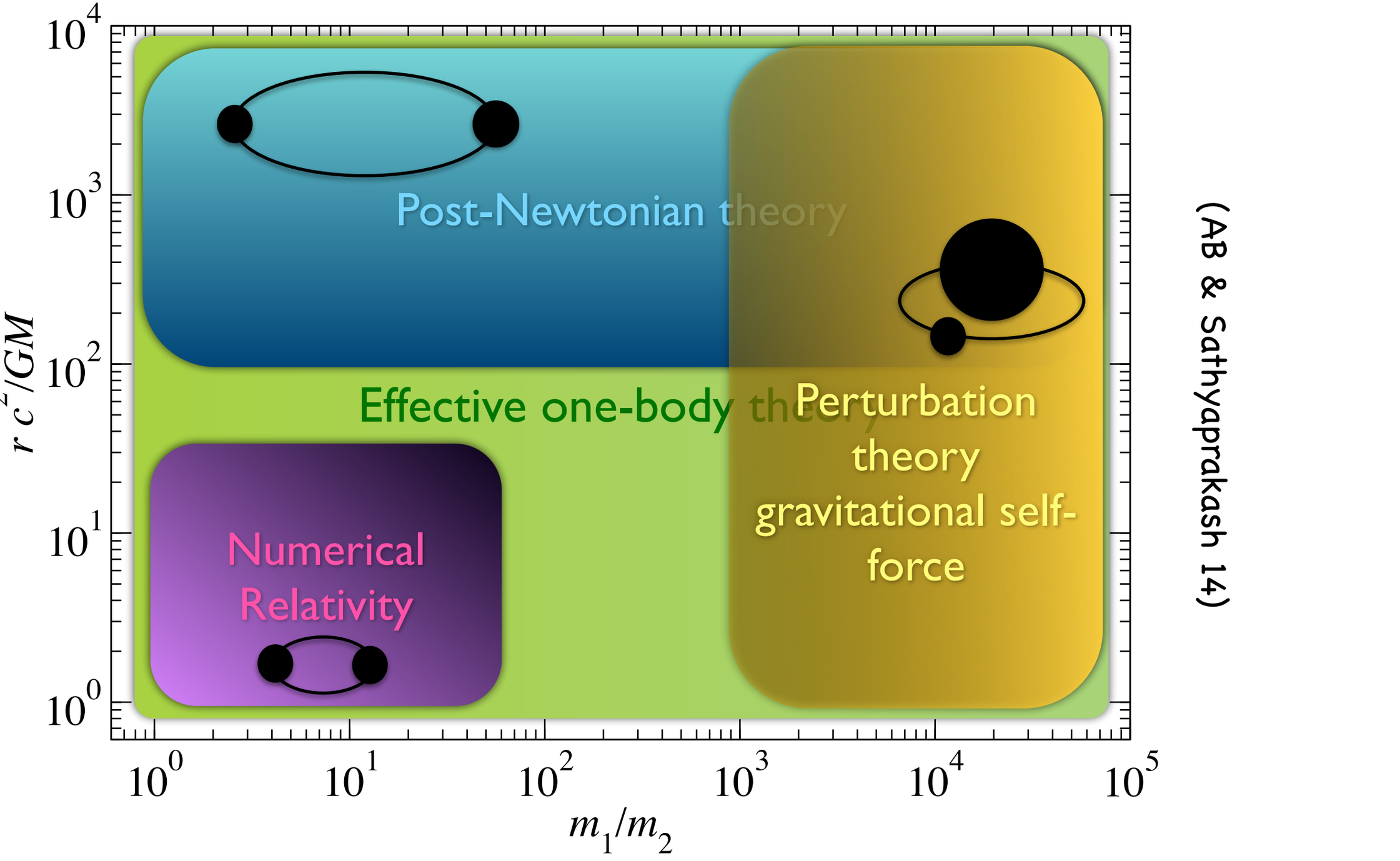
Methods to solve 2-body problem in General Relativity



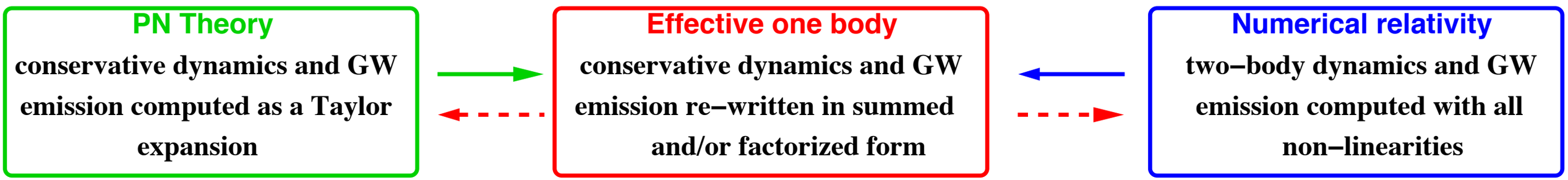
- Two parameters determine the **range of validity** of each method:

$$\frac{G\,M}{r\,c^2} \sim \frac{v^2}{c^2} \quad \& \quad \frac{m_2}{m_1}$$

The effective-one-body (EOB) approach

- **EOB** approach **introduced before NR** breakthrough

AB, Pan, Taracchini, Bohe', Shao, Barausse, Hinderer, Steinhoff; Damour, Nagar, Bernuzzi, Bini, Balmelli; Iyer, Sathyaprakash; Jaranowski, Schaefer;



- **EOB** model uses best information available in PN theory, but **resums PN terms** in suitable way to describe accurately dynamics and radiation during inspiral and plunge.

- **EOB** assumes **comparable-mass** description is **smooth deformation of test-particle limit**. It employs non-perturbative ingredients and **models analytically merger-ringdown** signal.

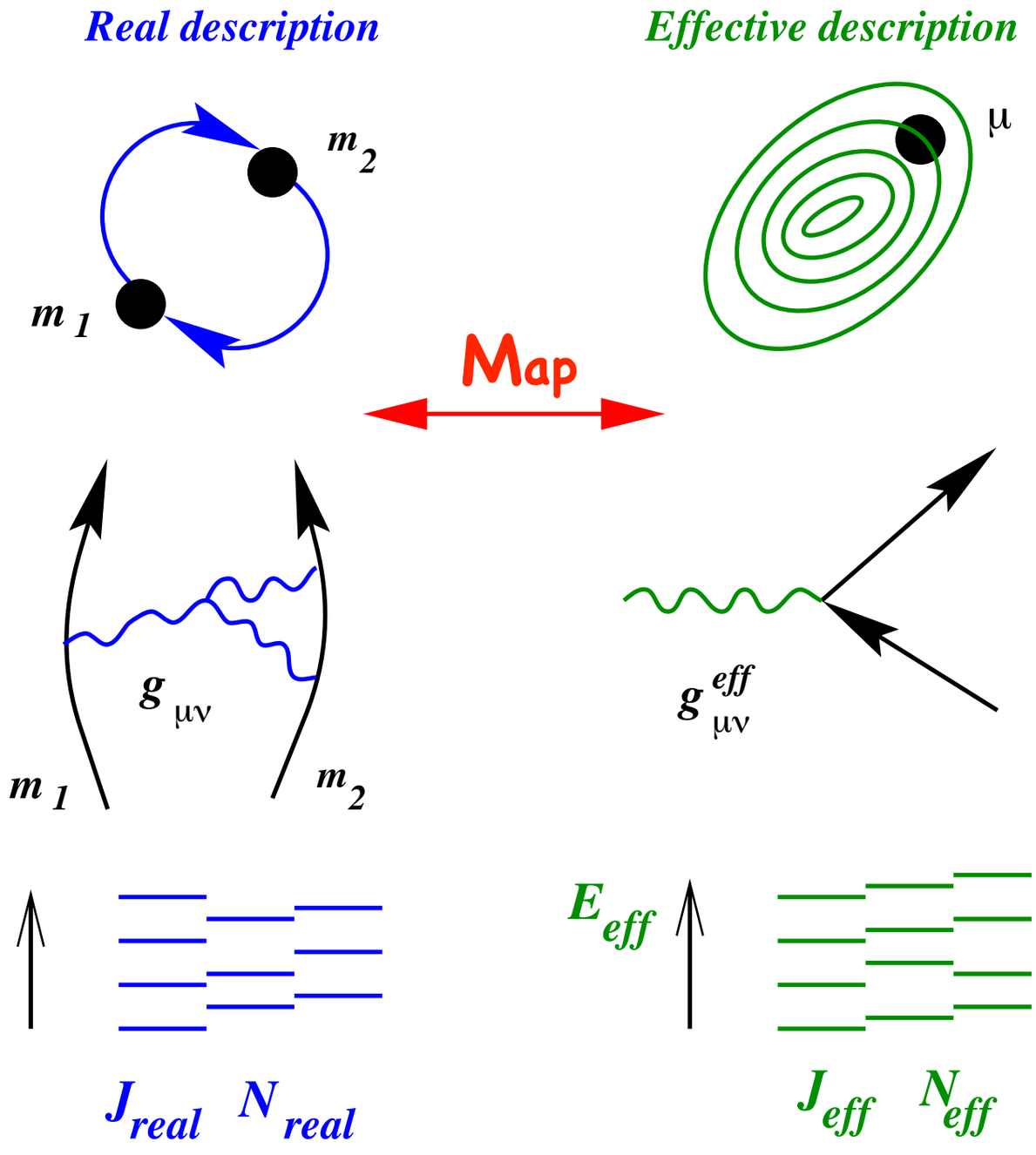
The effective-one-body approach in a nutshell

$$\nu = \frac{\mu}{M} \qquad 0 \leq \nu \leq 1/4$$

$$\mu = \frac{m_1 m_2}{M} \qquad M = m_1 + m_2$$

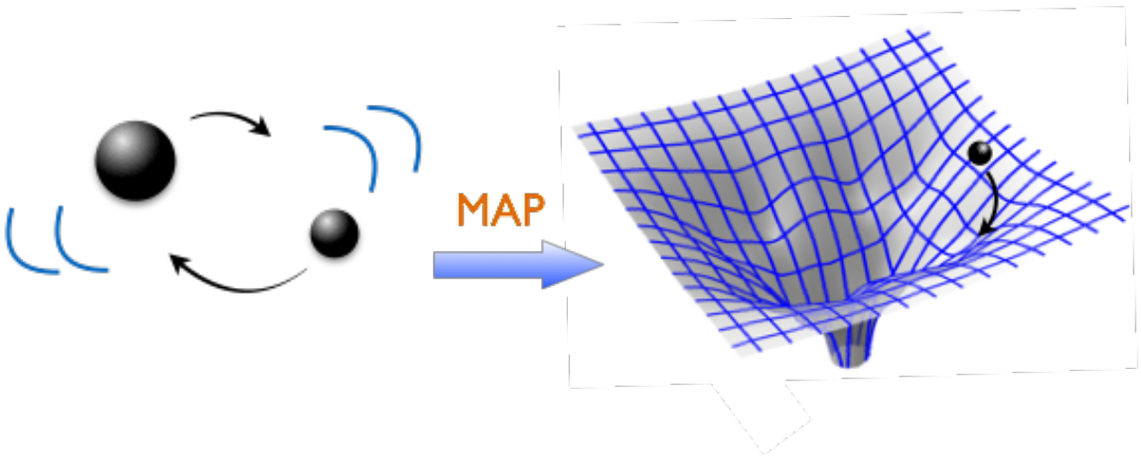
- **Two-body dynamics is mapped** into dynamics of **one-effective body** moving **in deformed black-hole spacetime**, deformation being the mass ratio.

- Some key **ideas** of EOB model were **inspired by quantum field theory** when describing energy of comparable-mass charged bodies.

