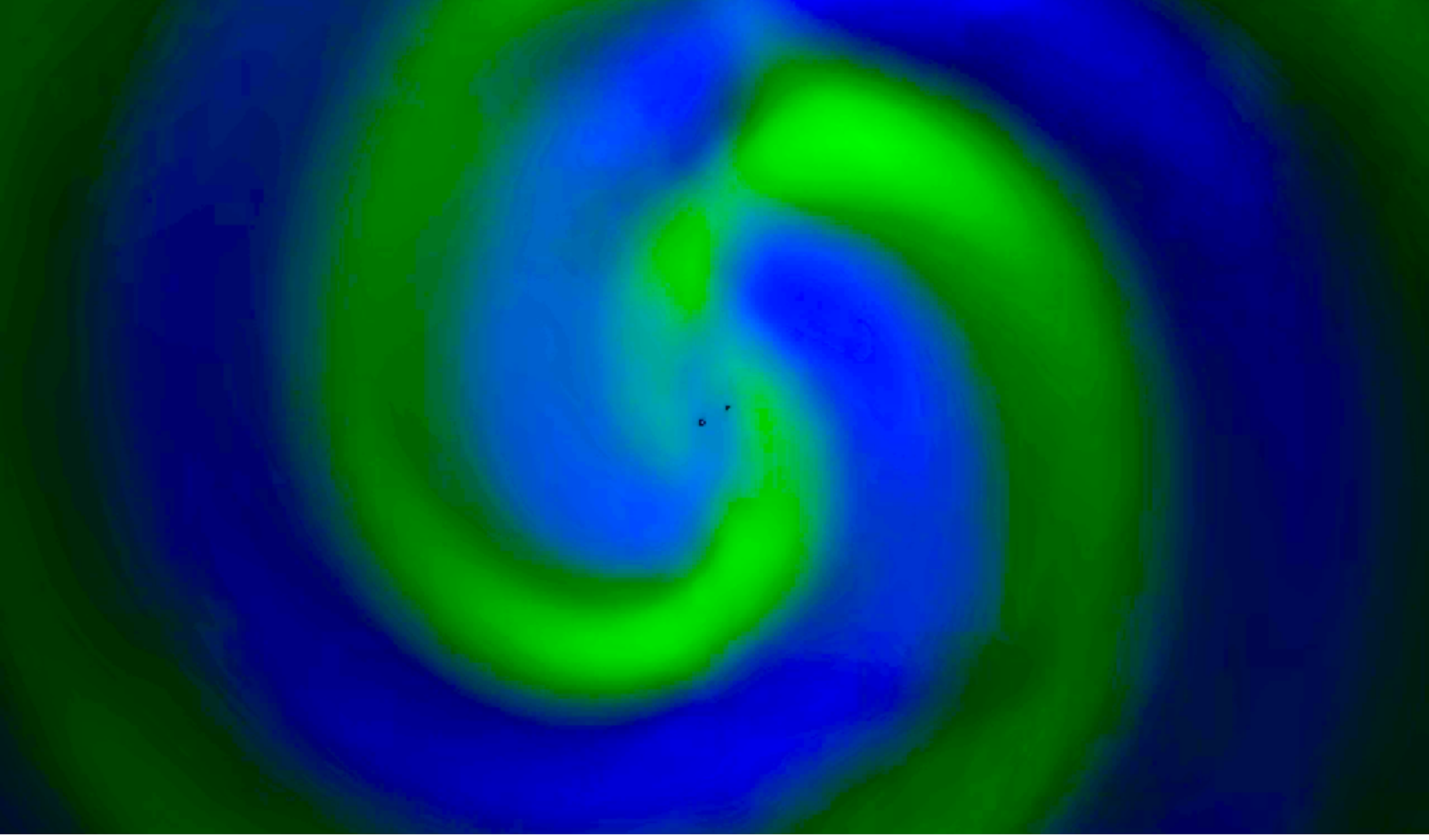


Numerical Relativity Examples

Ian Hinder, April 2017

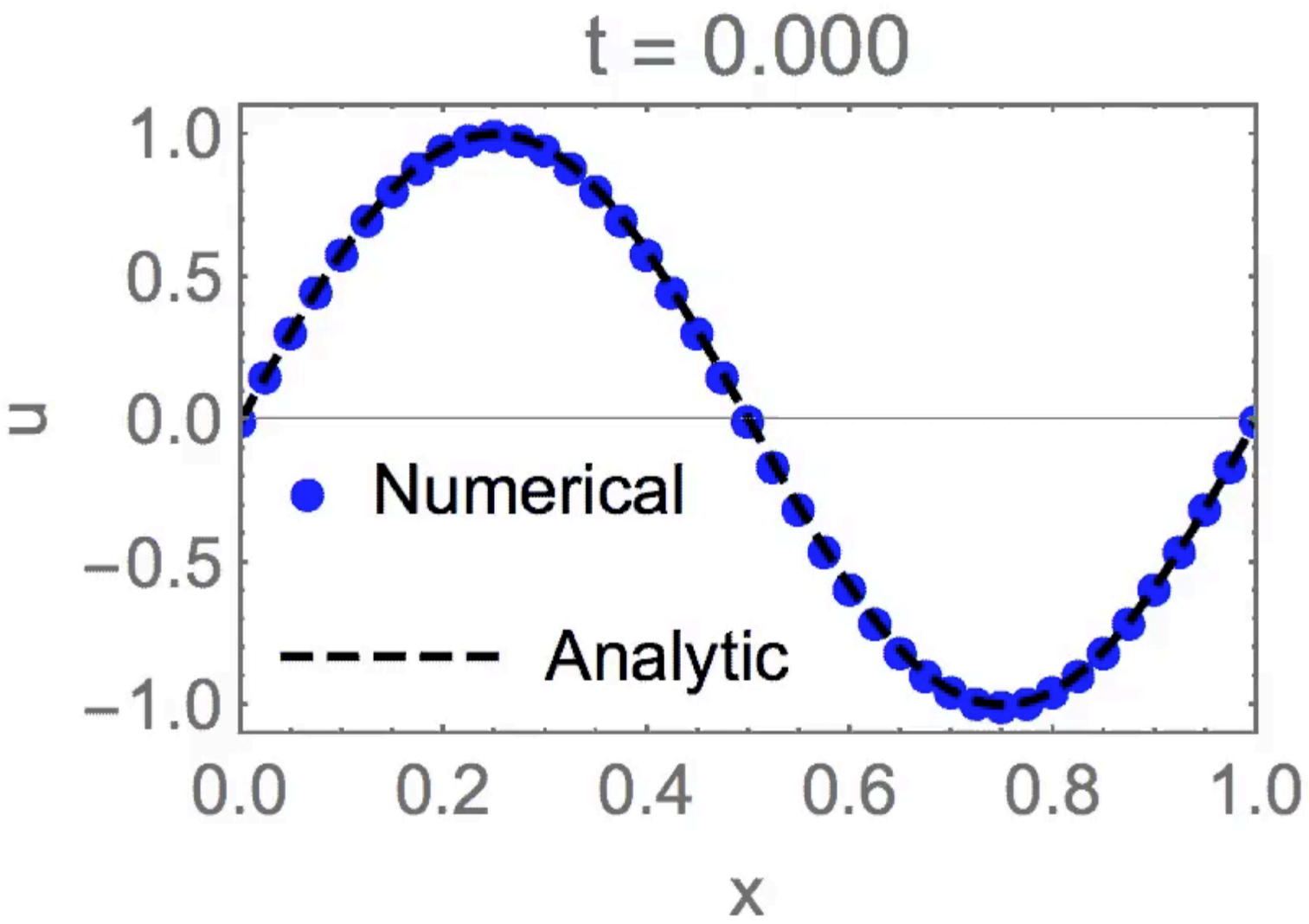
Example C code for advection equation

```c
#include <stdio.h>
#include <math.h>

const int N = 40+2;
const double h = 1./(N-2);
const double k = h/4.;
const double T = 1;
const double a = 1;

void output(double x[], double v[]) {
  for (int j = 0; j < N; j++) {
    printf("%.19f\t%.19f\n", x[j], v[j]);
  }
  printf("\n");
}

int main()
{
  double vn[N];  double x[N];

  for (int j = 0; j < N; j++) {
    x[j] = j * h;
    vn[j] = sin(2.*M_PI*x[j]);
  }

  output(x,vn);

  double vnp1[N];

  for (double t = 0; t < T; t+=k) {

    for (int j = 1; j < N-1; j++)
      vnp1[j] = vn[j] + k*a*(vn[j+1] - vn[j])/h;

    vnp1[0]   = vnp1[N-2];
    vnp1[N-1] = vnp1[1];

    for (int j = 0; j < N; j++)
      vn[j] = vnp1[j];

    output(x,vn);
  }
}
```

Output:

```
0.000000        0.000000
0.010000        0.062791
0.020000        0.125333
0.030000        0.187381
...

0.000000        0.015690
0.010000        0.078411
0.020000        0.140822
0.030000        0.202678
...
```

