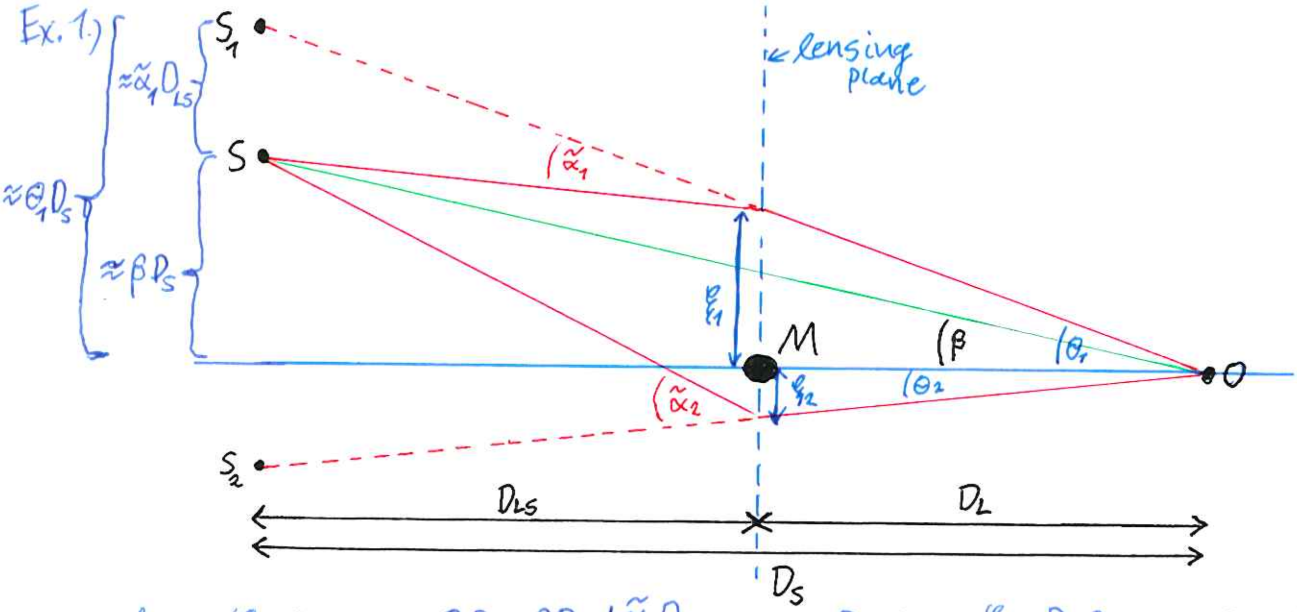
Ex. 1.)




from the figure: $\theta_i D_S = \beta D_S + \tilde\alpha_i D_{LS}$ , $D_S = D_L + D_{LS}$ , $\xi_i \approx D_L \theta_i$ $\quad i=1,2$

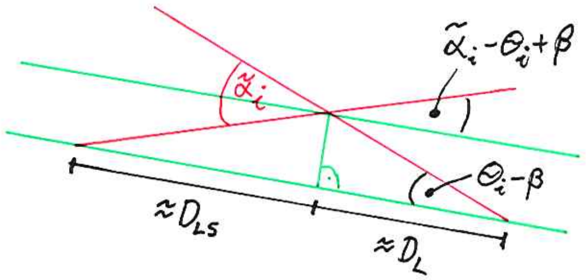
deflection angle: $\tilde\alpha_i = \frac{4M}{\xi_i} = \frac{4M}{D_L\theta_i}$

$$\rightsquigarrow \quad 0 = \theta_i - \beta - \frac{D_{LS}}{D_L D_S}\cdot\frac{4M}{\theta_i} = \theta_i - \beta - \frac{\theta_E^2}{\theta_i}$$

where $\theta_E := \sqrt{\frac{4MD_{LS}}{D_L D_S}}$ "Einstein radius"

solutions: $\boxed{\theta_{1,2} = \frac{1}{2}\left[\beta \pm \sqrt{\beta^2 + 4\theta_E^2}\right]}$

geometric time delay:




$$\tau_{geom} \approx \frac{D_L}{\cos(\theta_i-\beta)} + \frac{D_{LS}}{\cos(\tilde\alpha_i - \theta_i + \beta)} - \overset{D_L + D_{LS}}{\overset{\shortparallel}{D_S}}$$

$$\approx \frac{D_L}{2}(\theta_i-\beta)^2 + \frac{1}{2}(\tilde\alpha_i - \theta_i + \beta)^2 \cdot D_{LS}$$

$$\tilde\alpha_i = \frac{D_S}{D_{LS}}(\theta_i - \beta)$$

$$= \frac{1}{2}\Big[D_L + \underbrace{\left(\frac{D_S}{D_{LS}} - 1\right)^2}_{D_L^2/D_{LS}^2} D_{LS}\Big](\theta_i-\beta)^2$$

$$= \frac{1}{2}\cdot\frac{D_L D_S}{D_{LS}}\cdot(\theta_i - \beta)^2$$

## gravitational (Shapiro) time delay

Ray passes mass approximately along a straight line

$Z=0, \; y=\xi_i=\text{const}$

$ds^2=0 \rightsquigarrow dt \approx \left(1+\frac{2M}{r}\right)dx + \mathcal{O}\left(\frac{M^2}{r^2}\right)$, where $r=\sqrt{x^2+\xi_i^2}$

integrate:

$$\tau_{grav} = \int_{-D_{LS}}^{D_L} dx \frac{2M}{\sqrt{x^2+\xi_i^2}} = 2M\cdot\log\left(x+\sqrt{x^2+\xi_i^2}\right)\Big|_{x=-D_{LS}}^{D_L}$$

$$= 2M\cdot\log\Bigg[\underbrace{\frac{D_L+\sqrt{D_L^2+\xi_i^2}}{-D_{LS}+\sqrt{D_{LS}^2+\xi_i^2}}}_{\underset{\xi_i \ll D_L, D_{LS}}{\approx} \frac{D_L+D_L}{-D_{LS}+D_{LS}\left(1+\frac{1}{2}\frac{\xi_i^2}{D_{LS}^2}\right)} = \frac{4D_LD_{LS}}{\xi_i^2} \underset{\xi_i \approx D_L\Theta_i}{\approx} \frac{4D_{LS}}{D_L}\cdot\Theta_i^{-2}}\Bigg]$$

$$= -2M\Bigg[\log(\Theta_i^2) - \underbrace{\log\left(\frac{4D_{LS}}{D_L}\right)}_{\text{const.}}\Bigg]$$

**Remark:** rays take path of extremal time (Fermat's principle)

$$\frac{\partial}{\partial\Theta_i}(\tau_{geom}+\tau_{grav}) = \frac{D_LD_S}{D_{LS}}(\Theta_i-\beta) - \frac{4M}{\Theta_i} = \frac{D_LD_S}{D_{LS}}\left(\Theta_i-\beta-\frac{\Theta_E^2}{\Theta_i}\right) = 0$$

**magnification:** see Ref. 1, eq. 20

$$\mu_i := \frac{\Theta_i}{\beta}\cdot\frac{d\Theta_i}{d\beta} = \left(1-\frac{\Theta_E^4}{\Theta_i^4}\right)^{-1}$$

**example:** GW150914

$D_S = 10^9 y = 2D_L = 2D_{LS}$, $M = 10^{12} M_\odot = 0.16 y$, $\beta = 2\,\text{arcsec} = 10^{-5}$

$\rightsquigarrow \Theta_E = 2.5\cdot 10^{-5} = 5.2\,\text{arcsec}$

Ray 1: $\Theta_1 = 3\cdot10^{-5} = 6\,\text{arcsec}$, $\mu_1 = 1.9$, $\tau_{geom}+\tau_{grav} = 7.1y$

