HW#1 —Phys879—Spring 2017
Due before class, Tuesday, Feb. 13, 2017
Course webpage: http://www.aei.mpg.de/2000472

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Recommended readings:

1. LIGO Scientific Collaboration and Virgo Collaboration: Observation of Gravitational Waves from a Binary Black Hole Merger. Phys. Rev. Lett. 116(6), 061102 (2016). `arXiv:1602.03837`.
2. LIGO Scientific Collaboration and Virgo Collaboration: The basic physics of the binary black hole merger GW150914. Annalen der Physik, 529. `arXiv:1608.01940`.
3. A. Einstein: Über Gravitationswellen. In: Sitzungsberichte der Königlich Preussischen Akademie der Wissenschaften Berlin (1918), 154167. (English translation available at `http://einsteinpapers.press.princeton.edu/vol7-trans/25`.)

Exercises (prepared by A. Buonanno, T. Hinderer, J. Steinhoff & J. Vines):

1. **Basic physics of GW150914:** [20 pts.]

   Numbers of equations and figures below refer to arXiv:1608.01940, which slightly differ from the published version in Annalen der Physik.

   (a) *Power emitted in GWs:*

   Starting from Newtonian physics and Einstein's quadrupole formulas (4) and (5), fill in the details of the derivation of Eq. (27), the rate at which energy is emitted in the form of GWs from a binary on a circular orbit.

   In particular, give the intermediate expression for the GW strain tensor h_{ij} as a function of time.

   (b) *Frequency evolution and estimating the mass scale:*

   Combine the previous result with more Newtonian physics to derive Eq. (28), the relationship between the orbital frequency, its rate of change, and the masses. Derive also Eq. (8).

   This makes a certain assumption to allow combining Newtonian physics (which predicts a constant energy) with the quadrupole formula (which predicts that the energy decreases). Explain the assumption and check its applicability in this context.

   Use the result and Fig. 2 (which is essentially a Fourier transform of Fig. 1) to estimate the chirp mass, and the total mass assuming a 1:1 mass ratio, for GW150914.

   (c) *Drag force instead of GWs?:*

   What if the bodies in a binary experienced drag forces due to a gaseous environment? Could this explain the observed frequency-time dependence (prior to the peak), instead of energy loss due to GW emission? Assuming the drag force acting on the orbiting bodies is proportional to v^p where v is their velocity and p is some power (which is typically 1 or 2), derive an analog of Eq. (28), up to proportionality, and go on to find the frequency as a function of time—–i.e. find α and β for the relationships $\dot{\omega} \propto \omega^{\alpha}$ and $\omega^{\beta} \propto t$. Compare these with the values for GW emission.

   Extract the data points from Fig. 3, and obtain an estimate for β from the slope of a linear fit on a log-log plot.

   Does a drag force provide a viable alternative explanation of the frequency-time behavior?

(d) *Orbital separation, why BHs?:*

Given the estimate of the mass scale from above, use the GW frequency and Newtonian physics to estimate the distance between the centers of the orbiting bodies, first at the beginning of the signal, and then at the location of the peak amplitude. Explain why the latter estimate strongly suggests that the objects are black holes.

(e) *Distance to the source:*

Use the magnitude of the GW strain from Fig. 1 and your formula for the GW strain h from Problem 1a to derive an order-of-magnitude estimate for the distance of the source from the Earth.

How does this compare to the luminosity distance derived in Sec. 5?

2. **Orders of magnitude of gravitational-wave strength for sources on the Earth:** [5 pts.]

Use the leading, non-zero term in the multipolar expansion of the gravitational field, i.e., the quadrupolar term $G\,\ddot{I}_2/(c^4\,r)$, to *estimate* the amplitude of gravitational waves produced from the following earth-based events:

- A meteorite having diameter of 2 km and hitting the ground at a speed of 25 km/sec;
- A big chunk of piezoelectric material driven to oscillation at a frequency of 100 MHz.

To derive the above results take into account that gravitational waves exist only in the so-called wave zone, i.e., at a distance from the source which is at least equal to the reduced wavelength.

3. **Multipolar expansion of the far-zone gravitational field:** [5 pts.]

In the lecture the gravitational field is written in a multipolar expansion. Show that it is dimensionless. Assuming that the gravitational field is proportional to G/r (where r is the distance between the observer and the source, and G is the Newton constant), and that each term of the expansion depends only on c and on derivatives of the multipole mass-moments I_L and current-moments J_L, employ dimensional analysis to show that the multipolar expansion is unique.