Numerical vs Exact solution



- Error in speed: dispersion
- Error in amplitude: dissipation

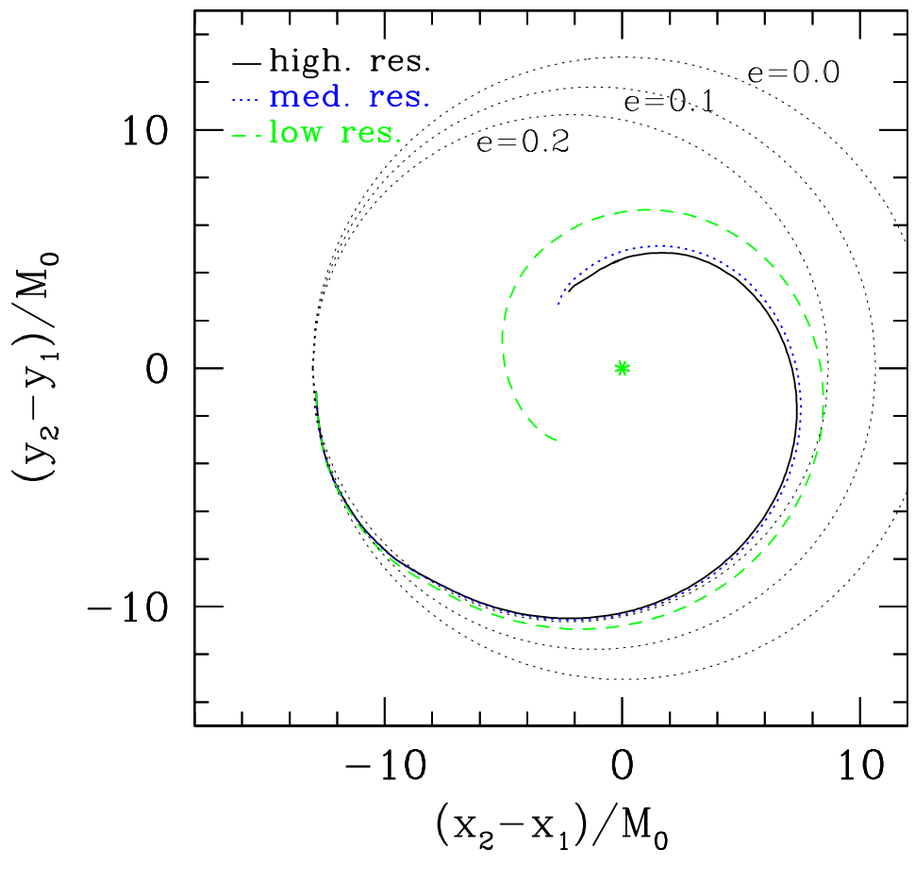
- Visualise output file:
  - Mathematica
  - IDL
  - Maple
  - Matlab
  - gnuplot
  - SciPy
  - Python (MatPlotLib)
  - ygraph
  - ...

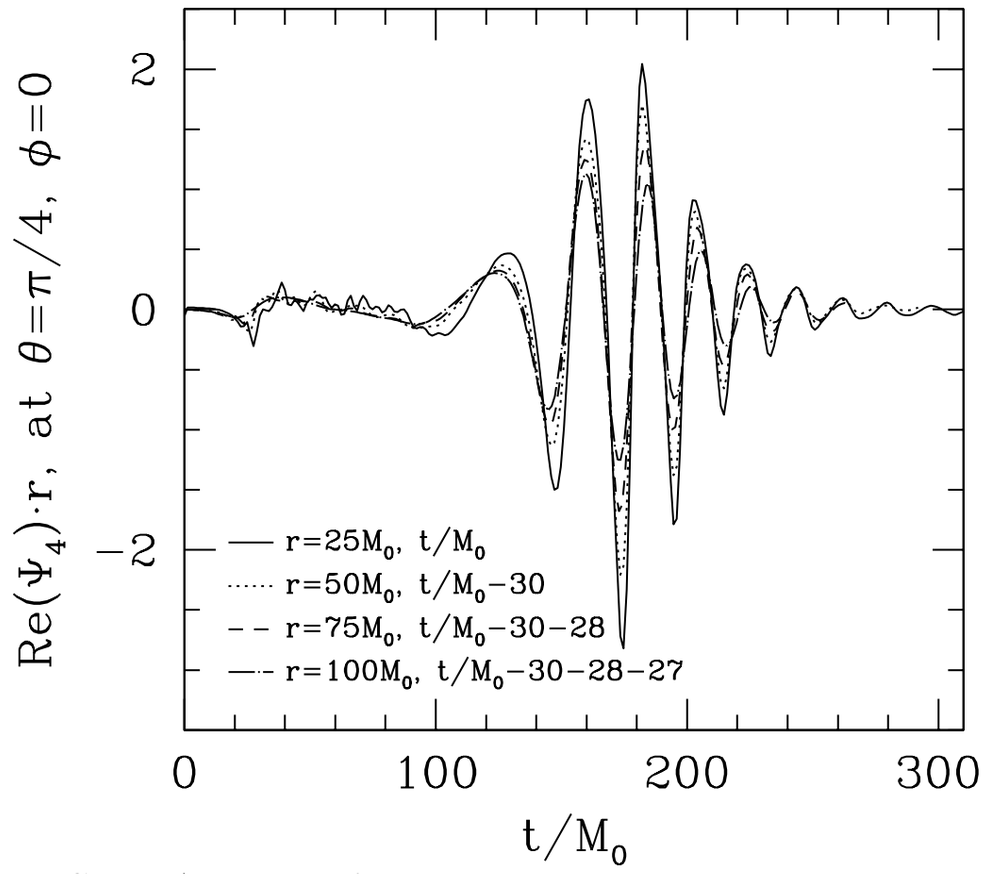
Selected milestones in Numerical Relativity - Pre-revolutionary

1959	Arnowitt, Deser and Misner - ADM formalism: **initial value problem** for GR
1964	Hahn and Lindquist: **first numerical solution** to the Einstein equations: attraction between two wormholes in axisymmetry, 51x51 grid points
1980s	Piran, Stark - **gravitational waves** in axisymmetry from formation of axisymmetric BH
1980s	Choptuik - **Critical collapse** with adaptive mesh refinement
1990s	Binary Black Hole Grand Challenge - **Head-on BBH collision**
to 2005	Development of **formulations**, **coordinate conditions**, **excision** techniques, wave extraction formalisms.
	Finite simulation lifetime, solutions unstable, much frustration