Ray 2: $\Theta_2 = -2\cdot10^{-5} = -4\,\text{arcsec}$, $\mu_2 = -0.9$, $\tau_{geom}+\tau_{grav} = 7.6y$

time delay between rays: $0.5y \gg 200\,\text{ms}$

Ex. 2) 

$$h^{\mu\nu} = (e_+^{\mu\nu} A_+ + e_\times^{\mu\nu} A_\times) e^{i k_\alpha x^\alpha}$$

with $e_+^{\mu\nu} = e_x^\mu e_x^\nu - e_y^\mu e_y^\nu$, $e_\times^{\mu\nu} = e_x^\mu e_y^\nu + e_y^\mu e_x^\nu$

$$\Downarrow \quad e_+^{\mu\nu} e_{+\mu\nu} = 2 = e_\times^{\mu\nu} e_{\times\mu\nu}, \quad e_+^{\mu\nu} e_{\times\mu\nu} = 0 \qquad (*)$$

$$T_{\mu\nu}^{GW} = \frac{1}{32\pi} \langle \partial_\mu h^{\alpha\beta} \partial_\nu h_{\alpha\beta} \rangle$$

$$= \frac{1}{32\pi} k_\mu k_\nu \langle [(e_+^{\alpha\beta} A_+ + e_\times^{\alpha\beta} A_\times) i e^{i k_\gamma x^\gamma} - (e_+^{\alpha\beta} A_+^* + e_\times^{\alpha\beta} A_\times^*) i e^{-i k_\gamma x^\gamma}]$$
$$\times [(e_{+\alpha\beta} A_+ + e_{\times\alpha\beta} A_\times) i e^{i k_\gamma x^\gamma} - (e_{+\alpha\beta} A_+^* + e_{\times\alpha\beta} A_\times^*) i e^{-i k_\gamma x^\gamma}] \rangle$$

use: $\langle e^{2 i k_\gamma x^\gamma} \rangle = 0 = \langle e^{-2 i k_\gamma x^\gamma} \rangle$ and $(*)$

$$= \frac{1}{8\pi} k_\mu k_\nu (|A_+|^2 + |A_\times|^2)$$

## Problem 3 Resonant mass detectors

We'll work in the center of mass frame with the two masses along the x-axis. For a given separation $\Delta$, the locations of the two masses will be

$$x_1 = -\frac{m_2}{m}\Delta, \qquad x_2 = \frac{m_1}{m}\Delta,$$

where $m$ is the sum of the masses. Hence, with an equilibrium separation of $L$, the masses will be perturbed by a gravitational wave to

$$x_1(t) = -\frac{m_2}{m}L - \frac{m_2}{m}x(t), \qquad x_2(t) = \frac{m_1}{m}L + \frac{m_1}{m}x(t).$$

By assumption, the gravitational wave varies on a length scale much smaller than the detector, so we can treat the equation of geodesic deviation to first order in $\xi$, the coordinate distance between two geodesics. In the proper detector frame, this geodesic deviation can be thought of as a Newtonian force on each mass given by

$$F_{1(2)} = \frac{m_{1(2)}}{2}\ddot{h}_{ij}^{\mathrm{TT}}\xi^j.$$

We consider deviations from a test mass initially at rest at the origin, which therefore remains stationary. Each mass can be though of as being acted on a force of

$$F_{1(2)} = -\frac{m_{1(2)}}{2}\omega^2 h\cos(\omega t)x_{1(2)}(t)\hat{\mathbf{x}}.$$

Including the spring and damping forces, the equations of motion for our two masses are

$$m_1\frac{d^2x_1}{dt^2} = \frac{m_1}{2}\omega^2 h\cos(\omega t)\frac{m_2}{m}L + kx(t) + b\frac{dx}{dt} + \mathcal{O}(h^2) \tag{1}$$
$$m_2\frac{d^2x_2}{dt^2} = -\frac{m_2}{2}\omega^2 h\cos(\omega t)\frac{m_1}{m}L - kx(t) - b\frac{dx}{dt} + \mathcal{O}(h^2) \tag{2}$$

Subtracting these two, we obtain the equation of motion for the relative separation.

$$\frac{d^2x}{dt^2} = -\frac{1}{2}\omega^2 hL\cos(\omega t) - \frac{k}{\mu}x(t) - \frac{b}{\mu}\frac{dx}{dt} + \mathcal{O}(h^2) \tag{3}$$

where $\mu$ is the reduced mass.