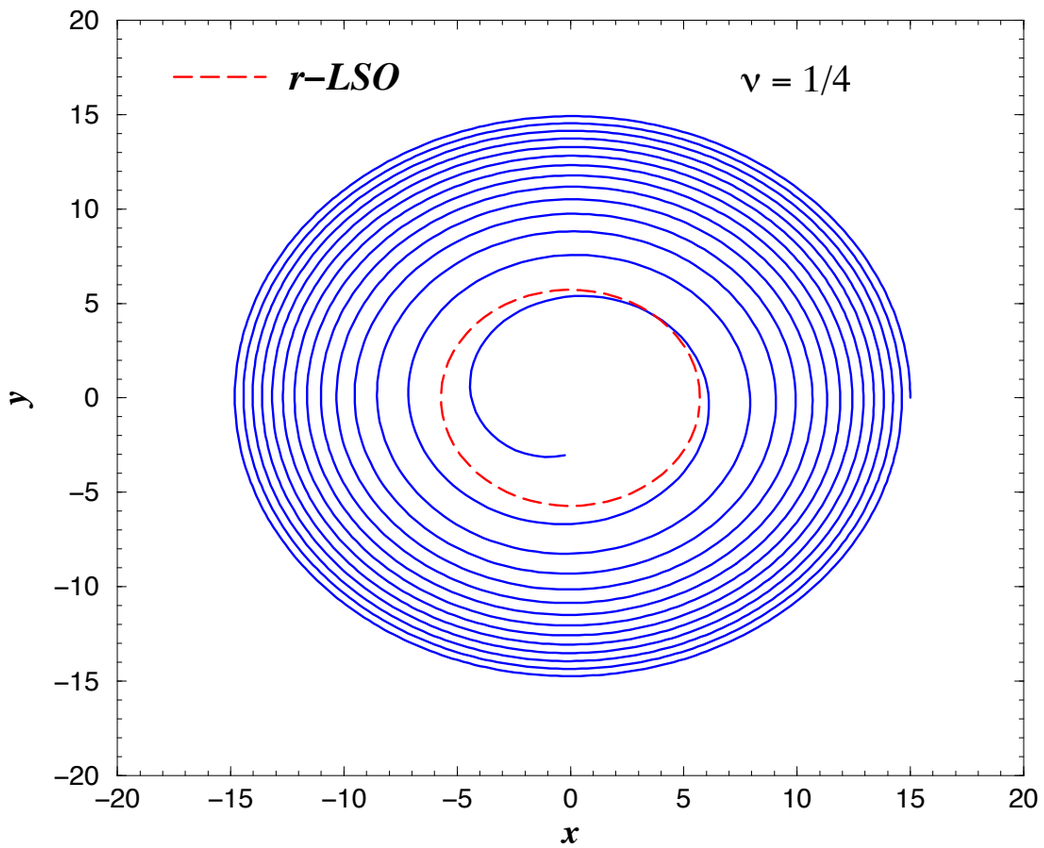


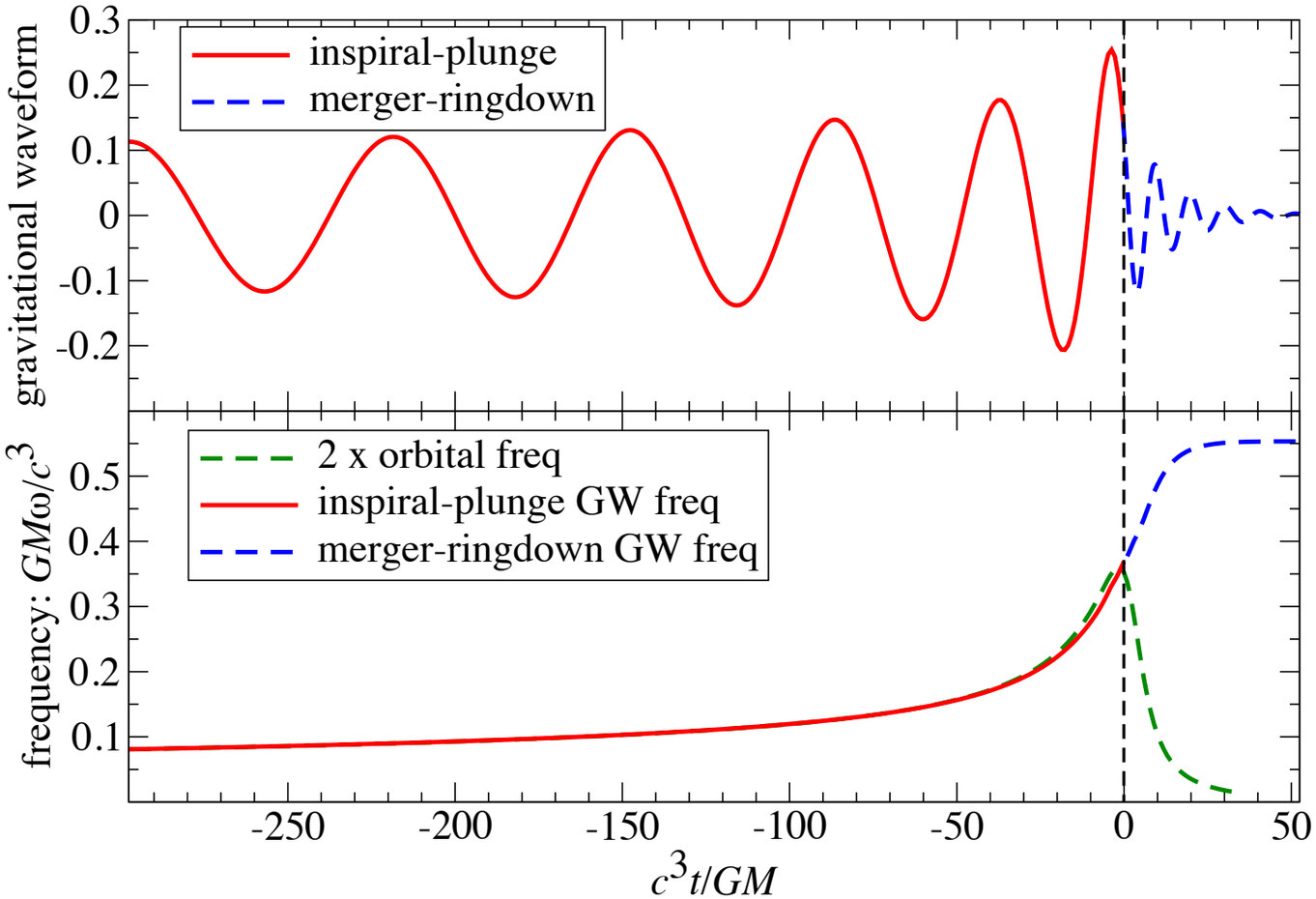
(AB & Damour PRD59 (1999) 084006)

EOB inspiral-merger-ringdown analytic waveform



$$E^2_{\rm real} = m_1^2 + m_2^2 + 2m_1 m_2 \left(\frac{E_{\rm eff}}{\mu} \right)$$





(AB & Damour, PRD59 (1999) 084006, PRD62 (2000) 064015)

EOB Hamiltonian: resummed PN conservative dynamics

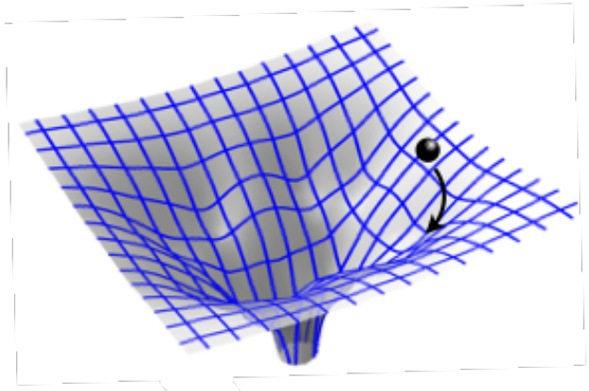
- Real Hamiltonian → Effective Hamiltonian

$$H^{\rm PN}_{\rm real} = H_{\rm Newt} + H_{\rm 1PN} + H_{\rm 2PN} + \cdots$$

$$H^{\nu}_{\rm eff} = \mu \sqrt{A_\nu(r)\left[1 + \frac{\mathbf{p}^2}{\mu^2} + \left(\frac{1}{B_\nu(r)} - 1\right)\frac{p_r^2}{\mu^2}\right]}$$

$$ds^2_{\rm eff} = -A_\nu(r)dt^2 + B_\nu(r)dr^2 + r^2 d\Omega^2$$

- EOB Hamiltonian: $H^{\rm EOB}_{\rm real} = M\sqrt{1 + 2\nu\left(\frac{{H_{\rm eff}}^{\nu}}{\mu} - 1\right)}$



(credit: Hinderer)

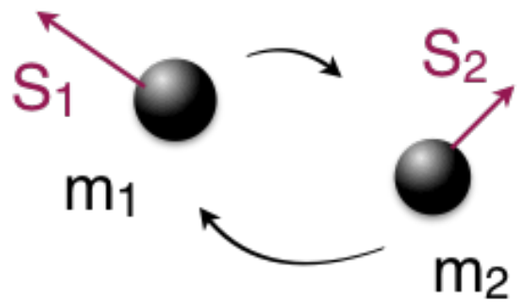
- Dynamics condensed in $A_\nu(r)$ and $B_\nu(r)$

- $A_\nu(r)$, which encodes the energetics of circular orbits, is quite simple:

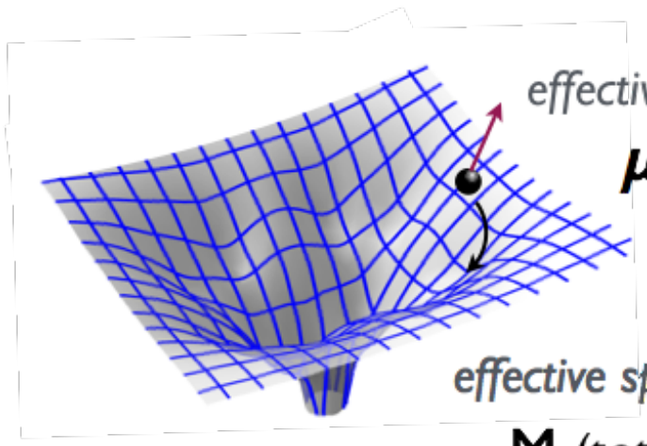
$$A_\nu(r) = 1 - \frac{2M}{r} + \frac{2M^3\nu}{r^3} + \left(\frac{94}{3} - \frac{41}{32}\pi^2\right)\frac{M^4\nu}{r^4} + \frac{a_5(\nu) + a_5^{\log}(\nu)\log(r)}{r^5} + \frac{a_6(\nu)}{r^6} + \cdots$$

EOB resummed spin dynamics & waveforms

(credit: Hinderer)



MAP



$$H^{\rm EOB}_{\rm real} = M\sqrt{1+2\nu\left(\frac{{H_{\rm eff}}^{\nu}}{\mu}-1\right)}$$

- EOB equations of motion:

$$\dot{\mathbf{r}} = \frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{p}} \qquad F \propto \frac{dE}{dt}, \quad \frac{dE}{dt} \propto \sum_{\ell m} |h_{\ell m}|^2$$

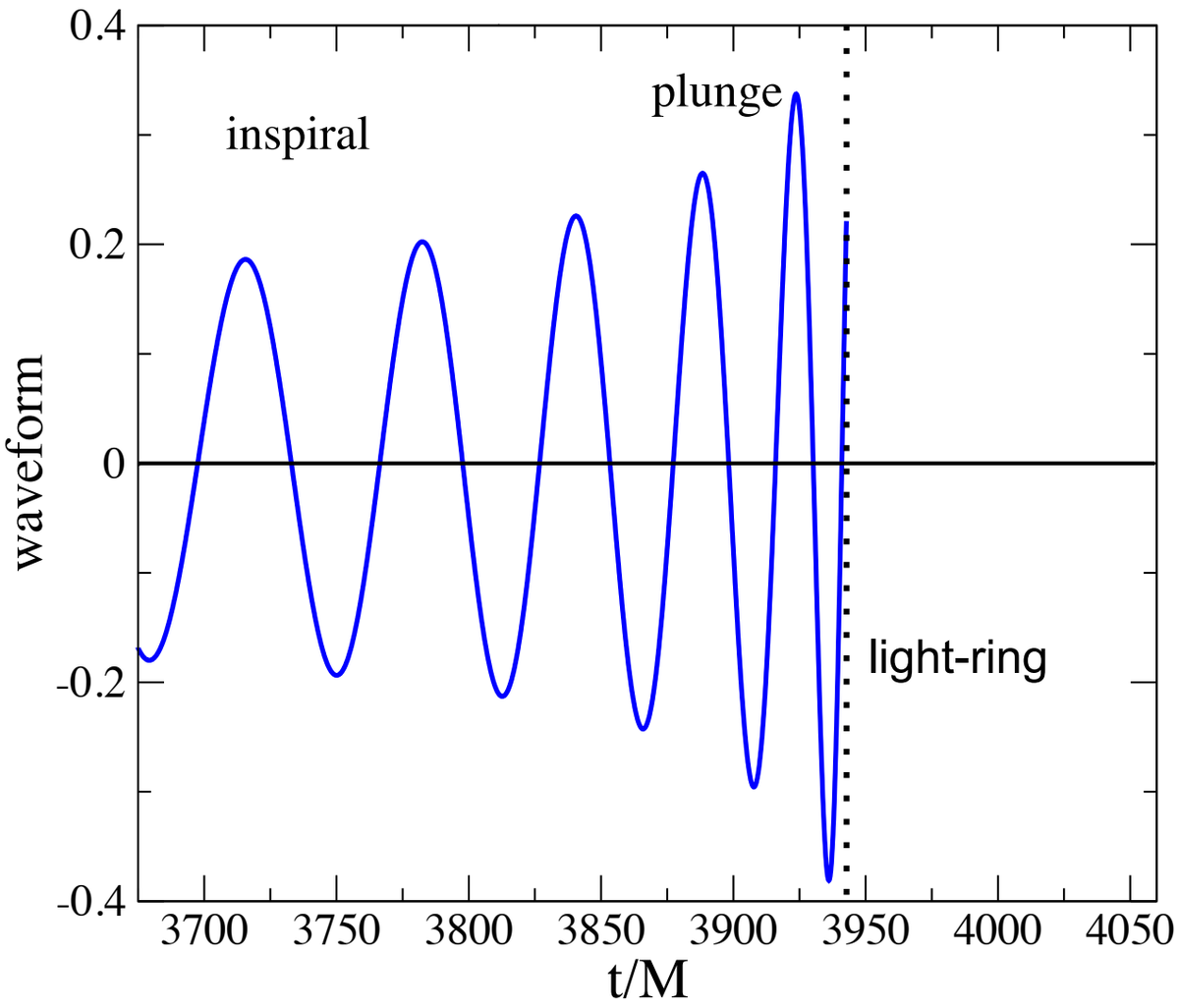
$$\dot{\mathbf{p}} = -\frac{\partial H^{\rm EOB}_{\rm real}}{\partial \mathbf{r}} + \mathbf{F} \qquad \dot{\mathbf{S}} = \{\mathbf{S}, H^{\rm EOB}_{\rm real}\}$$