Milestones of Numerical Relativity - The revolution begins

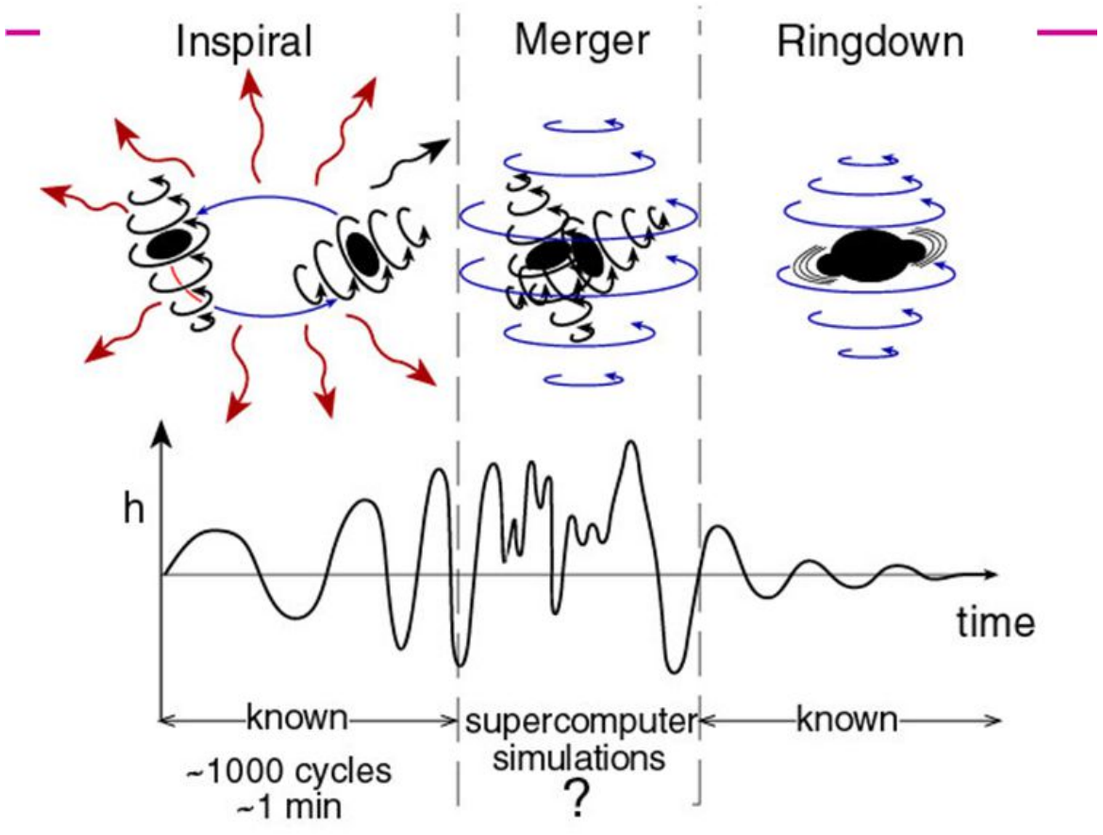
2005 **Pretorius** is the first to successfully evolve **more than one orbit of a BBH** through **merger and ringdown** and compute the **gravitational waveform**