As an ansatz, we assume that the separation is of the form $x(t) = A\cos(\omega t + \delta)$.

Plugging this in to (3), we derive the dependence of $A$ and $\delta$ on the given parameters.

$$-\omega^2 A\cos(\omega t+\delta) = -\frac{1}{2}hL\cos(\omega t) - \frac{k}{\mu}A\cos(\omega t+\delta) + A\omega\frac{b}{\mu}\sin(\omega t+\delta) \tag{4}$$

$$\Rightarrow \frac{1}{2}\omega^2 hL\cos(\omega t) = A\left(\left(\omega^2-\frac{k}{\mu}\right)\cos(\omega t+\delta)+\omega\frac{b}{\mu}\sin(\omega t+\delta)\right) \tag{5}$$

$$\frac{1}{2}\omega^2 hL\cos(\omega t) = A\sqrt{\left(\omega^2-\frac{k}{\mu}\right)^2+\omega^2\frac{b^2}{\mu^2}}\cos(\omega t+\delta-\phi) \tag{6}$$

$$\Rightarrow A = \frac{\omega^2 hL}{2\sqrt{\left(\omega^2-\frac{k}{\mu}\right)^2+\omega^2\frac{b^2}{\mu^2}}} \qquad \delta=\phi=\tan^{-1}\left(\frac{\omega\frac{b}{\mu}}{\omega^2-\frac{k}{\mu}}\right) \tag{7}$$

To find the resonant frequency, we must maximize $A$ with respect to the frequency. It is more convenient to instead minimize $\tilde{A}=A^{-2}$, which will provide the same result.

$$\frac{\partial\tilde{A}}{\partial\omega} \propto \left(\frac{b^2}{\mu^2}-\frac{2k}{\mu}\right)\omega^2+\frac{2k^2}{\mu^2}=0 \tag{8}$$

$$\Rightarrow\omega_r = \sqrt{\frac{k}{\mu}}\left(\frac{b^2}{2k\mu}-1\right)^{-\frac{1}{2}}. \tag{9}$$

The exact value of the amplitude and phase shift at resonance can be found by plugging this value into the previous result.

$$A_r = \frac{hLk\mu|2k\mu-b^2|}{b(2k\mu-b^2)\sqrt{4k\mu-b^2}}, \tag{10}$$

$$\delta_r = \tan^{-1}\left(\frac{\sqrt{4k\mu-2b^2}}{b}\right) \tag{11}$$

In the following results, an overbar denotes the value averaged over one oscillation.

$$\begin{aligned}
KE &= \frac{1}{2}m_1\left(\frac{dx_1}{dt}\right)^2 + \frac{1}{2}m_2\left(\frac{dx_2}{dt}\right)^2 \\
&= \left(\frac{1}{2}m_1\frac{m_2^2}{m^2} + \frac{1}{2}m_2\frac{m_1^2}{m^2}\right)\left(\frac{dx}{dt}\right)^2 \\
&= \tfrac{1}{2}\mu\left(\frac{dx}{dt}\right)^2 \\
&= \frac{1}{2}\mu A^2\omega^2\sin^2(\omega t+\delta) \\
\Rightarrow \overline{KE} &= \frac{1}{4}\mu A^2\omega^2
\end{aligned} \tag{12}$$

$$\begin{aligned}
PE &= \frac{1}{2}kx^2 \\
&= \frac{1}{2}kA^2\cos^2(\omega t+\delta) \\
\Rightarrow \overline{PE} &= \frac{1}{4}kA^2
\end{aligned} \tag{13}$$

The work done by the gravitational wave on both masses over one oscillation is given by

$$\begin{aligned}
W_{\rm GW} &= \int F\cdot\frac{dx}{dt}dt \\
&= -\int dt\left(\frac{m_1m_2}{2m}\omega^2hL\cos(\omega t)\frac{m_2}{m}\frac{dx}{dt} + \frac{m_1m_2}{2m}\omega^2hL\cos(\omega t)\frac{m_1}{m}\frac{dx}{dt}\right) \\
&= \frac{m_1m_2}{2m}\omega^3hLA\int dt\cos(\omega t)\sin(\omega t+\delta) \\
&= \frac{m_1m_2}{2m}\omega^3hLA\int dt\cos(\omega t)\cos(\omega t)\sin(\delta) - \cos(\omega t)\sin(\omega t)\cos(\delta) \\
&= \frac{m_1m_2}{2m}\pi\omega^2hLA\sin(\delta) \\
&= \frac{1}{2}\mu\pi\omega^2hLA\sin(\delta) \\
&= \pi b\omega A^2
\end{aligned} \tag{14}$$