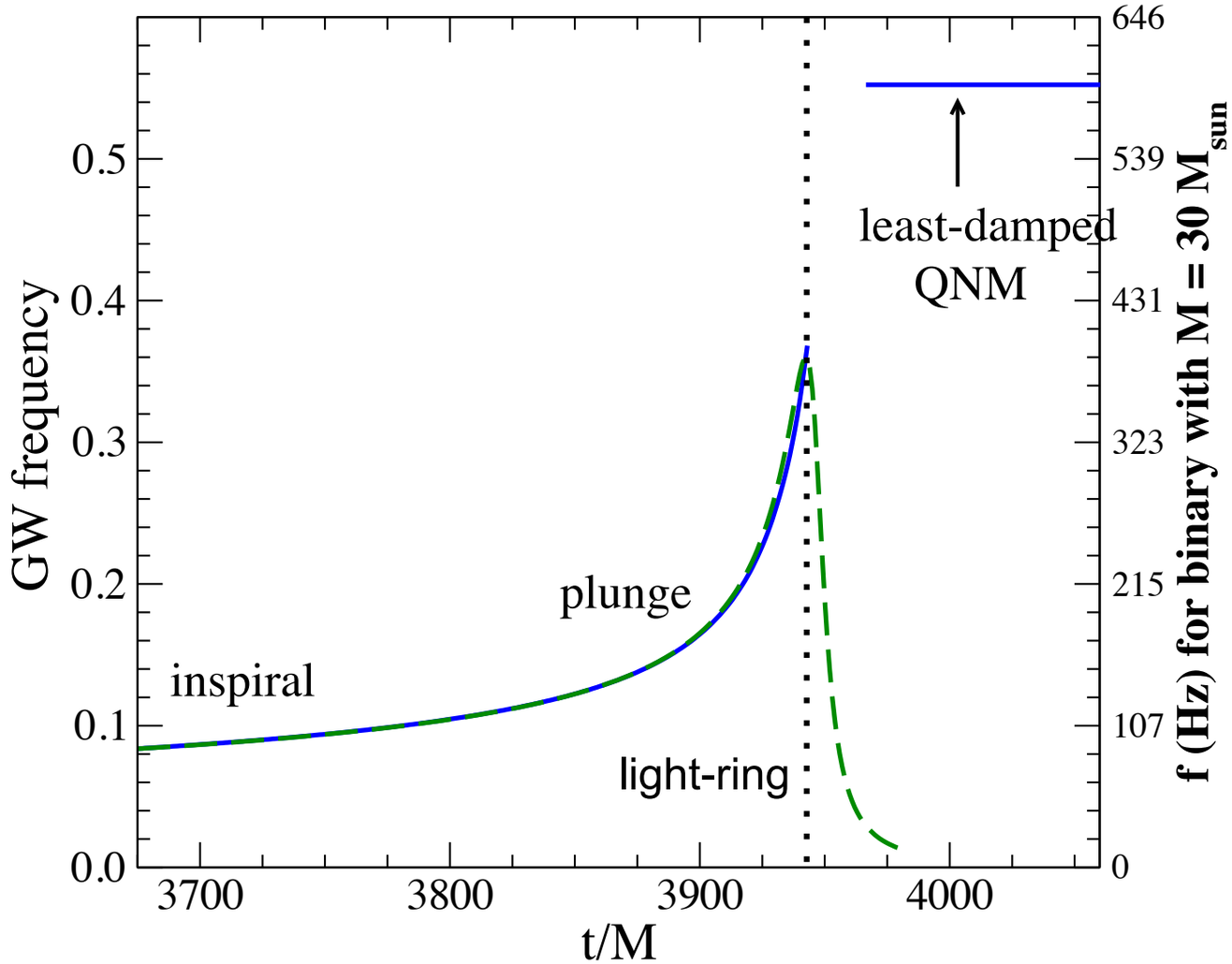
- EOB waveforms:

$$h^{\rm insp-plunge}_{\ell m} = h^{\rm Newt}_{\ell m}\, e^{-im\Phi}\, S_{\rm eff}\, T_{\ell m}\, e^{i\delta_{\ell m}}\, (\rho_{\ell m})^{\ell}\, h^{\rm NQC}_{\ell m}$$

EOB inspiral-plunge waveform & frequency

Evolve **two-body dynamics up to light ring** (or photon orbit) and then …



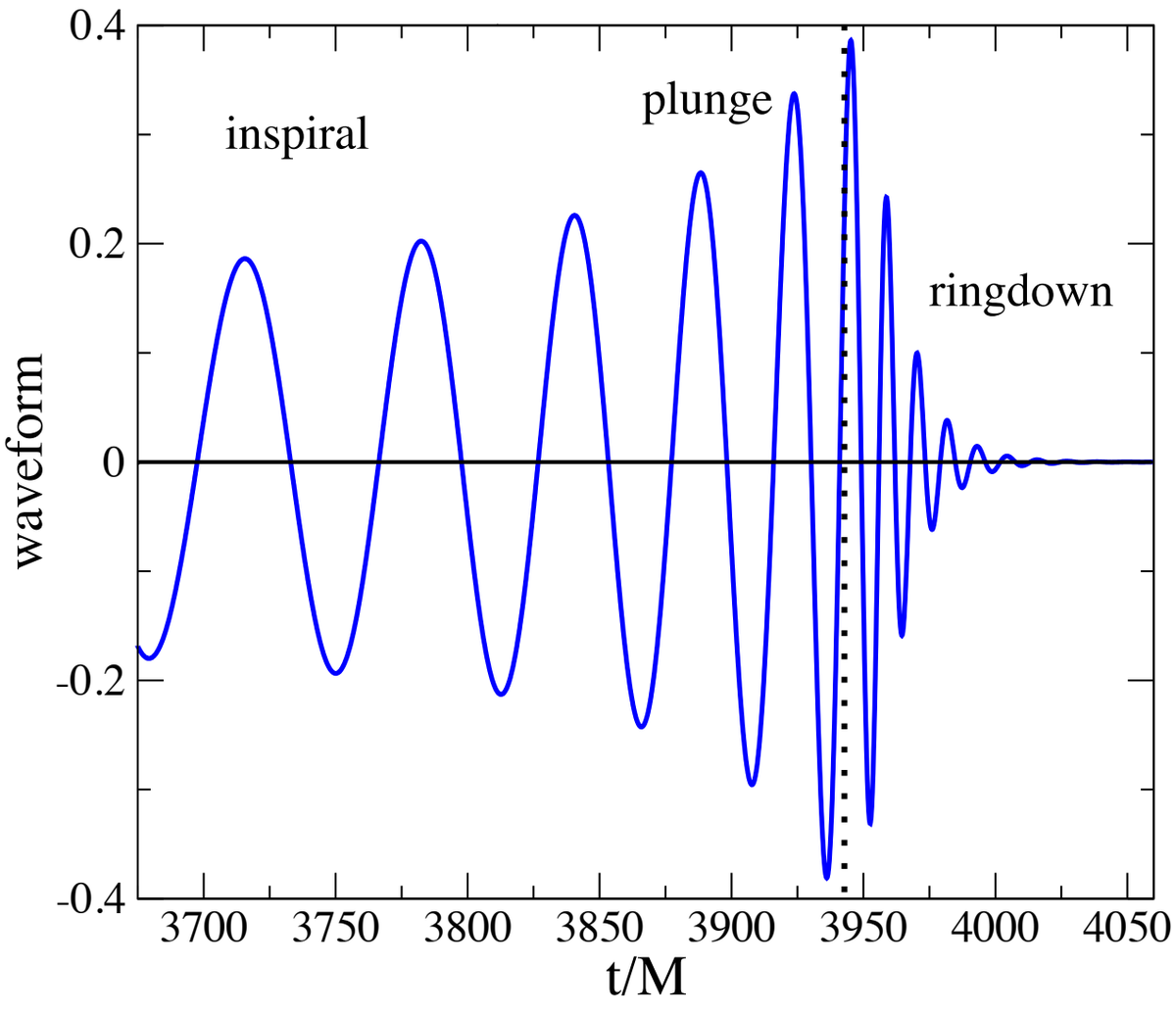


Quasi-normal modes excited at **light-ring crossing**

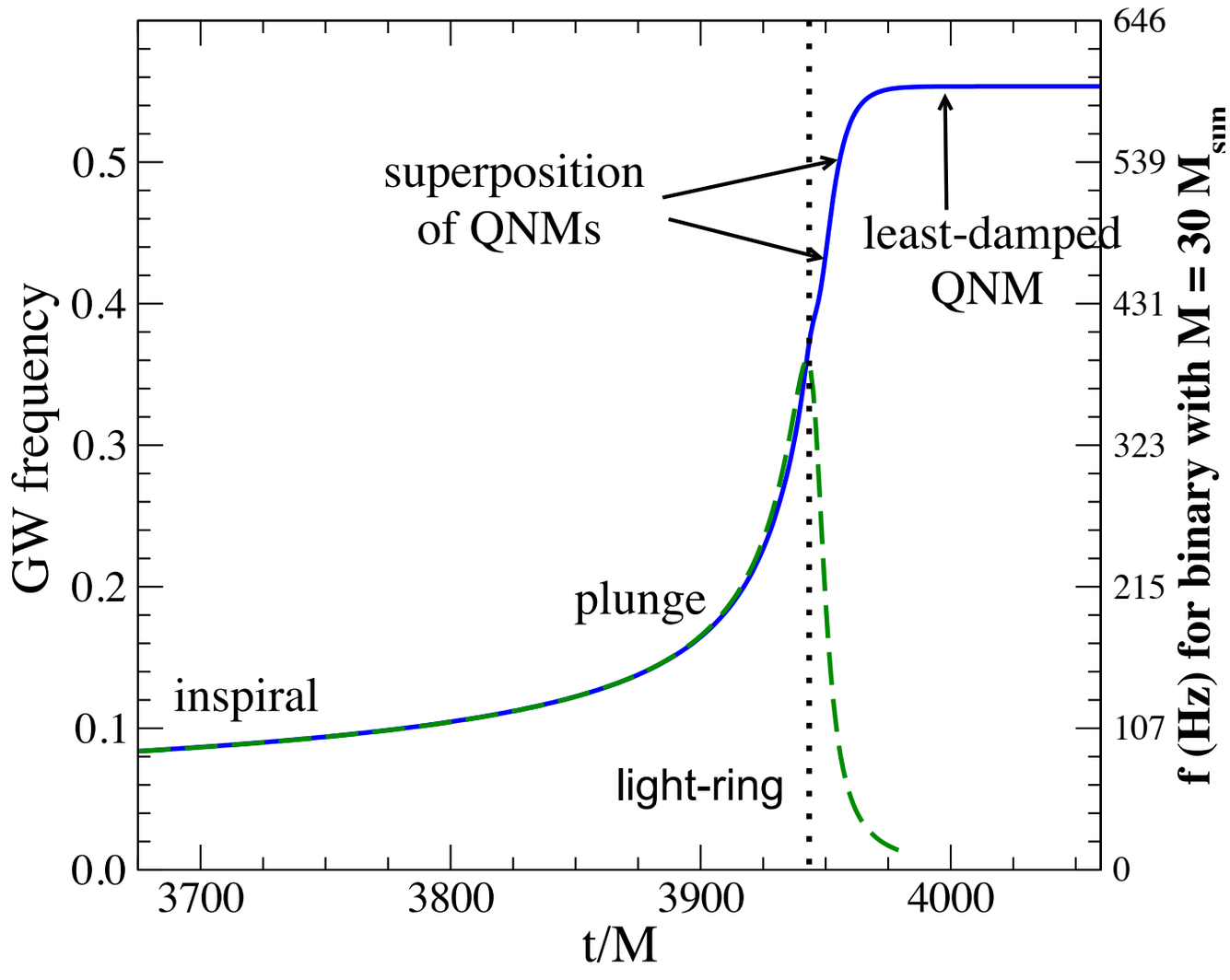
(Goebel 1972, Davis et al. 1972, Ferrari et al. 1984, Damour et al. 07, Barausse et al. 11, Price et al. 15)

EOB inspiral-merger-ringdown waveform & frequency

... attach **superposition** of **quasi-normal modes** of **remnant** black hole.



- Very **short/simple transition** plunge-merger-ringdown: "easy" to model!



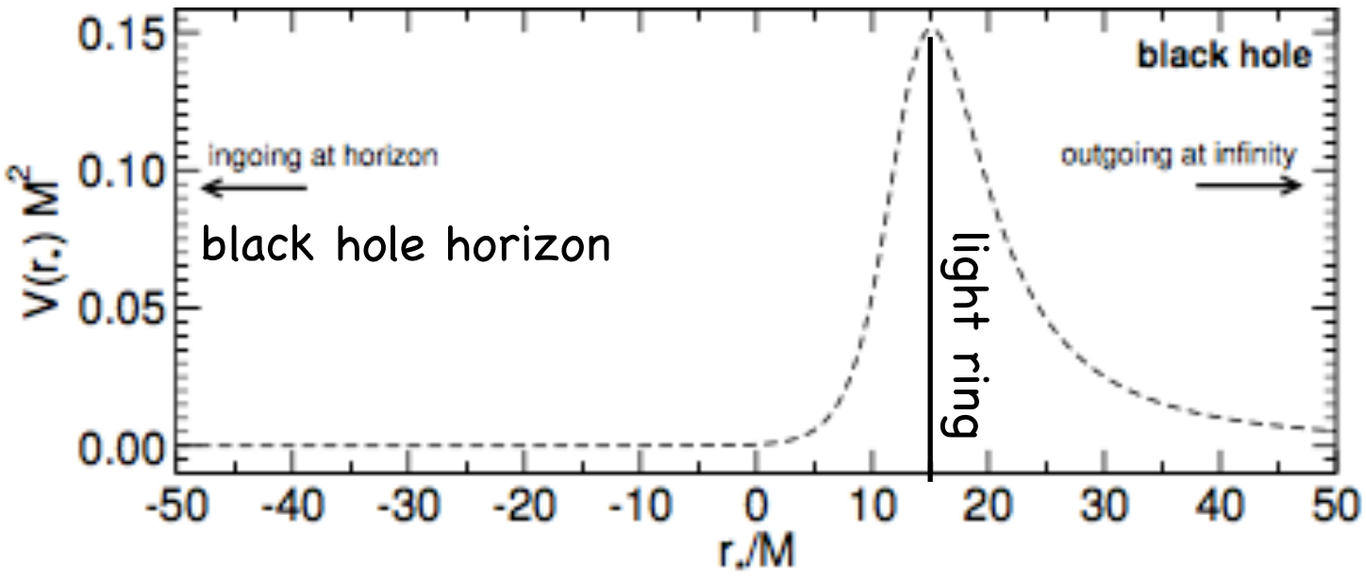
- Energy **quickly released** during merger: 2%-12%M

On the simplicity of merger signal

equation of gravitational perturbations in black-hole spacetime

(Regge & Wheeler 56, Zerilli 70, Teukolsky 72)

$$\frac{\partial^2\Psi}{\partial t^2} - \frac{\partial^2\Psi}{\partial r_\star^2} + V_{\ell m}\Psi = \mathcal{S}_{\ell m}$$



- **Peak** of black-hole potential **close to "light ring"**.

- Once particle is inside potential, **direct gravitational radiation** from its motion is **strongly filtered** by potential barrier (**high-pass filter**).