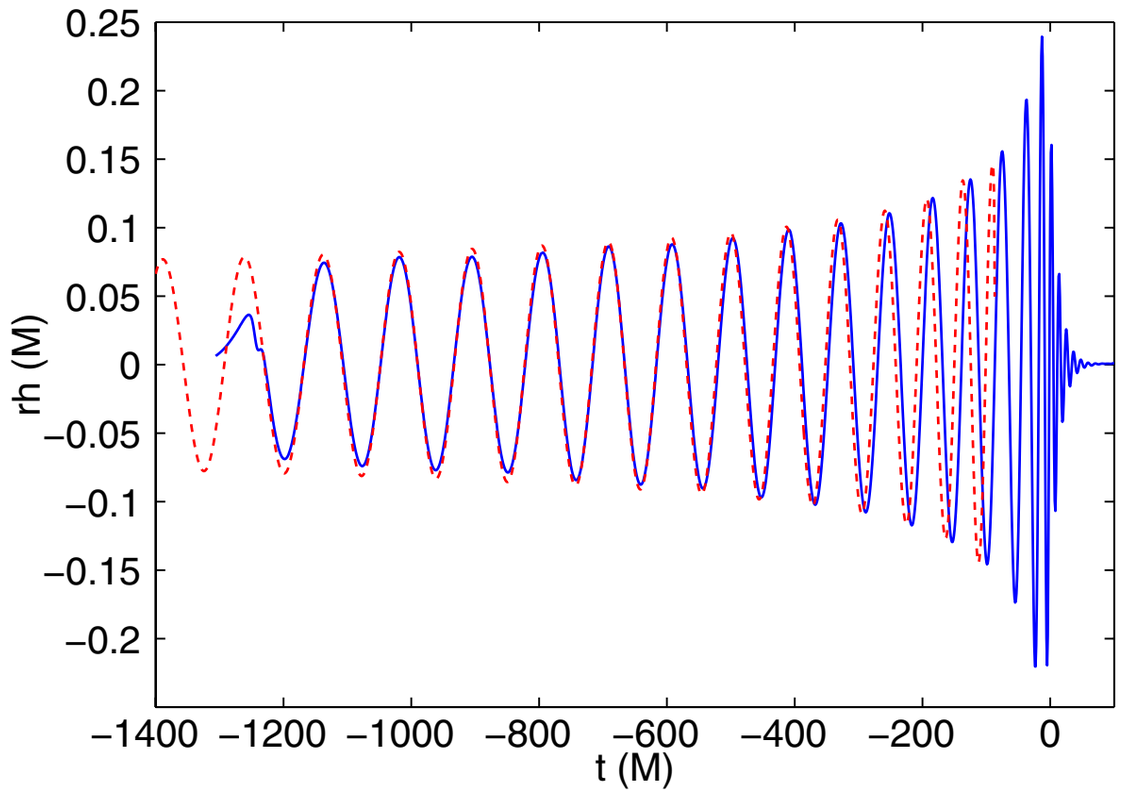


What does a BBH waveform look like?

- Before 2005: Kip Thorne's sketch



- After 2005: Numerical Relativity (e.g. Baker et al. 2007)



Selected milestones in Numerical Relativity - The Golden Age

2005 Pretorius, long-term stable method for orbit using **excision**, **finite difference** methods and **adaptive mesh refinement**, **generalised harmonic formulation**

2005 Goddard and Brownsville groups: **Moving puncture** method (no excision): **finite differences**, **BSSN formulation**

2007 Campanelli et al., Gonzalez et al. - Unexpectedly high "**super-kick**" of merging BHs for certain **spin** orientations

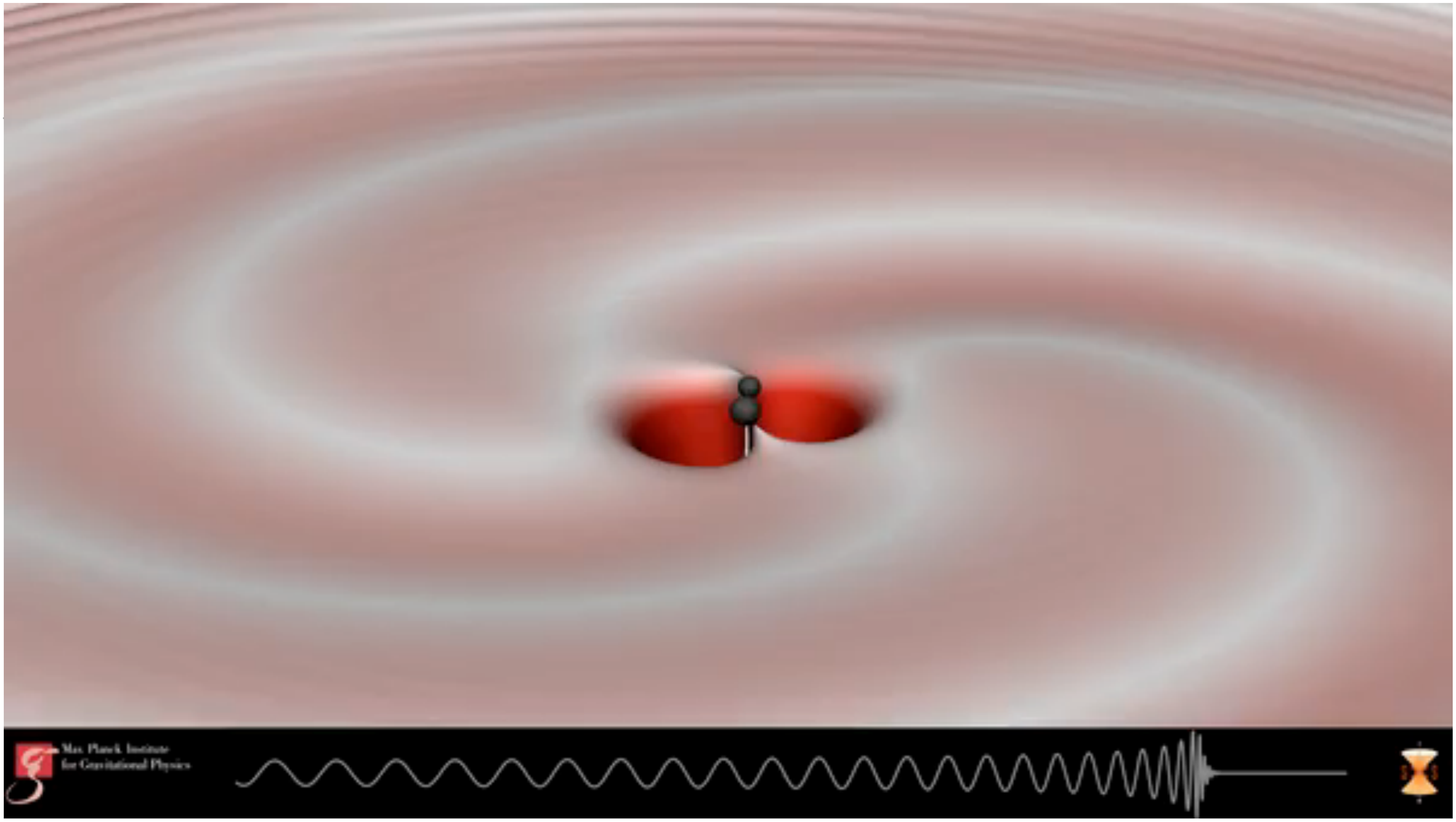
2011 Lovelace, Scheel, Szilagyi - Breaking the **high spin limit** (~0.93) of Bowen-York conformally flat initial data