By energy conservation, the dissipated energy must be equal to the (negative) average work done on the system by the the gravitational wave. Thus, the average rate of energy lost in a cycle is

$$\begin{aligned}
\overline{P}_{\rm damp} &= \frac{\omega}{2\pi}W_{\rm GW} \\
&= \frac{1}{2}b\omega^2A^2
\end{aligned} \tag{15}$$

Plugging in the given values $h = 10^{-21}$, $L = 1$ m, $\mu = 1000$ kg, $f = \sqrt{k/\mu}/(2\pi) = 1$ kHz, and $Q = \omega_r \times (\overline{PE} + \overline{KE})/\overline{P}_{\rm damp} = 10^6$, one finds that the amplitude at resonance is

$$A_r = 5 \times 10^{-16}\,\mathrm{m}, \tag{16}$$

and the total energy is

$$\overline{PE} + \overline{KE} = 5 \times 10^{-21}\,\mathrm{J}. \tag{17}$$

At room temperature (300 K), the average thermal energy is given by $E_{thermal} \approx k_B T \approx 10^{-21}$J. Thus, this gravitational wave is of the same magnitude of thermal noise in the detector, and thus would be very difficult to observe.

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**Problem 4 Attenuation of gravitational waves**

We work in the rest frame of the fluid, so that $u^\mu = (1, 0, 0, 0)$. We assume that the background spacetime is approximately flat and work within linearized gravity, i.e. to $\mathcal{O}(G^1)$, such that the metric is given by

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}. \tag{18}$$

We restrict our attention to transverse-traceless metric perturbations $h_{\mu\nu} = h^{\mathrm{TT}}_{\mu\nu}$.[1] We work in the transverse traceless frame, in which mass initially at rest remains at rest upon arrival of a gravitational wave, i.e. $u^\mu$ remains unchanged. To first order in $h$, the connection coefficients are given by

$$\Gamma^\mu_{\nu\rho} = \frac{1}{2}\eta^{\mu\sigma}(\partial_\nu h^{\mathrm{TT}}_{\rho\sigma} + \partial_\rho h^{\mathrm{TT}}_{\nu\sigma} - \partial_\sigma h^{\mathrm{TT}}_{\nu\rho}) + \mathcal{O}(h^2) \tag{19}$$

We calculate $\sigma$ to first order, noting that $u^\mu$ is constant.

$$\begin{aligned}\sigma_{\mu\nu} &= \tfrac{1}{2}(-\Gamma^\gamma_{\mu\nu}u_\gamma - \Gamma^\gamma_{\mu\nu}u_\gamma - u_\mu u^\alpha \Gamma^\gamma_{\alpha\nu}u_\gamma - u_\nu u^\alpha \Gamma^\gamma_{\alpha\mu}u_\gamma) - \tfrac{1}{3}(\eta_{\nu\mu} + h^{\mathrm{TT}}_{\mu\nu} + u_\mu u_\nu)\Gamma^\alpha_{\alpha\gamma}u^\gamma \\ &= \tfrac{1}{2}(2\Gamma^0_{\mu\nu} + u_\mu \Gamma^0_{0\nu} + u_\nu \Gamma^0_{0\mu}) - \tfrac{1}{3}(\eta_{\nu\mu} + u_\mu u_\nu)\Gamma^\alpha_{\alpha 0} + \mathcal{O}(h^2)\end{aligned} \tag{20}$$

Using (19), we find the only non-zero connection coefficient above to be

$$\Gamma^0_{\mu\nu} = \frac{1}{2}\frac{\partial}{\partial t}h^{\mathrm{TT}}_{\mu\nu}. \tag{21}$$