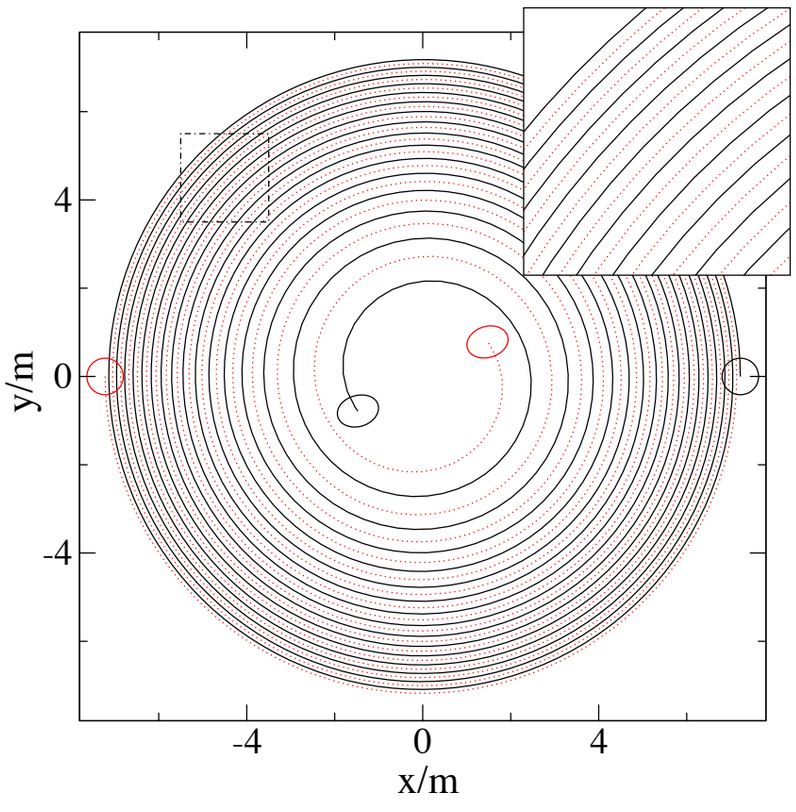
- Only **black-hole spacetime vibrations** (quasi-normal modes) **leaks out** black-hole potential.

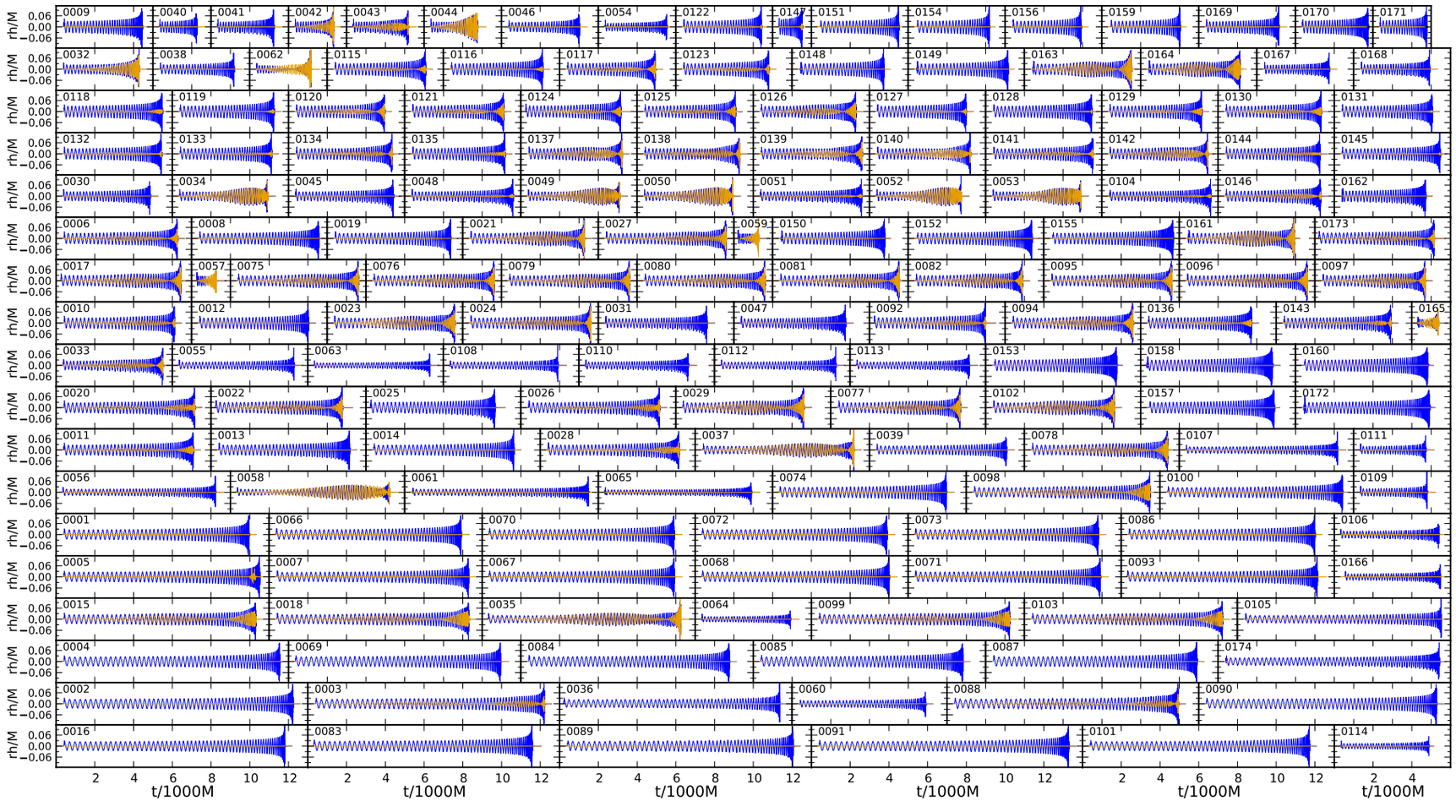
Numerical Relativity

- **Breakthrough** in 2005 (Pretorius 05, Campanelli et al. 06, Baker et al. 06)

Kidder, Pfeiffer, Scheel, Lindblom, Szilagyi; Bruegmann, Hannam, Husa, Tichy; Laguna, Shoemaker; ...

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{8\pi G}{c^4} T_{\mu\nu}$$

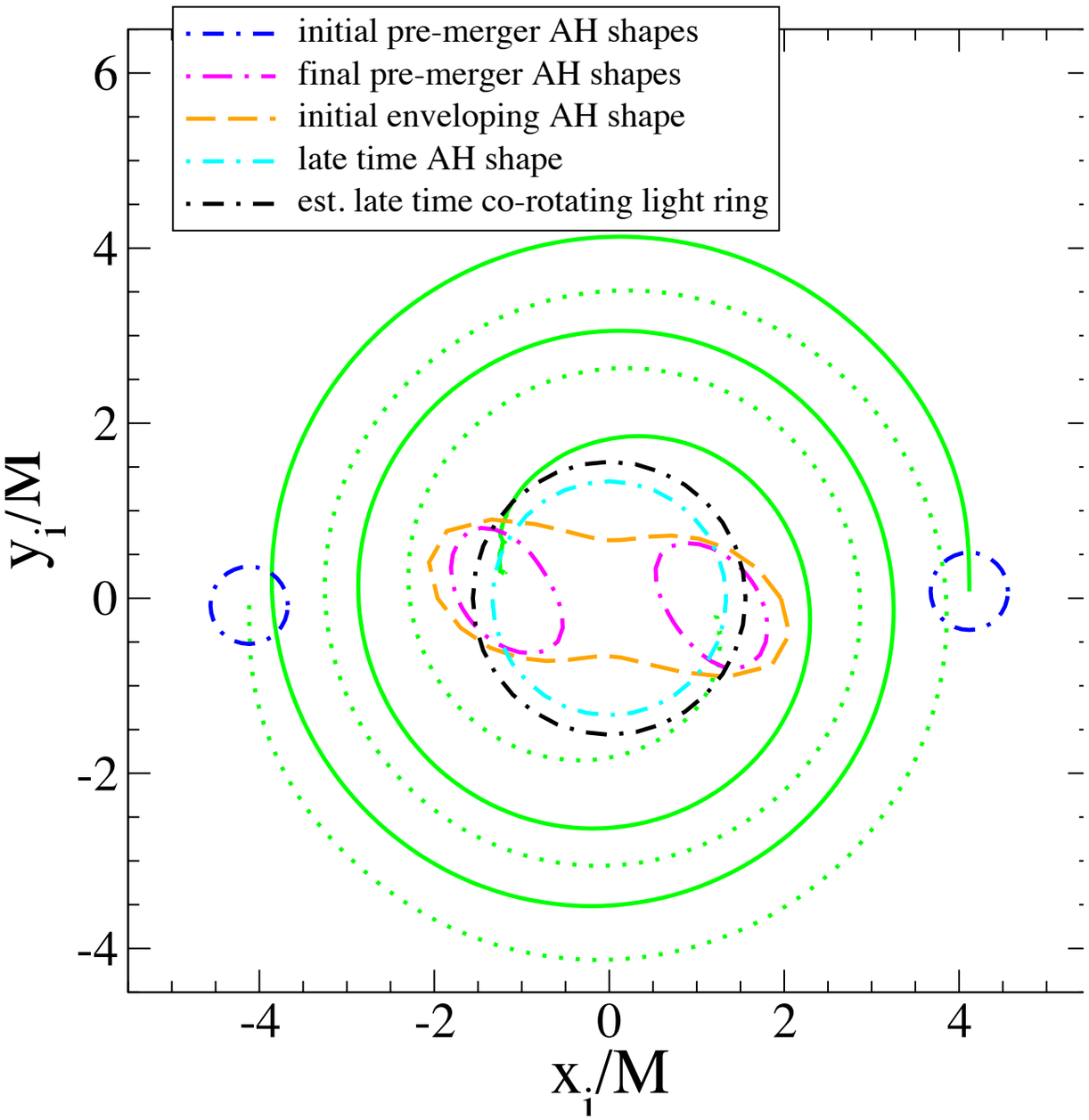


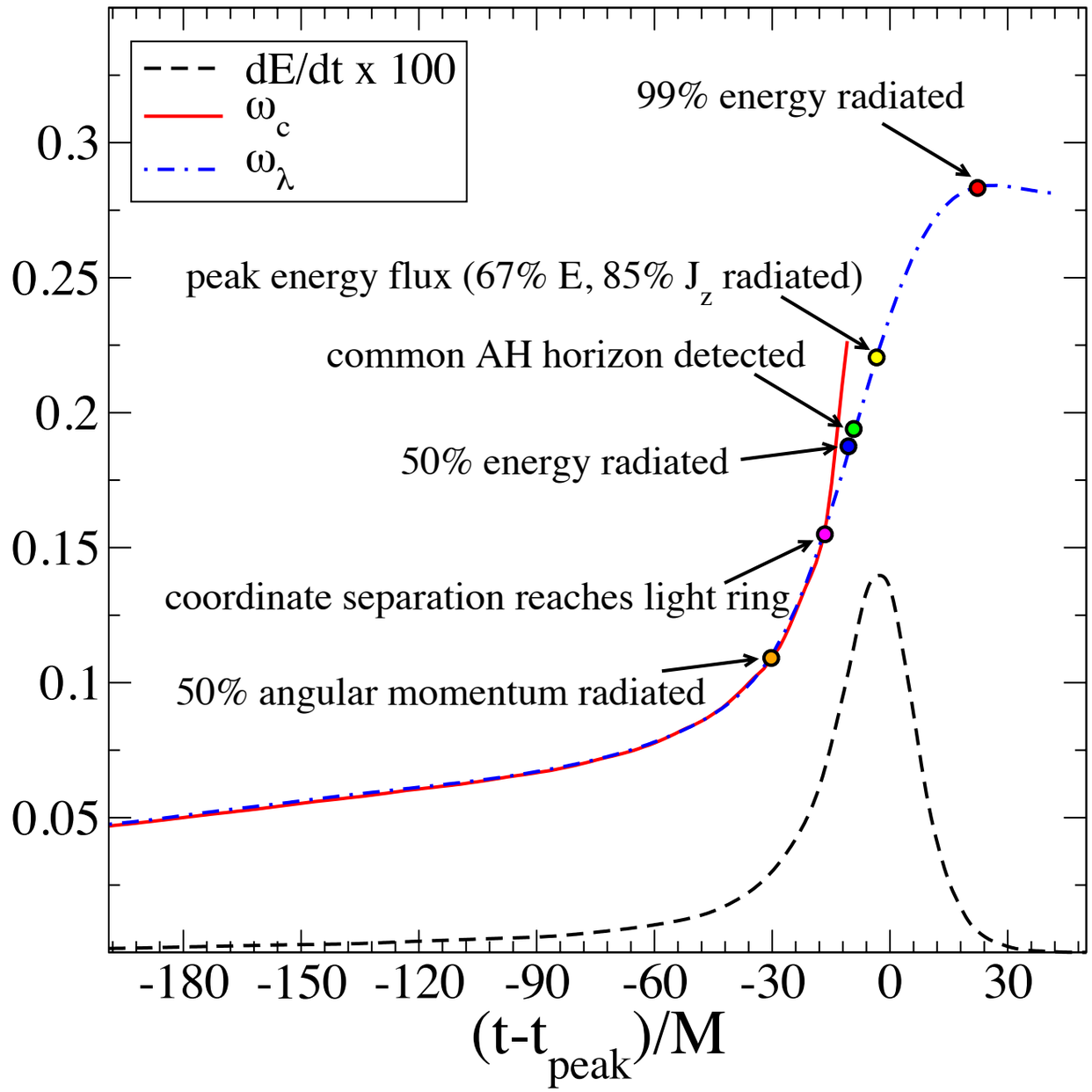


- **Simulating eXtreme Spacetime (SXS)** collaboration (Mroue et al. 13)

- **Numerical-Relativity & Analytical Relativity** collaboration (Hinder et al. 13)