2015 Waveform models built on NR results used in **LIGO** searches and parameter estimation for **first gravitational wave detection**

What can we do today?

- **Stable** evolutions of **moderate** BBH configurations:
  - Mass ratio $q = m_1/m_2 \lesssim 10$
  - Spins chi $= S/m^2 \lesssim 0.6$
  - Number of orbits $N \lesssim 15$

GW150914



Numerical simulation: S. Ossokine, A. Buonanno (Max Planck Institute for Gravitational Physics), Simulating eXtreme Spacetimes project. Scientific visualisation: R. Haas (Max Planck Institute for Gravitational Physics)

Open-source Numerical Relativity

- **Cactus** framework: open source, developed by **Ed Seidel**'s group at the **Albert Einstein Institute** in the late 90s
- Foundation of many NR codes today
- **Einstein Toolkit** is an entirely open source set of NR codes based around Cactus
- See einsteintoolkit.org/gallery.html for examples

- **GW150914 example**, including fully open parameter file, instructions, and **tutorials** for analysis and visualisation [Wardell, Hinder, Bentivegna]
- Simulate GW150914 on ~100 cores in a few days **yourself**!

Simulation data

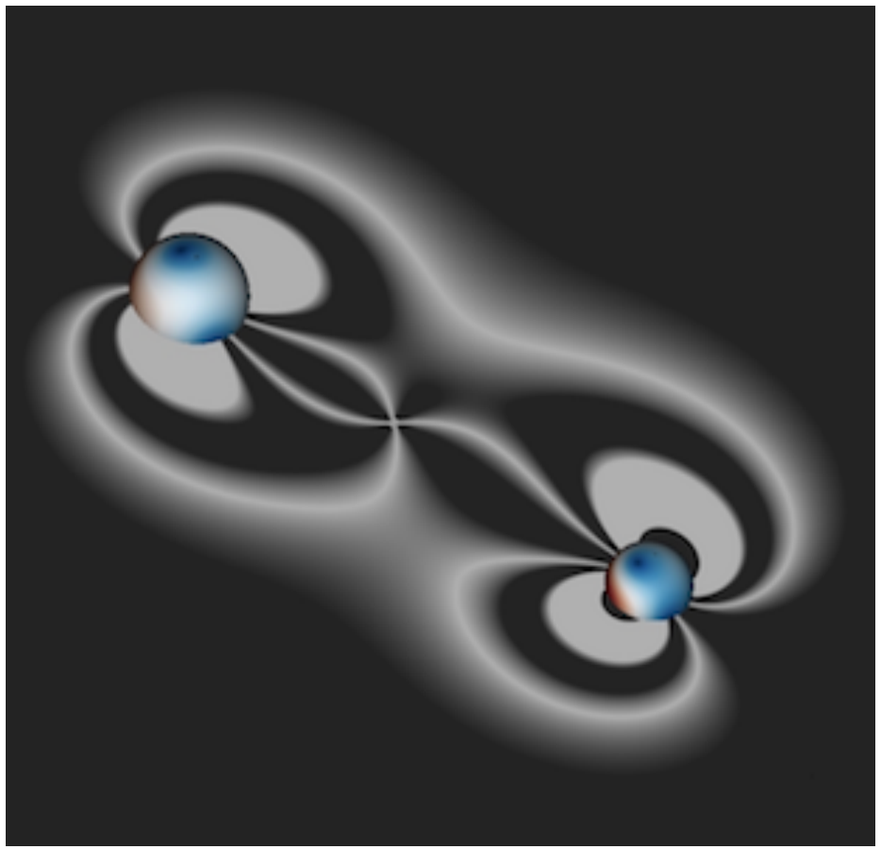
Lightweight simulation data with only a small number of iterations of 3D output is available for download from Zenodo:

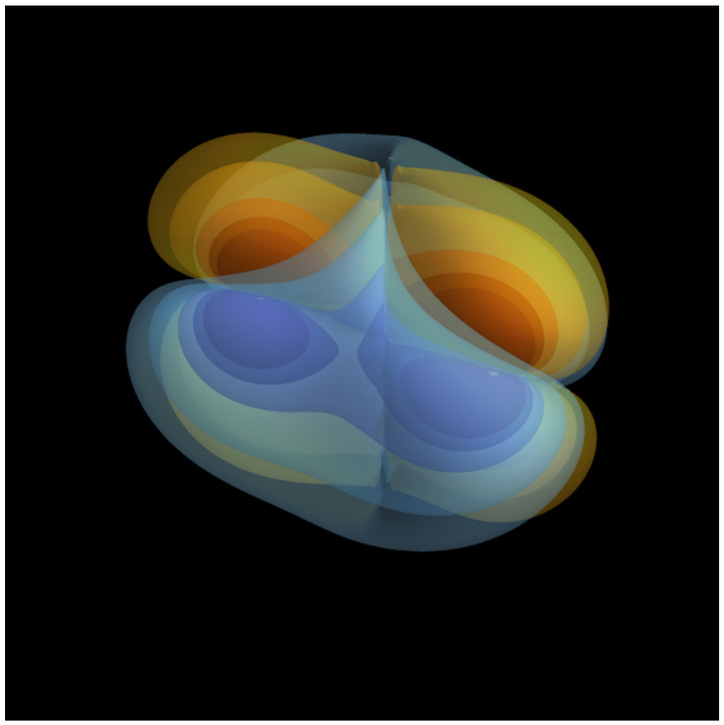
DOI 10.5281/zenodo.60213

The full simulation comprises several terabytes of data and can be made available upon request.

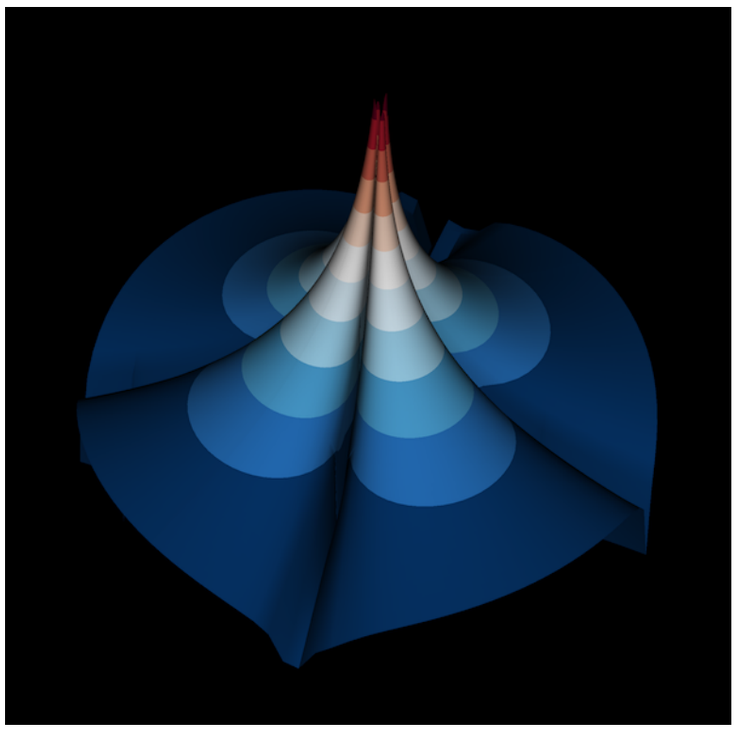
Images and movies

Horizons



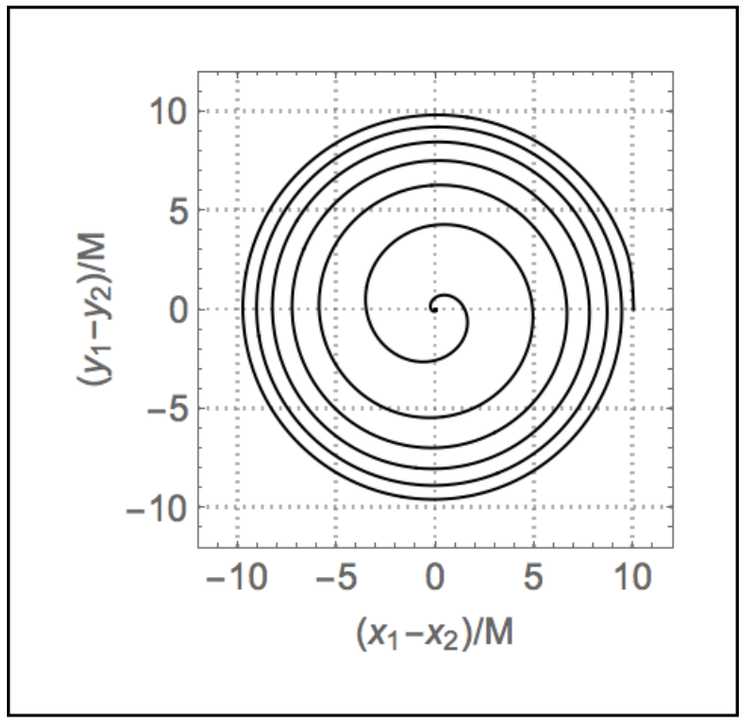


The real part of Ψ_4, the component of the Riemann tensor representing outgoing gravitational radiation.