Thus, we find that in the transverse traceless gauge, the sheer reduces to

$$\sigma_{ij} = \frac{1}{2}\frac{\partial}{\partial t}h^{\mathrm{TT}}_{ij} \tag{22}$$

[1] Unlike in vacuum, we do not have the gauge freedom to remove all non-transverse-traceless components of $h_{\mu\nu}$ *a priori* (see page 8 of the Maggiore book). Within a viscous fluid, an additional "sound wave mode" can occur in $h_{\mu\nu}$ tied to wavelike perturbations of the fluid [1]. For simplicity, we consider only transverse-traceless metric perturbations.

which is solely spatial because $h^{\mathrm{TT}}$ only has spatial components.

In linearized gravity, the strain satisfies the wave equation

$$\Box\bar{h}_{\mu\nu} = -16\pi G T_{\mu\nu}, \tag{23}$$

where $\bar{h}_{\mu\nu} \equiv h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h^{\alpha}{}_{\alpha}$. Using the given form of the stress-energy tensor, (23) becomes

$$\Box h_{ij}^{\mathrm{TT}} = -16\pi G\eta \frac{\partial}{\partial t} h_{ij}^{\mathrm{TT}}. \tag{24}$$

We consider a solution of the form $h_{ij} = h_{ij}(0)e^{-i(\omega t - kz)}$. At a fixed location in the fluid, we expect the amplitude of the wave to be constant as there are no non-periodic time-dependent sources, i.e. $\omega$ will be real. We plug this ansatz in to (24) and solve for $k = k_R + ik_I$ where $k_R$, $k_I$ are real.

$$\begin{aligned}
(\omega^2 - k^2)h_{ij} &= 16\pi G\eta(i\omega)h_{ij} \\
\Rightarrow \omega^2 = k_R^2 - k_I^2, \qquad k_R k_I &= -8\pi G\eta \\
\Rightarrow k_R^2 k_I^2 &= (8\pi G\eta)^2\omega^2 \\
\Rightarrow (\omega^2 + k_I^2)k_I^2 &= (8\pi G\eta)^2\omega^2 \\
\Rightarrow k_I^2 = \frac{1}{2}\left(-\omega^2 + \sqrt{\omega^4 + 4(8\pi G\eta)^2\omega^2}\right)&.
\end{aligned} \tag{25}$$

In the eikonal limit, we expect $k_R \approx \omega \gg k_I$. For gravitational waves observable with current detectors, this regime is reached for fluids with viscosity

$$\frac{2(8\pi G\eta)}{\omega c^2} \ll 1 \Rightarrow \eta \ll 1.7 \times 10^{27} \left(\frac{f_{\mathrm{GW}}}{10\mathrm{Hz}}\right) \frac{\mathrm{kg}}{\mathrm{m}\cdot\mathrm{s}} \tag{26}$$

In this limit

$$k_I^2 \approx \frac{1}{2}(2(8\pi G\eta)^2) \tag{27}$$

$$\Rightarrow k_I = 8\pi G\eta. \tag{28}$$

Restoring the factors of c, we find an attenuation length of $l = k_I^{-1} = \frac{c^3}{8\pi G\eta}$. For chocolate, the attenuation length is approximately $6.8 \times 10^{16}$ lightyears.

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# References

[1] A. R. Prasanna. Propagation of gravitational waves through a dispersive medium. *Phys. Lett.*, A257:120–122, 1999.

Solutions

**Challenge #1**: Make your own estimate of the rate per volume of BH-BH mergers (expressed in number per Gpc$^{-3}$ yr$^{-1}$), including the 90% credible interval, based on the three events reported thus far (for these purposes we assume that LVT151012 was a real event). The first Advanced LIGO run had 49 total days in which both detectors were taking data, so that will be our baseline time. Potentially relevant numbers are: GW150914 was at a distance of 420 Mpc (we'll ignore the uncertainties for simplicity) and had a signal to noise ratio of 23.7; GW151226 was at a distance of 440 Mpc and had a signal to noise ratio of 13.0; LVT151012 was at a distance of 1 Gpc and had a signal to noise ratio of 9.7. Suppose that the threshold for announcing a detection is a signal to noise ratio of 12.0 (recall that LVT151012 was a marginal detection), and remember that for a given event the distance scales as the reciprocal of the signal to noise ratio.