The (plunge and) merger in first NR simulations

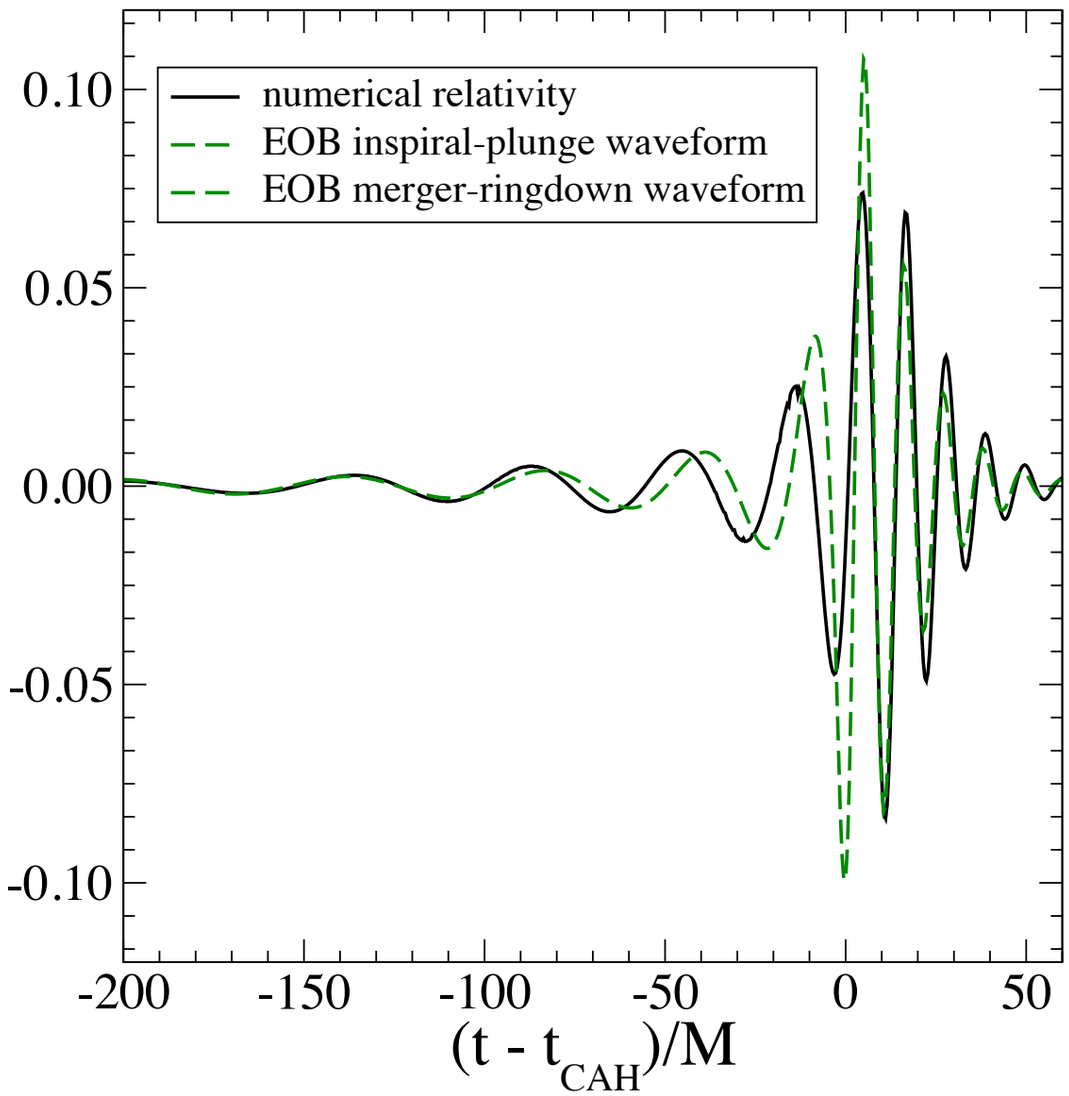




(AB, Cook & Pretorius 07)

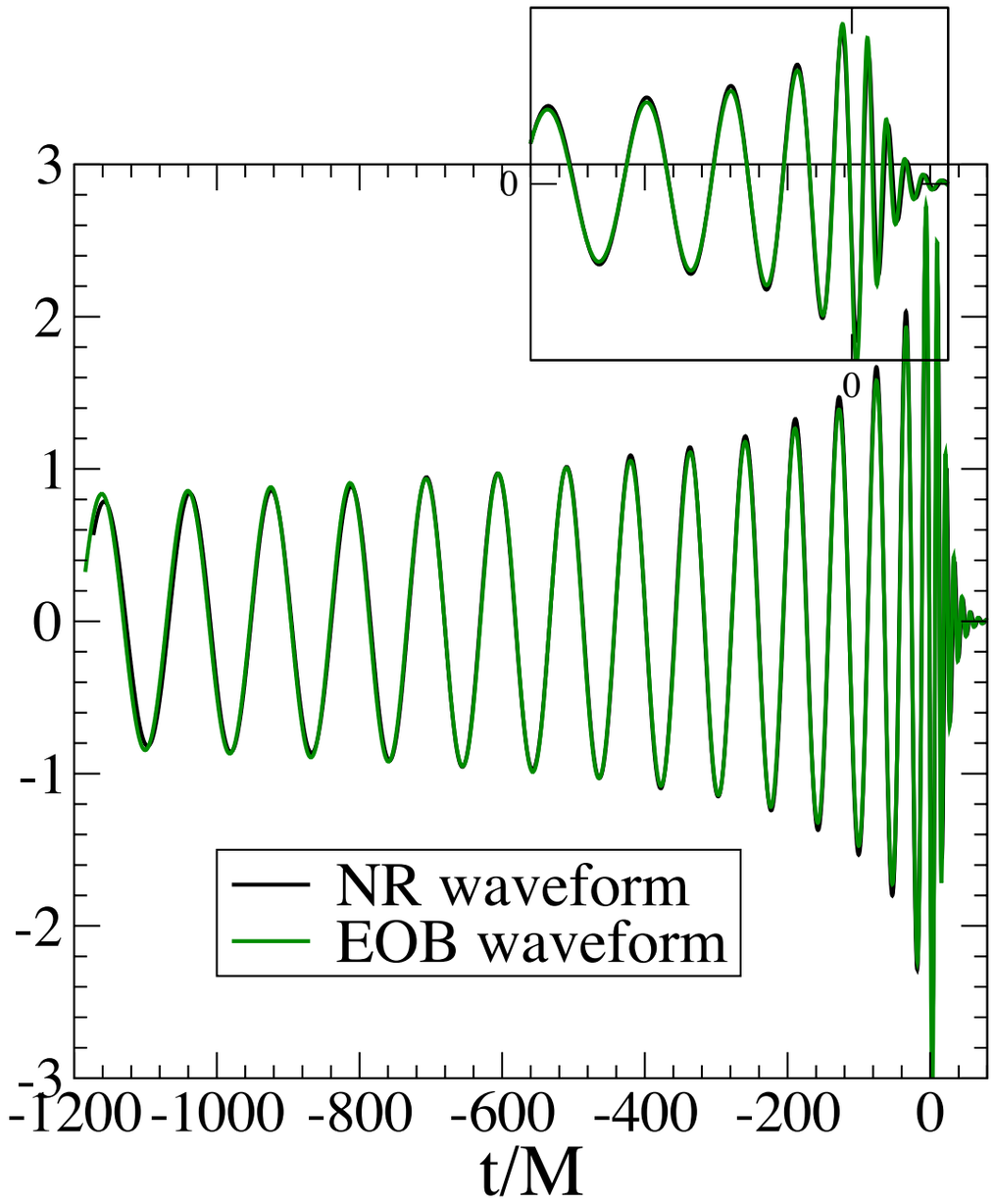
First comparisons/calibrations between NR and EOB model

- Uncalibrated EOB waveform at 3.5PN order



(AB, Cook & Pretorius 07)

- Calibrated EOBNR waveform



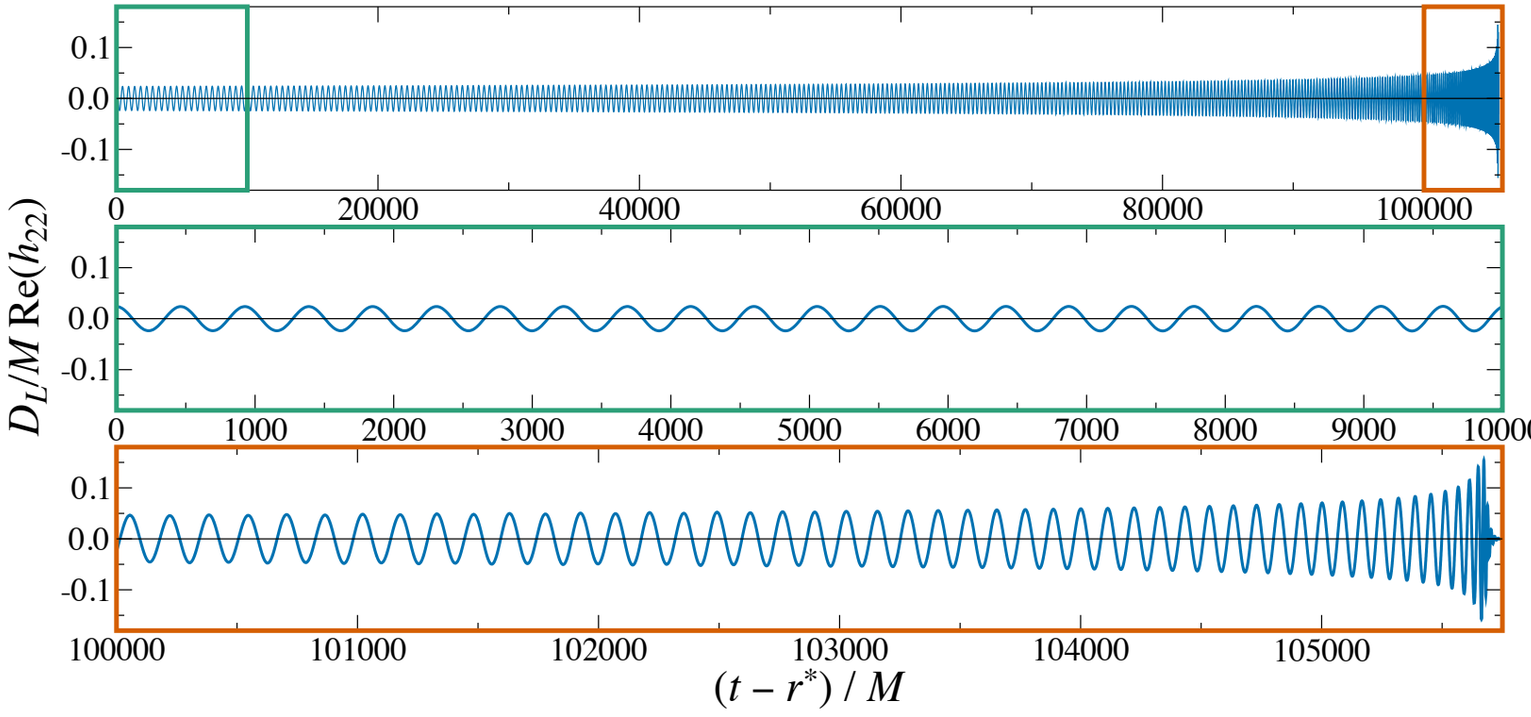
(AB, Pan, Baker, Centrella, Kelly at al. 08)

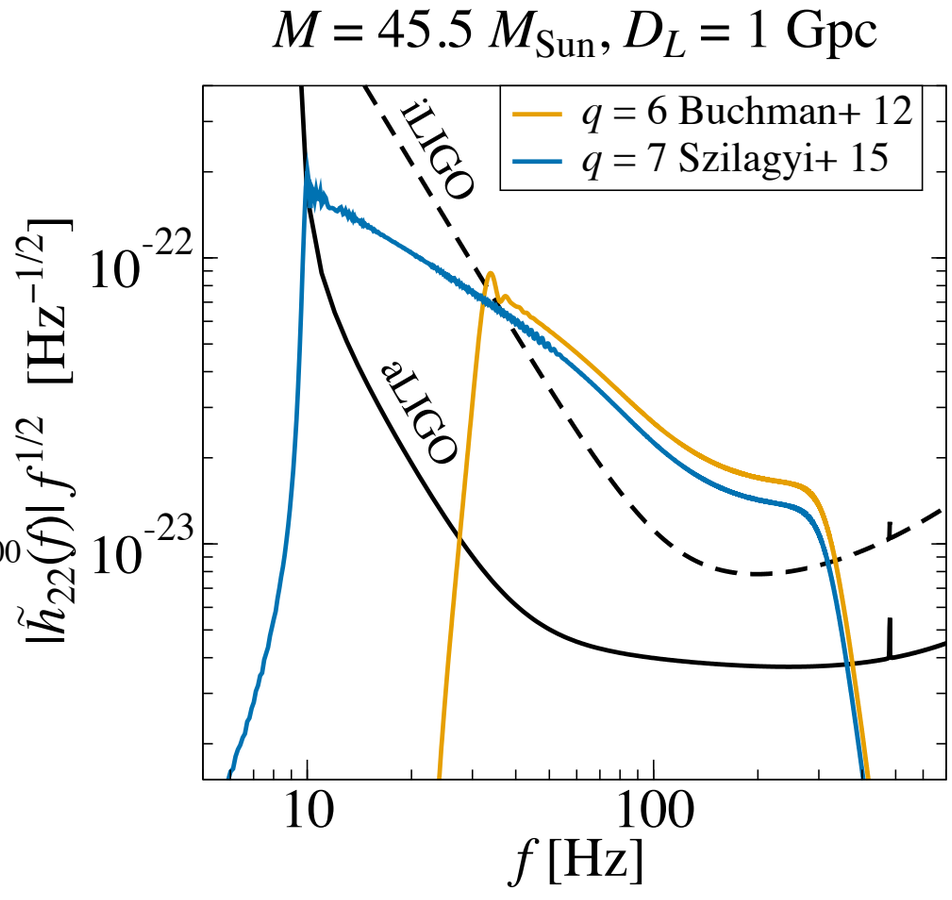
- EOBNR waveforms used in first LIGO **searches**

Numerical relativity reaching into post-Newtonian territory

(Szilagyi, Blackman, AB, Taracchini et al. 15)

- **376 GW cycles**, zero spin & mass ratio 7
  (8 months, a few millions CPU hours!)