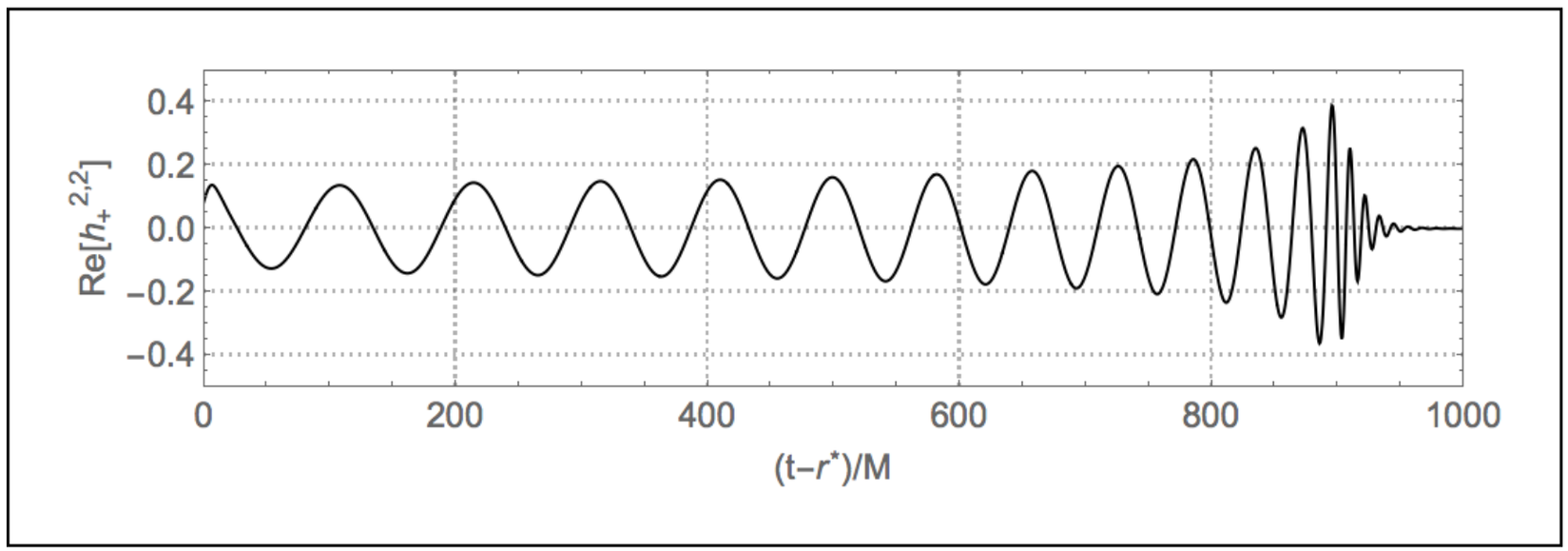
Elevation plot of the magnitude of Ψ_4 on the equatorial plane at $t = 0$.

Horizon coordinate trajectories

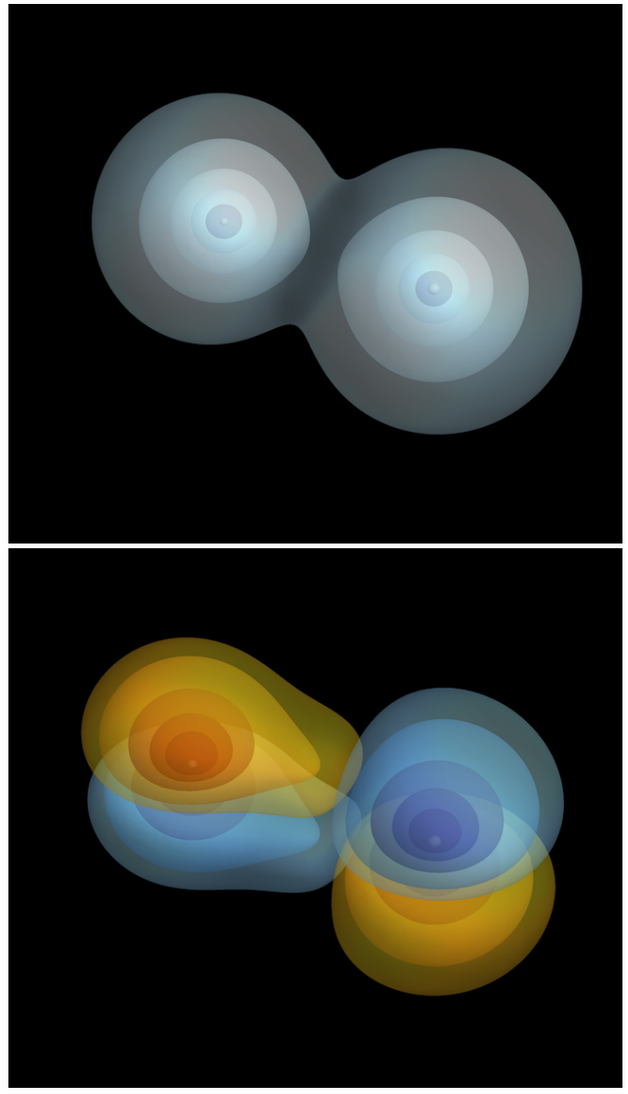


Coordinate tracks of the centroids of the apparent horizons showing inspiral of the binary due to emission of energy and angular momentum in gravitational waves

Gravitational waveform



Curvature scalars



einsteintoolkit.org