a) With no other information, what would be your best estimate for the rate per volume based on each of the events individually (i.e., without combining them or estimating uncertainties)?

b) How should you estimate the uncertainties for each event individually? More specifically, how would you calculate the 90% credible interval for the rate based on each event individually?

c) How should you combine the information from the three events? Do this without, then with, the uncertainties included.

d) Suppose now that you are given the information that one of the events (pick any of them) was in a direction to which Advanced LIGO was unusually sensitive. What effect, if any, would this have on your best estimate of the rate based on that event (i.e., would it decrease your best estimate, increase your best estimate, or leave it unchanged)?

e) Same question as d), but with regard to the orientation: suppose that one of the events was known to have its binary orbital axis pointed nearly towards us, which means that we see a high amplitude compared to the orientation-averaged amplitude. What effect would this have on your best estimate of the rate from that event alone?

**Answers:**

a) Recall from the discussion in the notes about the way to estimate the rates of NS-NS mergers that the contribution from an individual source (or event in our case) goes inversely with the time of observation and inversely with the volume in which the event *could* have been seen. The logic is that, based on the Copernican principle, weak events could have many similar but more distant events that we didn't see, whereas strong events would have to be much farther away to remain undetected. With that in mind, and recalling that the amplitude goes like the reciprocal of the distance, we note that GW150914 could have been seen out to $r_{\rm max} = 420$ Mpc $\times$ $(23.7/12) \approx 0.83$ Gpc; GW151226 could have been seen out to $r_{\rm max} = 440$ Mpc $\times$ $(13/12) \approx 0.48$ Gpc; and LVT151012 could have been seen out to $r_{\rm max} = 1000$ Mpc $\times$ $(9.7/12) \approx 0.8$ Gpc. Thus the best estimates for rate per Gpc$^3$ per year are $(365/49)1/(4\pi/3(0.83)^3) = 3$ for GW150914, $(365/49)1/(4\pi/3(0.48)^3) = 16$ for GW151226, and $(365/49)1/(4\pi/3(0.8)^3) = 3.5$ for LVT151012.

b) For each event individually we are certainly in the Poisson-dominated regime. We see one event. In a Poisson distribution, the probability of seeing $d$ (a non-negative integer) events given an expected number $m$ (a positive real number) of events is

$$P(d|m) = \frac{m^d}{d!}e^{-m} \ . \tag{1}$$

You can confirm that this expression is properly normalized: $\sum_{d=0}^{\infty} P(d|m) = 1$ for any $m > 0$ and $\int_0^\infty P(d|m)dm = 1$ for any $d \geq 0$. In our case $d = 1$. If we want the middle 90% of the probability distribution for $m$ we therefore need $m_1$ and $m_2$ such that:

$$\int_0^{m_1} p(1|m)dm = 0.05 \tag{2}$$

and

$$\int_{m_2}^{\infty} p(1|m)dm = 0.05 \ . \tag{3}$$

Because $p(1|m) = me^{-m}$, the solutions are $m_1 \approx 0.356$ and $m_2 \approx 4.74$. There is, however, a subtlety in this calculation. It assumes implicitly that all values of $m$ are, a priori, equally likely. This might not be the case; for example, it is at least equally plausible that we should assume that all values of the *log* of $m$ are equally plausible. If we do this then we get different values of $m_1$ and $m_2$. In any case, we simply multiply our $m_1$ and our $m_2$ by our best value to get the 90% range.

c) In both cases you simply add the results from the individual events.

d) and e). The volumetric rate (rate per volume per time) is for all events. Thus the representative signal would be one that was averaged over sky direction and orientation. If we saw a given event in a direction to which Advanced LIGO was unusually sensitive, then the intrinsic event would be weaker than average, which would mean that the volume out to which we could see it would be smaller. Thus that event would have a *greater* weight than if it had been average. Similarly, if the orientation was favorable, the orientation-averaged amplitude would be lower, which would mean that the event would receive greater than average weight.