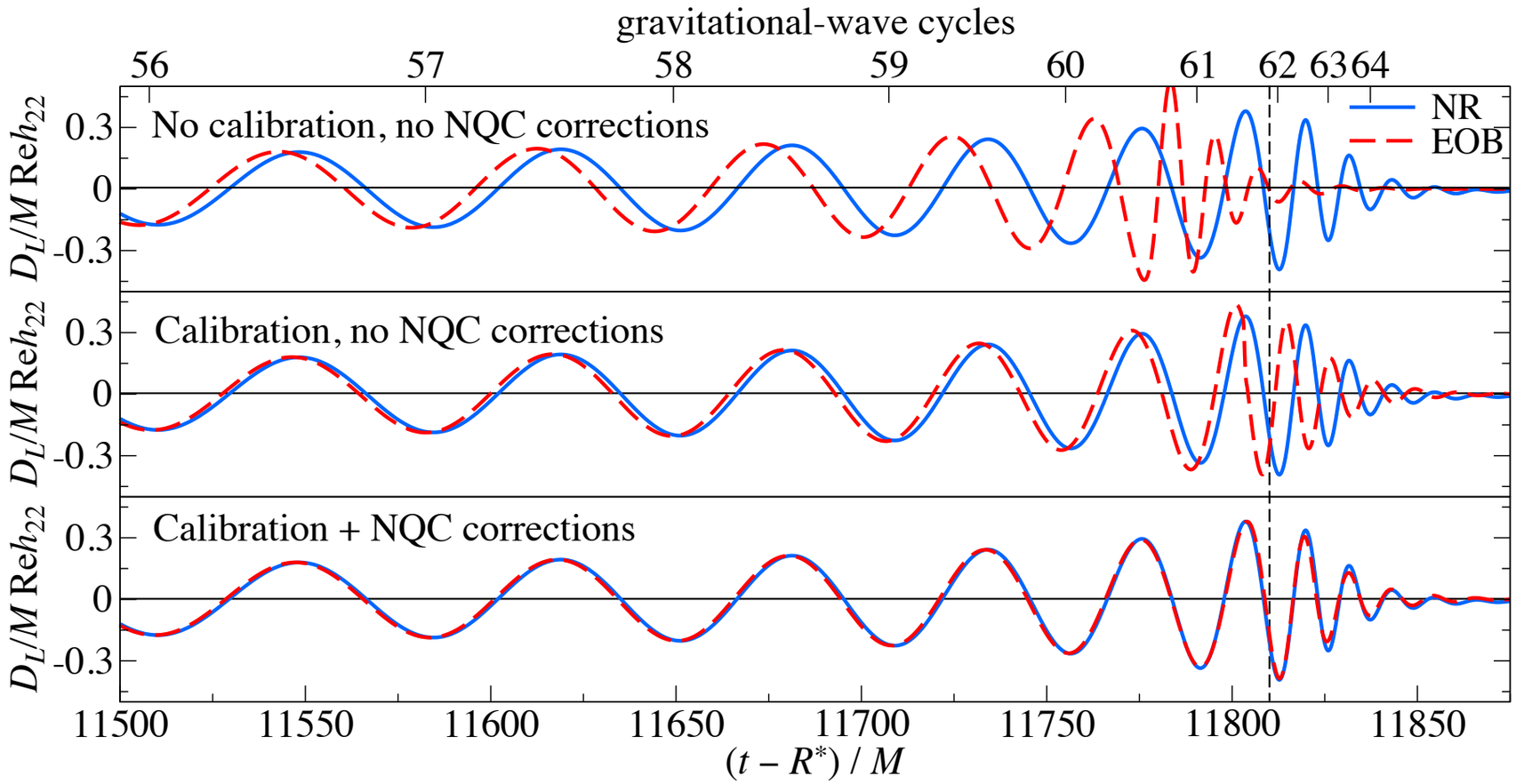


- Except for this case, **current waveforms** in NR catalogues have **30-80 GWs**, thus they **cover only** a **portion of** the detector **bandwidth**.

Completing EOB waveforms using NR/perturbation theory information

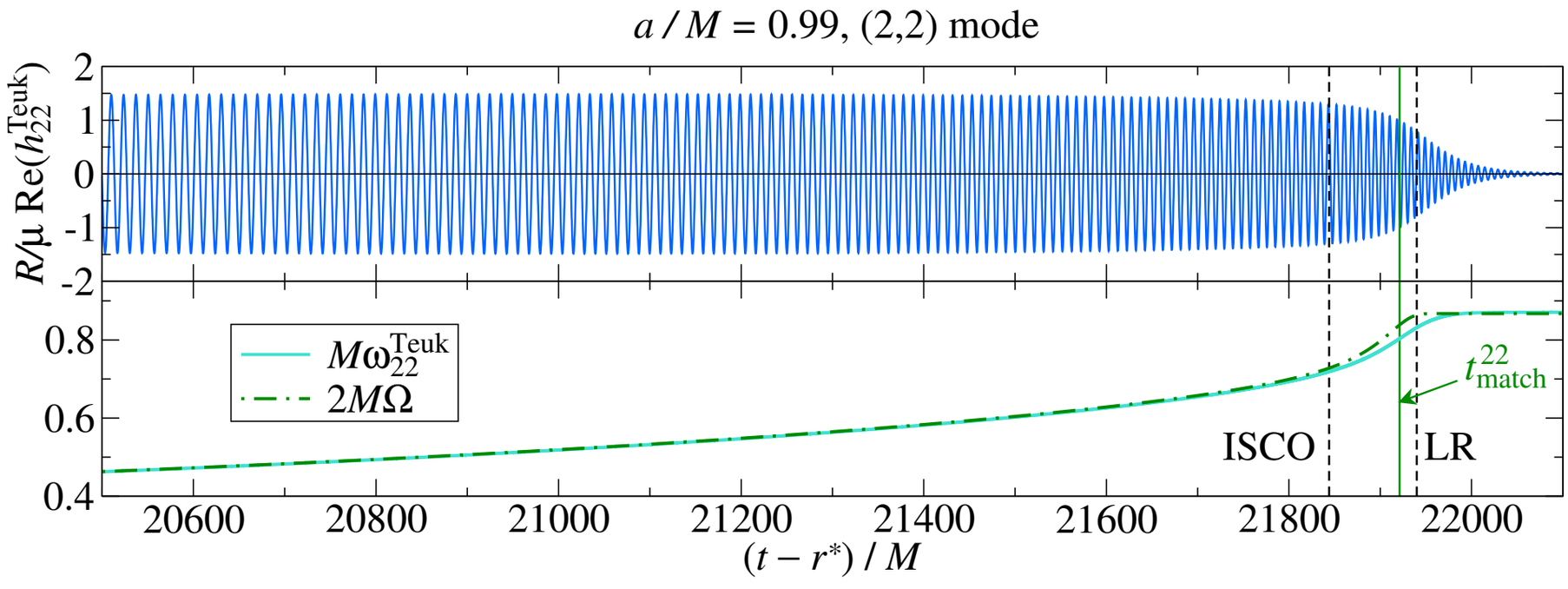
- We calibrate to inspiral-merger-ringdown **NR** waveforms.

- We incorporate **gravitational self-force** results.



(credit: Taracchini)

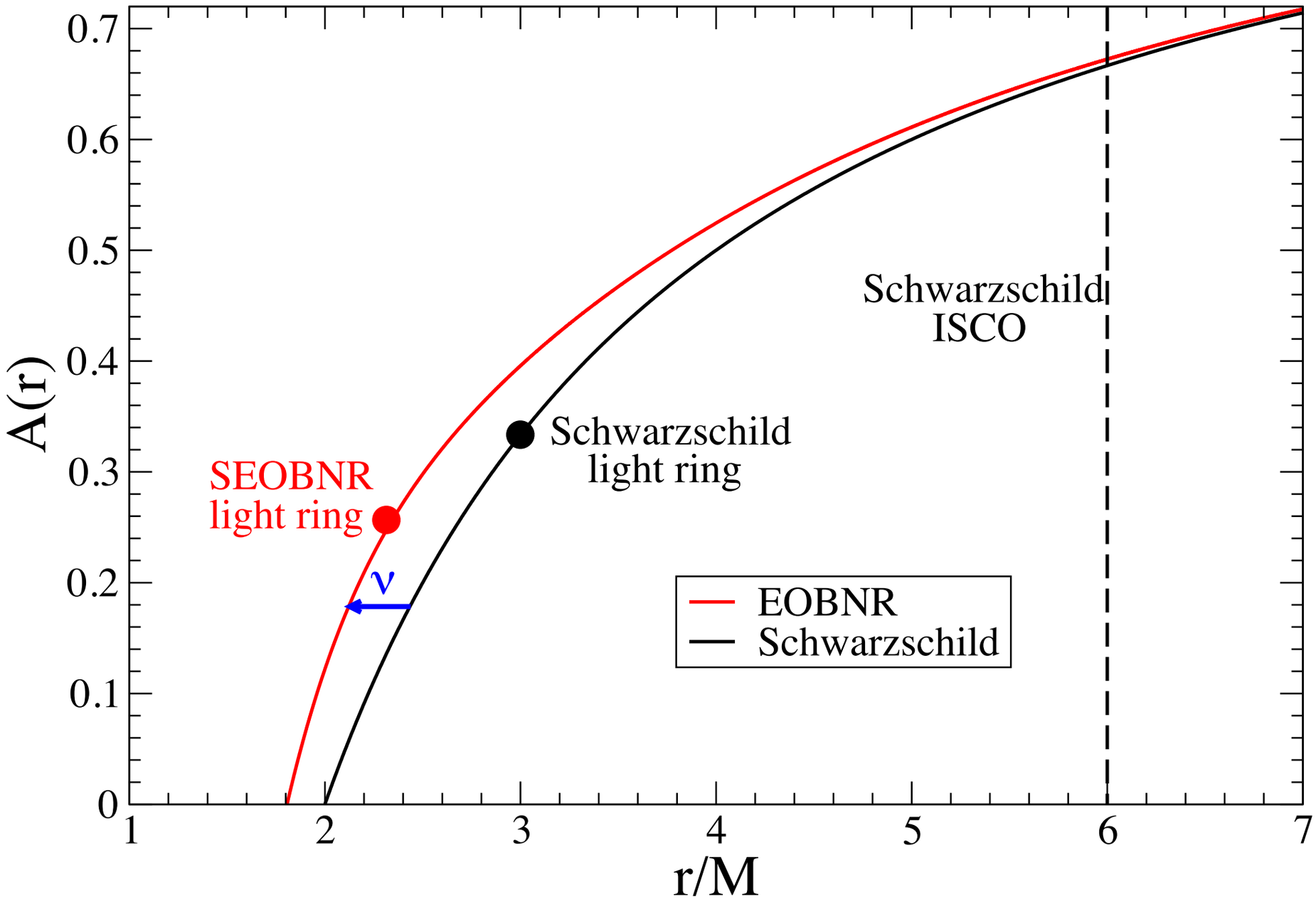
- We calibrate to merger-ringdown **Teukolsky-based** waveforms.



(Taracchini et al. 13)

Strong-field effects in binary black holes

Finite mass-ratio effects make **gravitational** interaction **less attractive**

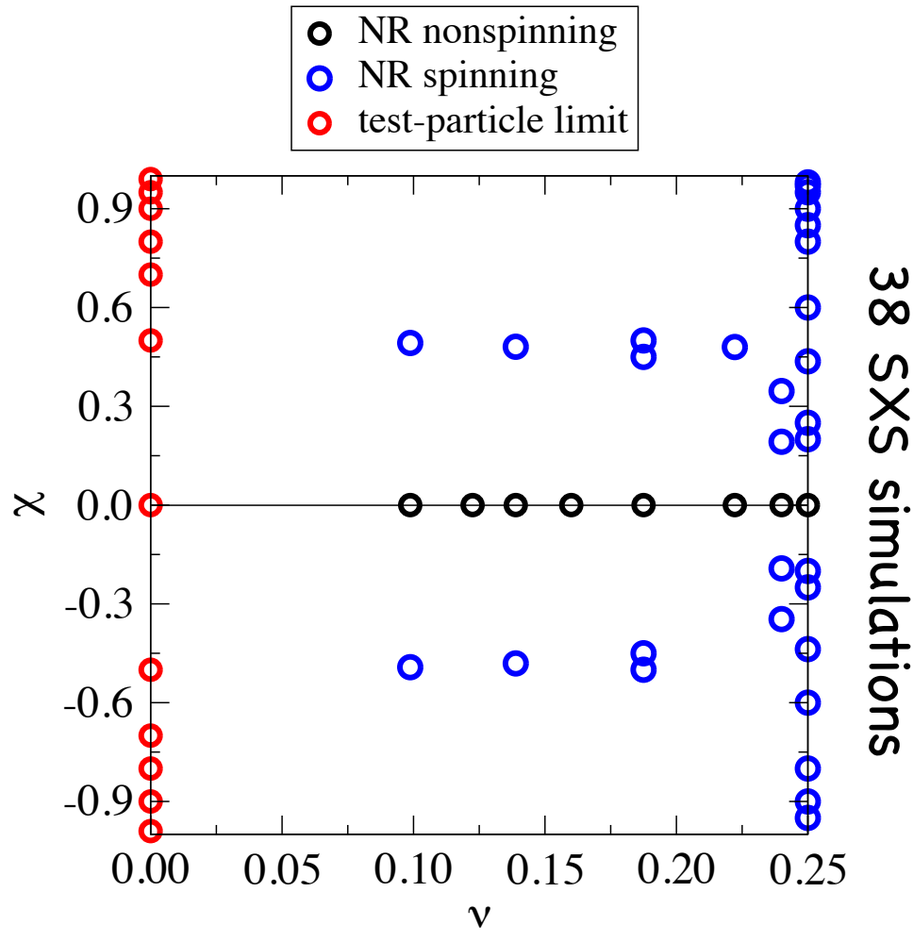


EOBNR waveforms used in LIGO O1 modeled-search

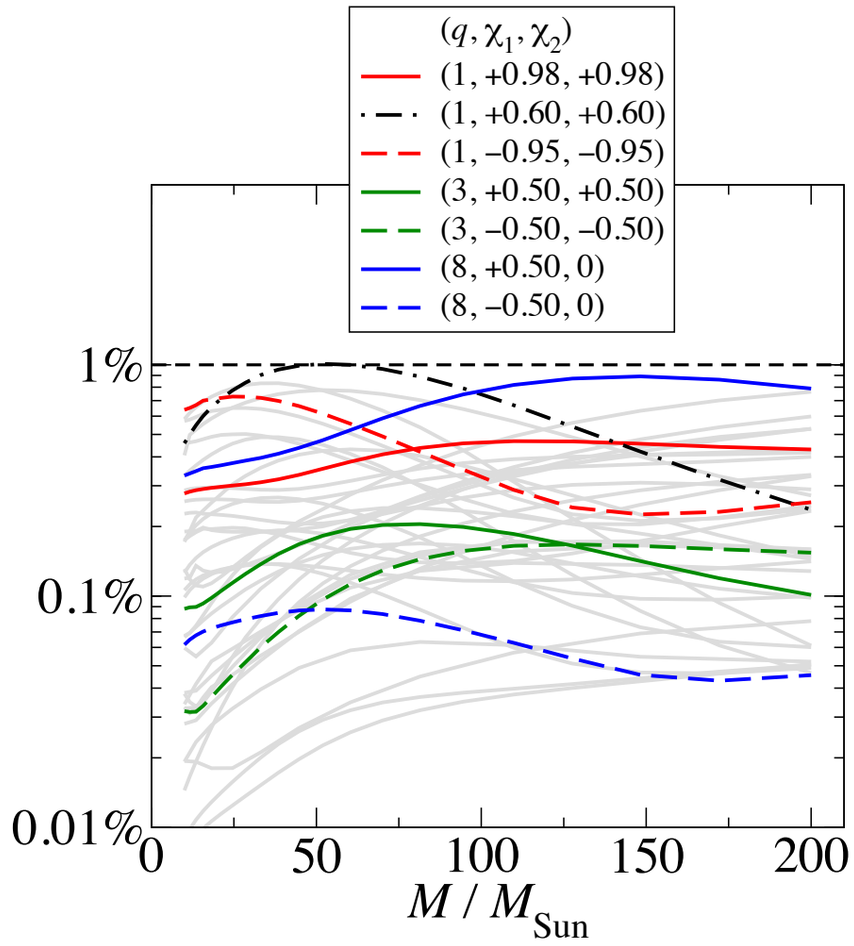
- EOB waveforms with **nonprecessing spins** calibrated to NR waveforms

  (Taracchini, AB, Pan, Hinderer & Simulating eXtreme Spacetimes (SXS) collaboration 14)

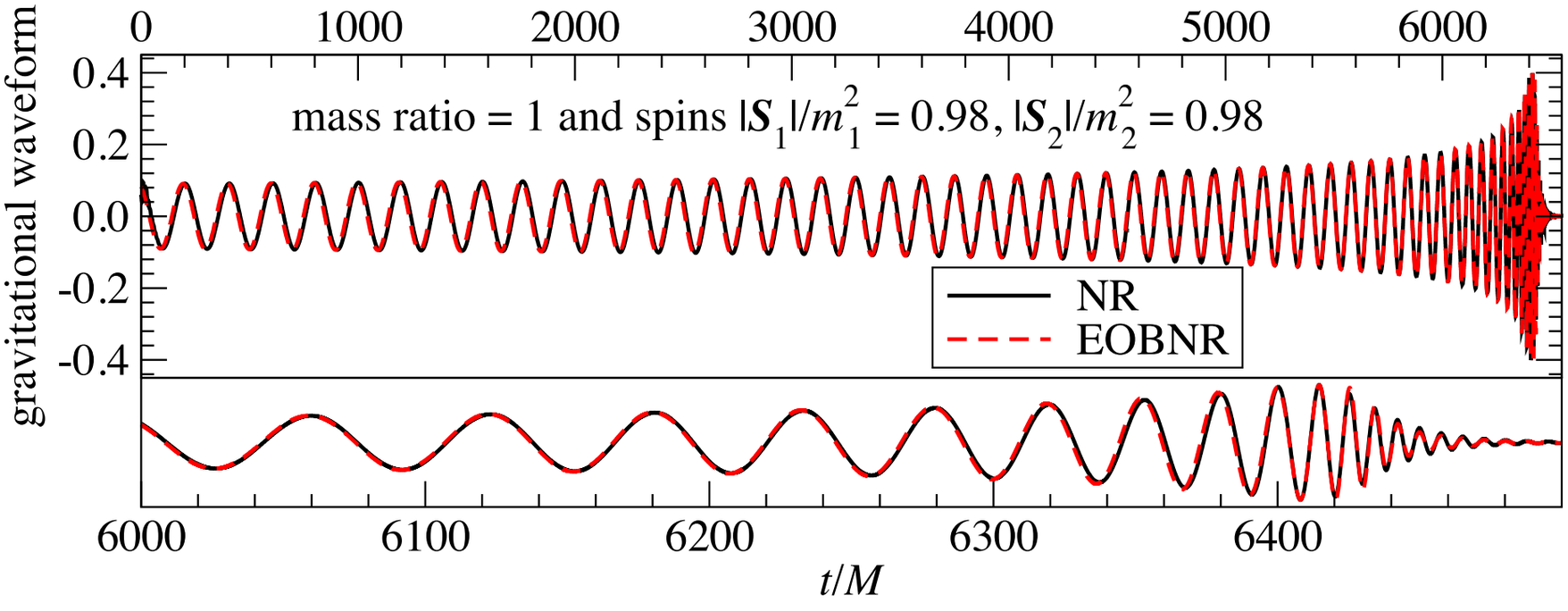
  (Barausse & AB 10, 11, Barausse et al. 09, Damour & Nagar 09, Pan et al. 08, Damour et al. 08)



38 SXS simulations



loss in event rates



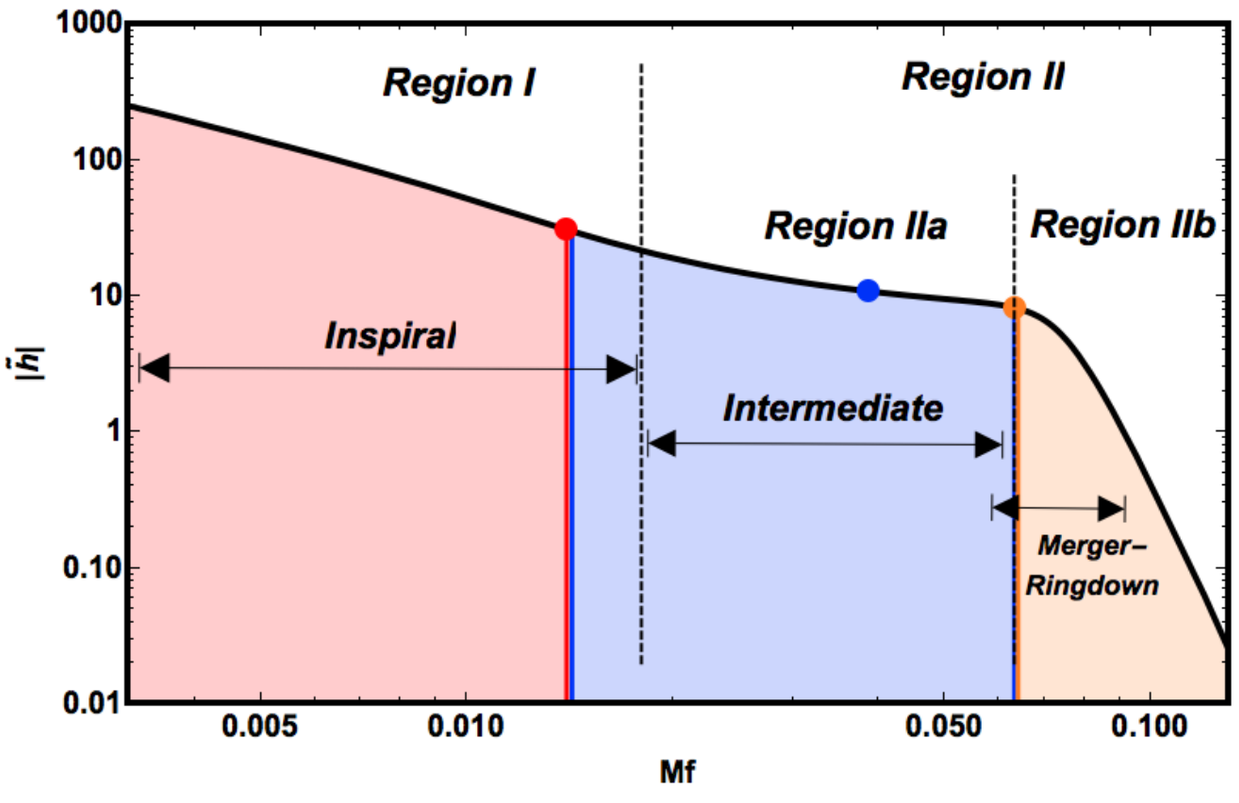
(Taracchini et al. 14)

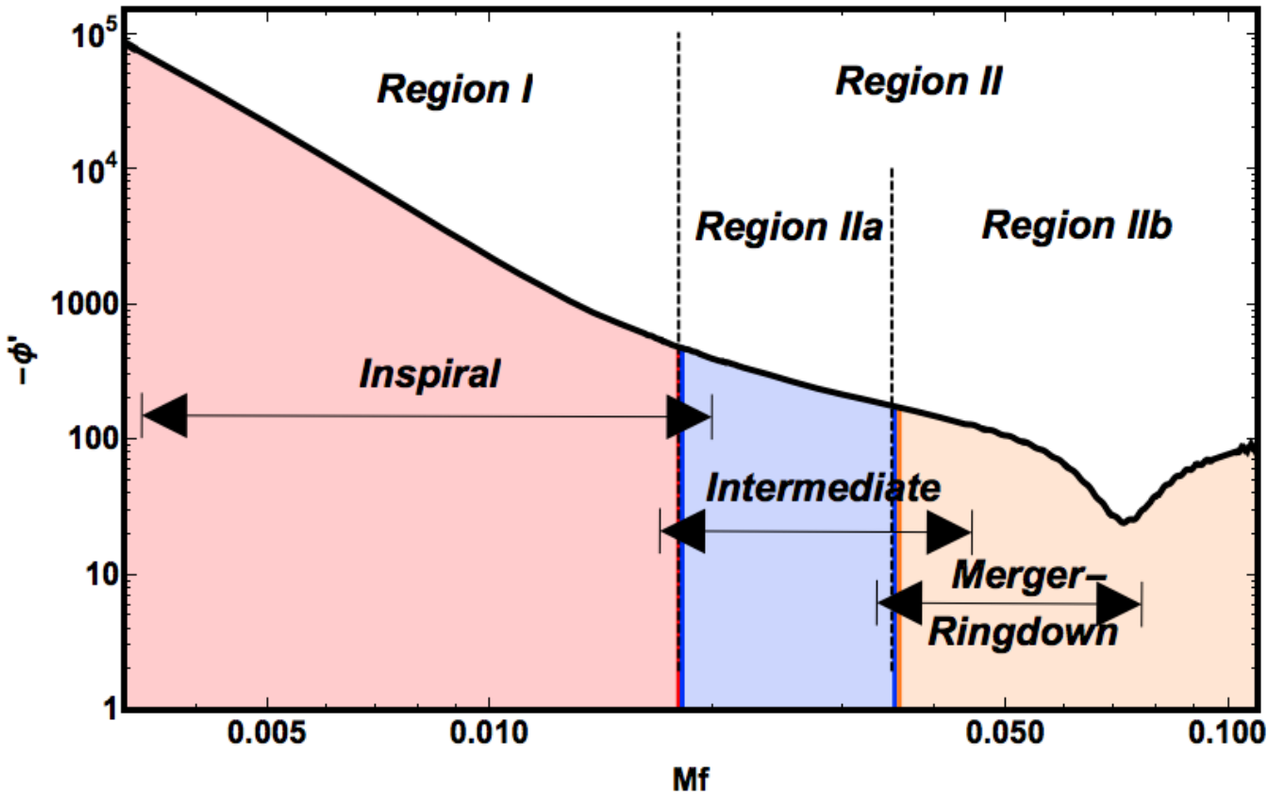
Inspiral-merger-ringdown phenomenological waveforms

- First works in mid-late 2000 (Ajith et al. 07, Pan et al. 07, Santamaria et al. 10)

- **Fast, frequency-domain** waveform model hybridizing and fitting EOB & NR (Khan et al. 15; Husa et al. 15)

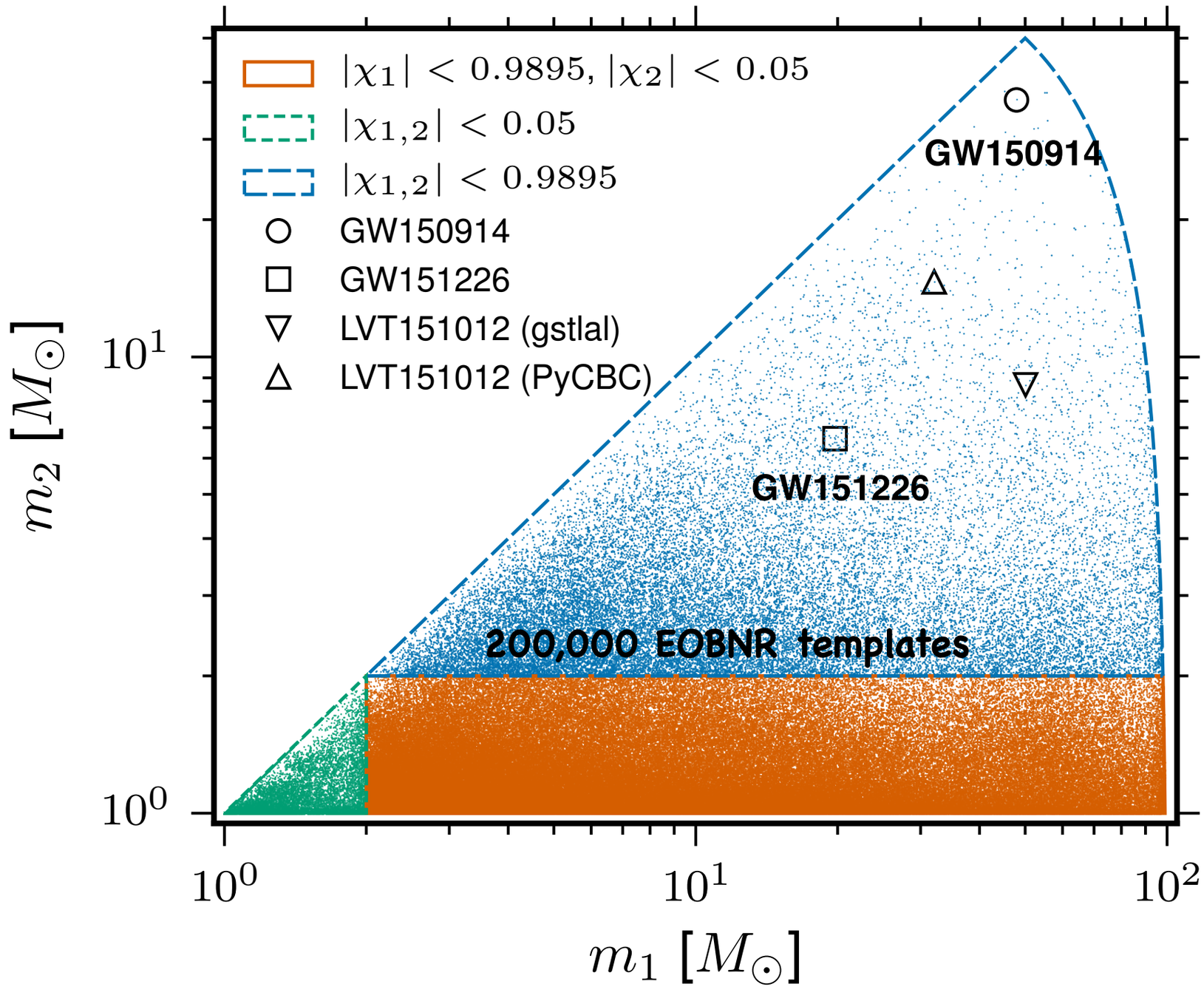
$$\tilde{h}(f;\lambda_i) = \mathcal{A}(f;\lambda_i)\, e^{i\phi(f;\lambda_i)}$$





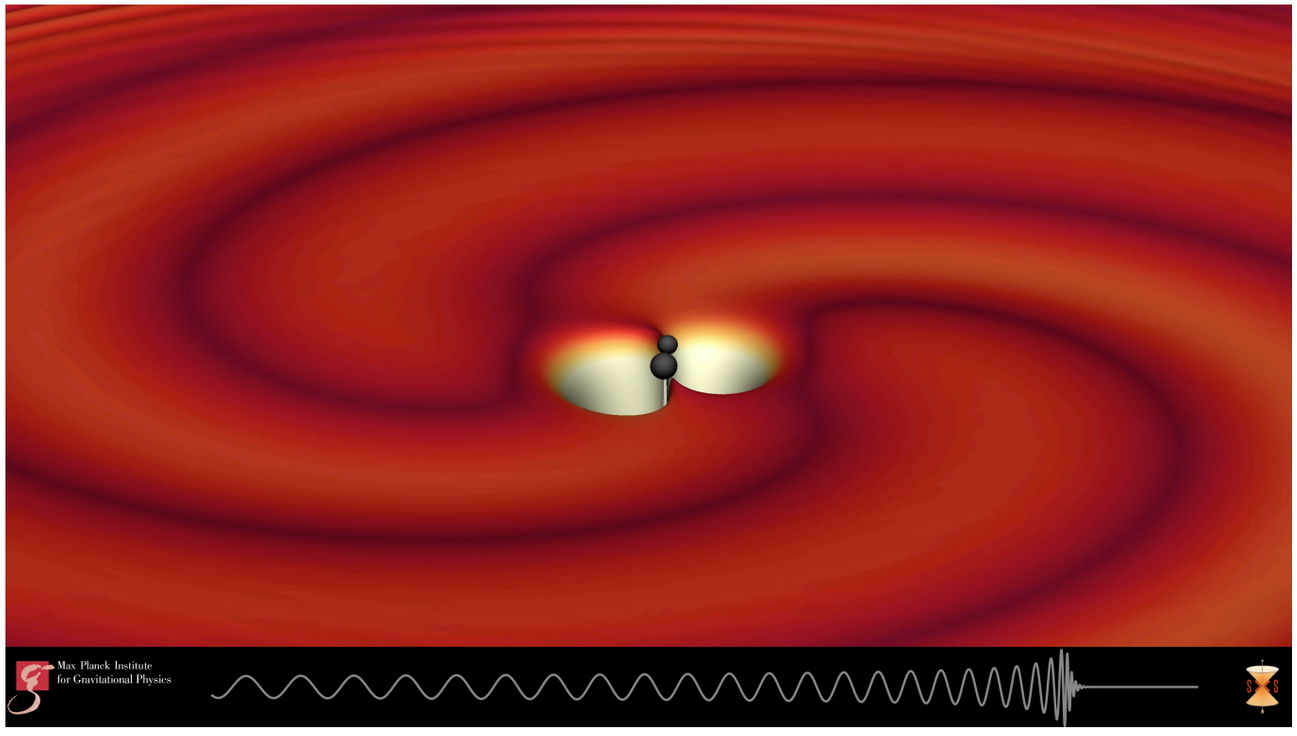
EOBNR waveforms used in modeled search in LIGO O1

- **Matched filtering** employed (Abbott et al. arXiv:1606.04856)



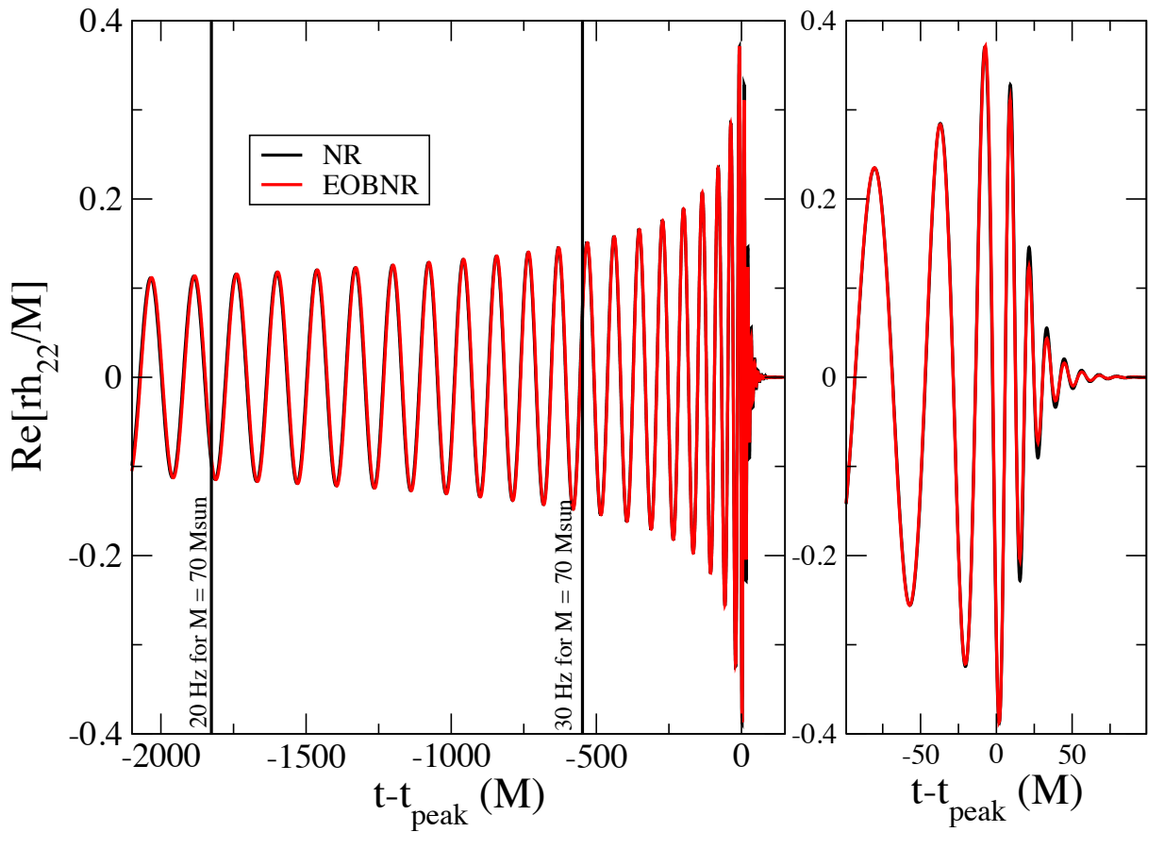
Numerical-relativity simulation of a binary black-hole merger with parameters close to GW150914

(visualization credit: Haas @ AEI)



(Ossokine, AB & SXS project)

(Ossokine, AB & SXS project)



- **Waveform models** very closely **match** the **exact solution** from Einstein equations around GW150914 & GW151226.

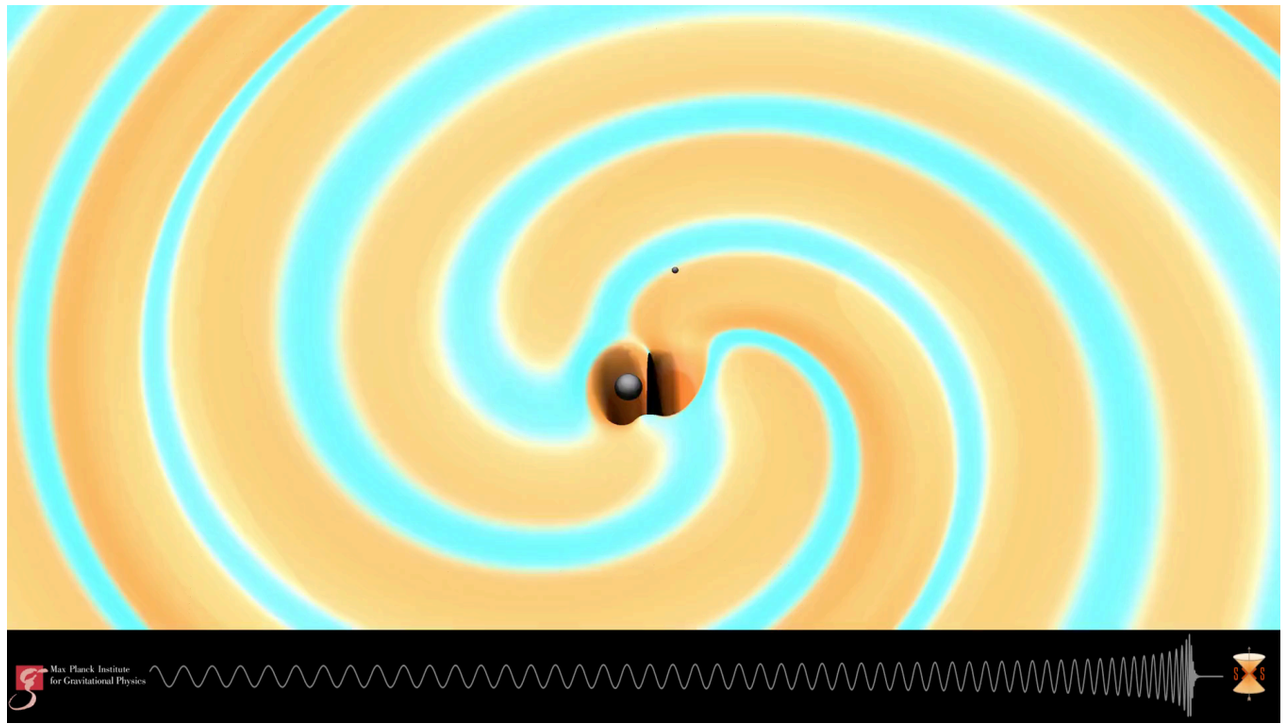
- **Systematics** due to modeling **are smaller than statistical** errors.

(see also Abbott et al. arXiv:1611.07531)

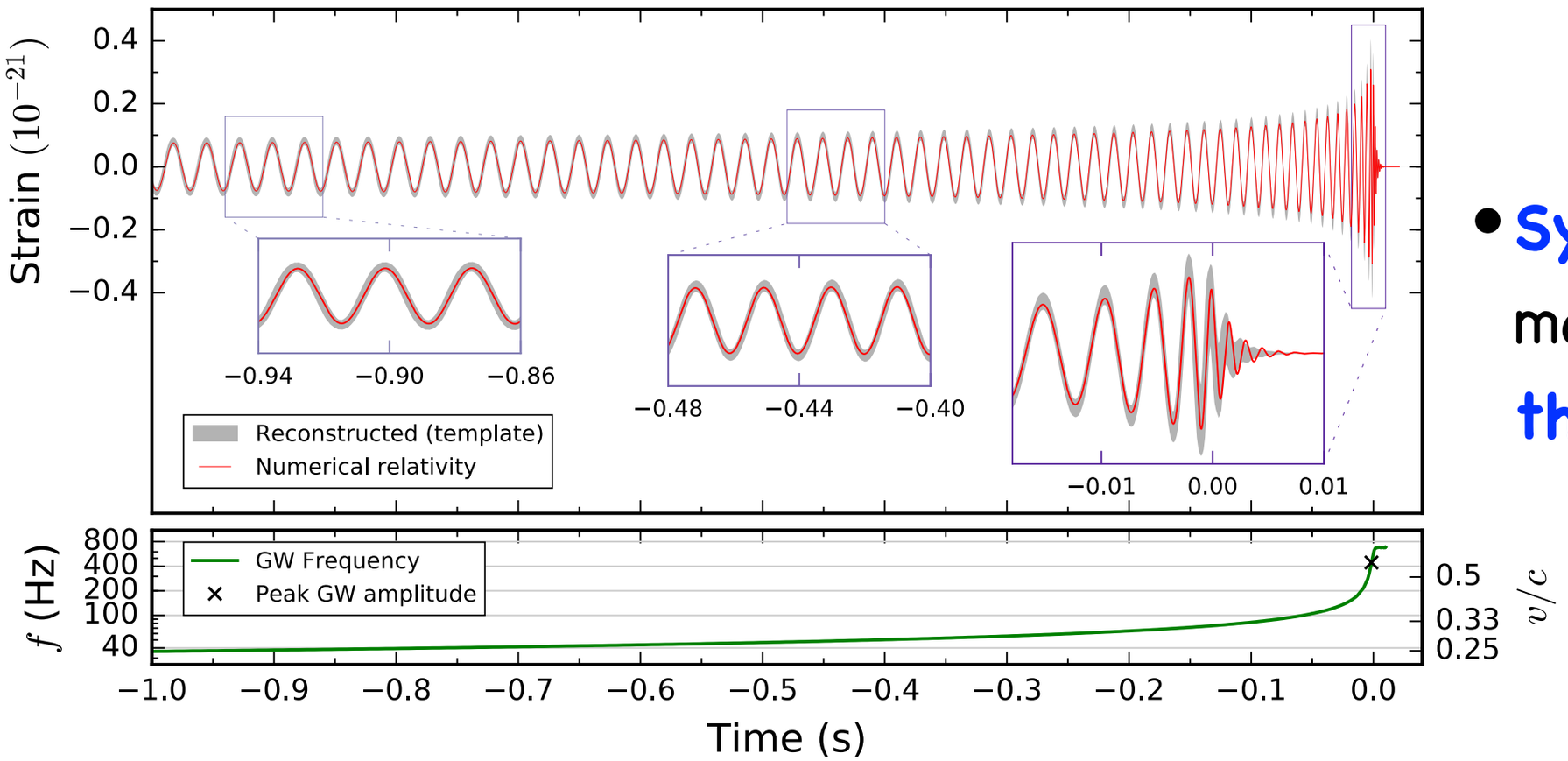
Numerical-relativity simulation of a binary black-hole merger with parameters close to GW151226

(visualization credit: Dietrich, Haas @AEI)

(Ossokine, AB & SXS project)



(Abbott et al. PRL 116 (2016) 241103)



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